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DESCRIPTION OF GIANT DIPOLE RESONANCES IN THE SELF-CONSISTENT MODEL WITH SEPARABLE EFFECTIVE INTERACTION, INCLUDING ONE-PARTICLE CONTINUUM

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1. INTRODUCTION

In the theoretical study of the nuclear structure, in particular, of the giant resonances and their decay properties we are faced with a number of related problems. Among these are: The self-consistent formulation of the theory, the compleness the one-particle basis and the inclusion of many-particle configurations, which are essential for description of the damping of resonance states. Let us discuss briefly these problems.

In a self-consistent theory the Hamiltonian and the equations of motion should satisfy certain requirements which have its origin both in the space-time properties and the symmetry of nuclear interactions. In the event of spontaneous breaking symmetry these requirements result in certain consistency conditions for the quantities invented in the theory.

In this paper we use the spontaneous breaking of the translational invariance from which the consistency relation follows between the mass operator (the shell-model potential), the irreducible two-particle interaction amplitude (the effective interaction) and the density matrix. Various formulations of this condition were given in refs.[1-4]. It has been used in papers[5,6] for the self-consistent description of the ground state and low-energy excited states in closed-shell nuclei within the framework of the theory of finite Fermi-systems. The consistency conditions can be used for constructing the effective interaction provided the variations both of the density matrix and the corresponding potential are known. In refs.[7-9] the effective interaction was chosen in the separable form and then applied to the self-consistent description of various excited states. Self-consistency requirement is naturally fulfilled in the Hartree-Fock approach. In recent years this approach has been used for describing the giant resonances, employing the density-dependent Skyrme forces (see, e.g., refs.[10-12]). A common feature of all those methods is an explicit elimination of "spurious" states associated with the spontaneous breaking of symmetry. Besides that, the self-consistency allows one to diminish the number of arbitrary parameters in the theory.

The commonly used nuclear one-particle potentials (phenomenological, Hartree-Fock, or those found from the consistency conditions) are finite so that the set of bound single-particle

states is not complete. For the sake of completeness this set should be supplemented with the one-particle continuum. The completeness of the basis is important for some reasons. First, the effective fields accompanying the nuclear excitations are not smooth. They have usually the well developed surface peak (its nature is well understood now[5]) and their matrix elements decrease slowly with increasing excitation energy, so that the correct numerical results cannot be obtained with the truncated basis including 2-3 shells near the Fermi-surface. Second, the continuum states play an important role as intermediate ones for various reactions, such as (γ, n), (γ, p), etc., and for the electromagnetic transitions to nuclear states above the nucleon threshold. The giant multipole resonances lie, as a rule, in this range of excitation energies. Without the proper inclusion of the one-particle continuum, which is essentially responsible for the decay of the resonance states to open channels, the correct theoretical description of the high-energy excitations seems to be impossible. An appropriate method for treating the one-particle continuum was developed in refs.[13,14] It has been used, in particular, for calculating the cross section for excitation of the giant dipole resonance in ^{40}Ca by neutral currents in inelastic neutrino scattering (V.Yu.Rusinov and S.A.Fayans. Report presented at the Session of Acad. of Sci. USSR, Leningrad, 1975.).

From the very nature of the giant resonances which give an essential contribution to the sum rule, it is clear that their wave functions are superpositions of a large number of particle-hole (p-h) configurations. In such a picture the resonance width is entirely due to the nucleon escape to the continuum (escape-width). In medium and heavy nuclei the calculated escape-width of resonance states is much smaller than the observed one (see, e.g., refs.[10-12]). The reason for such a difference is that with increasing the excitation energy the probability for the decay of p-h states over multiparticle configurations becomes larger. This results in spreading width for resonance states [15] (see also ref.[16]). Such a process is accompanied both by the enrichment of the excitation spectrum and some redistribution of the transition strengths (see, e.g., refs.[17-19]).

In this paper we consider the first two of the above-mentioned problems, using the self-consistent model [9], that is formulated in the coordinate representation[13]. This allows one to include correctly the one-particle continuum when treating the giant dipole resonance and thus to calculate the escape-widths. The use of the separable effective interaction is

a compromise between the simplicity of the mathematical expressions and numerical calculations on one hand, and the desirable physical characteristics on the other. Such an interaction exhibits some of the most important features of the so-called "realistic" effective interaction, e.g., the surface character and the density dependence (implicitly). In subsequent works we intend to use more realistic forces and to carry out the fully self-consistent calculations. Still one should bear in mind that the results obtained until now with the separable interaction [9] agree qualitatively both with the experimental data and the theoretical calculations [5,10] in which the "realistic" interaction has been employed. The main purpose of this paper is to reveal the effects associated with the one-particle continuum, using the self-consistent approach to the dipole excitations of nuclei.

2. BASIC EQUATIONS

We shall describe the nuclear excitations in terms of the effective field $V(\vec{r},\omega)$, that arises as a response of the system to the external field $V_0(\vec{r})$ [20]

$$V^i(\vec{r},\omega) = e_q^i V_0^i(\vec{r}) + \sum_k \int \mathcal{F}^{ik}(\vec{r},\vec{r}_1) A^k(\vec{r}_1,\vec{r}_2,\omega) \times V^k(\vec{r}_2,\omega) d\vec{r}_1 d\vec{r}_2 , \tag{1}$$

where e_q^i is the local charge of a quasiparticle with respect to the field V_0, $\mathcal{F}^{ik}$ represents the effective interaction $(i,k = n,p)$ and A^k is the p-h propagator defined by

$$A(\vec{r}_1,\vec{r}_2,\omega) = \int G^q(\vec{r}_1,\vec{r}_2;\epsilon - \frac{\omega}{2}) G^q(\vec{r}_2,\vec{r}_1;\epsilon + \frac{\omega}{2}) \frac{d\epsilon}{2\pi i}$$

$$= \sum_{\lambda\lambda'} \phi_\lambda^*(\vec{r}_1)\phi_{\lambda'}(\vec{r}_1) \frac{n_\lambda - n_{\lambda'}}{\epsilon_\lambda - \epsilon_{\lambda'} - \omega} \phi_{\lambda'}^*(\vec{r}_2)\phi_\lambda(\vec{r}_2). \tag{2}$$

Here, ω is the excitation energy; G^q, the quasiparticle Green function, subject to the equation

$$(\epsilon - p^2/2m - u(\vec{r})) G^q(\vec{r},\vec{r}',\epsilon) = \delta(\vec{r}-\vec{r}'), \tag{3}$$

$u(\vec{r})$ is the self-consistent field, ϕ_λ and ϵ_λ are the quasiparticle wave functions and energies, respectively; and n_λ, the occupation numbers. Note that eq. (1) does not contain the velocity-dependent forces, but the non-locality in the p-h channel is taken into account.

In the case of spherical nuclei all the quantities in eq. (1) are expanded over spherical harmonics that allows one to separate the angular variables. Then for a given partial wave L the radial part of eq. (1) is [5]

$$V^i_L(r,\omega) = e^i_q V^i_{0L}(r) + \sum_k \int \mathcal{F}^{ik}_L(r,r_1) \times A^k_L(r_1,r_2,\omega) V^k_L(r_2,\omega) r_1^2 r_2^2 \, dr_1 dr_2 , \tag{4}$$

where the spin-dependent and spin-orbital forces are neglected.

An additional equation that connects the effective interaction with the self-consistent field arises from the translational invariance condition. In the case of spherical symmetry it has the form

$$\frac{\partial u^i}{\partial r} = \sum_k \int \mathcal{F}^{ik}_1(r,r_1) \frac{\partial \rho^k}{\partial r_1} r_1^2 dr_1 , \tag{5}$$

where ρ^k are the nucleon density distributions. In the absence of the spin-orbital potential the following identity takes place [5]:

$$\frac{\partial \rho^i}{\partial r} = \int A^i_1(r,r_1, \omega = 0) \frac{\partial u^i}{\partial r_1} r_1^2 dr_1 . \tag{6}$$

Let us suppose that the self-consistent field is given as a sum of the isoscalar u^0, isovector u^1, and Coulomb V_c terms

$$u(r) = u^0(r) + u^1(r) r_Z + V_c(r) \frac{1}{2}(1 - r_Z), \tag{7}$$

with $r_Z = +1$ for neutrons and -1 for protons. Then the effective interaction is sought in the form

$$\mathcal{F} = \mathcal{F}^0 + \mathcal{F}^1(\vec{r}\cdot\vec{r}') + \mathcal{F}^c \frac{1}{4}(1 - r_Z)(1 - r'_Z) \tag{8}$$

from which it follows

$$\mathcal{F}^{nn} = \mathcal{F}^0 + \mathcal{F}^1 \; ; \qquad \mathcal{F}^{np} = \mathcal{F}^{pn} = \mathcal{F}^0 - \mathcal{F}^1 \; ;$$

$$\mathcal{F}^{pp} = \mathcal{F}^0 + \mathcal{F}^1 + \mathcal{F}^c \; . \tag{9}$$

We suppose that the consistency relation between V_c and $\mathcal{F}^c$ can be separated from eq.(5)

$$\frac{\partial V_c}{\partial r} = \int \mathcal{F}^c(r, r_1) \frac{\partial \rho^p}{\partial r_1} r_1^2 dr_1 \; . \tag{10}$$

Then the consistency relations for u^0 and u^1 are

$$\frac{\partial u^0}{\partial r} = \int \mathcal{F}^0(r, r_1) \frac{\partial \rho^c}{\partial r_1} r_1^2 dr_1 \; , \tag{11}$$

$$\frac{\partial u^1}{\partial r} = \int \mathcal{F}^1(r, r_1) \frac{\partial \rho^1}{\partial r_1} r_1^2 dr_1 , \tag{12}$$

where $\rho^0 = \rho^n + \rho^p$ and $\rho^1 = \rho^n - \rho^p$. Note that only the dipole component $L = 1$ is present in eqs.(10)-(12). In these equations and below index L is omitted.

The most important suggestion concerns the choice of the effective interaction in the separable form[9], that allows one to determine uniquely from eqs.(10)-(12) its radial form factors and strength parameters

$$\mathcal{F}^m(r, r') = \kappa_m \frac{\partial u^m}{\partial r} \cdot \frac{\partial u^m}{\partial r'} , \tag{13}$$

$$\kappa_m^{-1} = \int \frac{\partial u^m}{\partial r} \cdot \frac{\partial \rho^m}{\partial r} r^2 dr , \tag{14}$$

where index $m = 0, 1, c$ $(\rho^c \equiv \rho^p)$. Now the integral equations (4) for the effective field are transformed into the system of algebraic equations. For simplicity we suppose that the isovector potential is proportional to the isoscalar one[21]

$$u^1(r) = -\eta \frac{N-Z}{A} u^0(r), \tag{15}$$

where η is the isovector parameter. As a result, for the effective field we obtain

$$V^n(r,\omega) = e_q^n V_0^n(r) + \frac{\partial u^0}{\partial r}(\kappa_{nn}\xi_n + \kappa_{np}\xi_p),$$

$$V^p(r,\omega) = e_q^p V_0^p(r) + \frac{\partial u^0}{\partial r}(\kappa_{nn}\xi_p + \kappa_{np}\xi_n) + \frac{\partial V_c}{\partial r}\kappa_c\xi_c, \tag{16}$$

where the strength parameters are defined by

$$\kappa_{nn} = \kappa_{pp} = \kappa_0 + (\eta\frac{N-Z}{A})^2\kappa_1,$$
$$\kappa_{np} = \kappa_0 - (\eta\frac{N-Z}{A})^2\kappa_1. \tag{17}$$

The quantities ξ_n, ξ_p and ξ_c that depend only on ω are defined by the system of the algebraic equations

$$\begin{pmatrix} \kappa_{nn}B_n - 1 & \kappa_{np}B_n & 0 \\ \kappa_{np}B_p & \kappa_{nn}B_p - 1 & \kappa_c B_c \\ \kappa_{np}B_c & \kappa_{nn}B_c & \kappa_c D_c - 1 \end{pmatrix} \begin{pmatrix} \xi_n \\ \xi_p \\ \xi_c \end{pmatrix} = \begin{pmatrix} e_q^n C_n \\ e_q^p C_p \\ e_q^p C_c \end{pmatrix}. \tag{18}$$

Here, the following notations are used

$$B_i(\omega) = (\frac{\partial u^0}{\partial r}A^i(\omega)\frac{\partial u^0}{\partial r}); \qquad B_c(\omega) = (\frac{\partial u^0}{\partial r}A^p(\omega)\frac{\partial V_c}{\partial r});$$

$$C_i(\omega) = (\frac{\partial u^0}{\partial r}A^i(\omega)V_0^i); \qquad C_c(\omega) = (\frac{\partial V_c}{\partial r}A^p(\omega)V_0^p); \tag{19}$$

$$D_c(\omega) = (\frac{\partial V_c}{\partial r}A^p(\omega)\frac{\partial V_c}{\partial r}),$$

where the common brackets indicate the integration over all coordinates.

Putting zero the determinant of eqs. (18) we obtain the bound spectrum of 1^- excitations (the poles of the effective field). For the excitation energy ω above the nucleon threshold the p-h propagator acquires the imaginary part, associated with the nucleon escape to continuum, and the effective field becomes complex. The continuous excitation spectrum is described usually by the imaginary part of the polarization operator with respect to a given external field V_0 (this quantity is often referred to as the strength function):

$$S(\omega) = -\frac{1}{\pi} \operatorname{Im} \sum_i (e_q^i V_0^i A^i(\omega) V^i(\omega)). \tag{20}$$

The quantity $S(\omega)$ can be written as a sum over the eigenstates $|s>$:

$$S(\omega) = \sum_s |<0|V_0|s>|^2 \delta(\omega - \omega_s). \tag{21}$$

Various energy-weighted sum rules can be obtained from eqs. (20) and (21) by integrating over the excitation energy

$$\sigma_n = \int \omega^{n+1} S(\omega) d\omega \equiv \sum_s \omega_s^{n+1} |<0|V_0|s>|^2 , \tag{22}$$

$$n = 0, \pm 1, \pm 2, \ldots$$

3. THE TREATMENT OF THE ONE-PARTICLE CONTINUUM

The proper inclusion of the one-particle continuum is based on the calculation of the p-h propagator in the coordinate space. For a given partial wave eq. (2) reads

$$A_L(r_1, r_2, \omega) = \sum_{j\ell, j'\ell'} B^L_{j\ell j'\ell'} \sum_n N_{n\ell j} R_{n\ell j}(r_1) R_{n\ell j}(r_2)$$
$$\times [G_{\ell' j'}(r_1, r_2; \epsilon_{n\ell j} + \omega) + G_{\ell' j'}(r_1, r_2; \epsilon_{n\ell j} - \omega)], \tag{23}$$

where $R_{n\ell j}$ and $\epsilon_{n\ell j}$ are the radial wave functions and one-particle energies; $N_{n\ell j}$, the occupation numbers; and $B^L_{j\ell j'\ell'}$, the angular coefficients:

$$B^L_{j\ell j'\ell'} = \frac{1}{4\pi}(2\ell+1)(2\ell'+1)(2j+1)(2j'+1) \begin{Bmatrix} \ell & \ell' & L \\ 0 & 0 & 0 \end{Bmatrix}^2 \begin{Bmatrix} \ell & j & \frac{1}{2} \\ j' & \ell' & L \end{Bmatrix}^2 . \tag{24}$$

The sum in eq.(23) runs only over occupied levels and the one-particle Green function in the coordinate representations is given by

$$G_{\ell j}(r_1, r_2; \epsilon) = 2m\, y^{(1)}_{\ell j}(r_<, \epsilon) y^{(2)}_{\ell j}(r_>, \epsilon)/\hbar^2 r_1 r_2 W_{\ell j}(\epsilon), \qquad (25)$$

where $r_<$ and $r_>$ denote the lesser and the greater of r_1 and r_2 and $y^{(1)}$ and $y^{(2)}$ are two linearly independent solutions of the Shrödinger equation with the following asymptotic behaviour for $\epsilon > 0$:

$$y^{(1)}_{\ell j}|_{r \to \infty} \sim \sin(kr - \beta \ln 2kr - \frac{\pi \ell}{2} + \delta_{\ell j} + \sigma_\ell),$$

$$y^{(2)}_{\ell j}|_{r \to \infty} \sim \exp\{i(kr - \beta \ln 2kr - \frac{\pi \ell}{2} + \delta_{\ell j} + \sigma_\ell)\}, \qquad (26)$$

where $\beta = (Z-1)e^2 m/\hbar^2 k$ is the Coulomb parameter; $\sigma_\ell = \arg\Gamma(1 + \ell + i\beta)$, the Coulomb phase; and $\delta_{\ell j}$, the nuclear phase in the single-particle potential. The quantity $W_{\ell j}(\epsilon)$ denotes the Wronskian of $y^{(1)}$ and $y^{(2)}$. For ω below the nucleon threshold solutions $y^{(1)}$ and $y^{(2)}$ are subject to the boundary conditions

$$y^{(1)}_{\ell j}|_{r \to 0} \sim r^{\ell+1}; \quad y^{(2)}_{\ell j}|_{r \to \infty} \sim \exp(-\sqrt{2m|\epsilon|}\, r/\hbar). \qquad (27)$$

4. THE ESCAPE-WIDTH AND THE TRANSITION DENSITY FOR THE ISOLATED RESONANCES

In the p-h excitation spectrum above the nucleon threshold there are narrow resonances of two types. First, corresponding to transitions between bound one-particle levels and, second, the transitions from bound to quasi-discrete levels. When the effective interaction is switched off the former resonances have no width, while the width of the latter is associated with the probability of penetration of a particle through the potential barrier (centrifugal and Coulomb ones). The effective interaction leads to the energy shift of the resonances. Due to the admixture of the continuum the resonances of the first type acquire the escape-width. For the same reason the width of the resonances of the second type increases.

It is difficult to estimate the width of the narrow resonance directly from $S(\omega)$, because this requires calculations with a very small step $\Delta\omega$. We have used, instead, a simple approximate method that allows one to calculate the total escape-width as well as partial widths for different open channels.

Consider the equation for the nucleon-nucleon interaction amplitude[20] in a symbolic form

$$\Gamma = \mathcal{F} + \mathcal{F} A \Gamma . \tag{28}$$

For the excitations above the nucleon threshold the p-h propagator becomes complex: $A = A_1 + iA_2$. The amplitude Γ can be presented by a sum of terms having the poles in a complex energy plane. For an isolated resonance with energy $\omega_s = \omega_{1s} - i\gamma_s/2$ it is possible to separate Γ into the regular part Γ_R (a smooth function of energy near the pole) and the pole term

$$\Gamma = \Gamma_R + g_s \cdot g_s / (\omega - \omega_s), \tag{29}$$

where g_s is the creation amplitude of the excited state $|s>$. Evidently, γ_s has the meaning of the resonance width.

Let us introduce an auxiliary amplitude Γ' connected only with the real part of the propagator

$$\Gamma' = \mathcal{F} + \Gamma' A_1 \mathcal{F} . \tag{30}$$

Multiplying the l.h.s. of eq.(28) by $1+\Gamma' A_1$ we get

$$\Gamma = \Gamma' + i\Gamma' A_2 \Gamma . \tag{31}$$

The amplitude Γ' has the poles $\tilde{\omega}_s$ on the real energy axis. In the vicinity of $\tilde{\omega}_s$ it can be written as

$$\Gamma' = \Gamma'_R + \tilde{g}_s \cdot \tilde{g}_s / (\omega - \tilde{\omega}_s). \tag{32}$$

Substituting (29) and (32) into eq.(31) and letting $\omega = \omega_s$ we obtain

$$\tilde{g}_s \cdot \tilde{g}_s + i\tilde{g}_s \cdot g_s A_2(\tilde{\omega}_s)[\Gamma_R(\tilde{\omega}_r) + g_s \cdot g_s / (\tilde{\omega}_s - \omega_s)] = 0. \tag{33}$$

The regular part $\Gamma_R(\tilde{\omega}_s)$ can be represented by the sum

$$\Gamma_R(\tilde{\omega}_s) = \sum_{s' \neq s} g_{s'} \cdot g_{s'} / (\tilde{\omega}_s - \omega_{s'}), \tag{34}$$

where $\omega_{s'}$ are the poles of the total amplitude Γ, $g_{s'} \cdot g_{s'}$ being the residues. The mean value of (34) can be estimated as $\Gamma_R(\tilde{\omega}_s) \sim g_s \cdot g_s / D$, where D is the average energy spacing between resonances with the same quantum numbers. Since $|\tilde{\omega}_s - \omega_s| \approx \gamma_s$ the regular part $\Gamma_R(\tilde{\omega}_s)$ in eq.(33) can be neglected with the accuracy of γ_s / D. Hence, $g_s \approx \tilde{g}_s$ and $\omega_s - \tilde{\omega}_s = i(\tilde{g}_s A_2(\tilde{\omega}_s) \tilde{g}_s)$ from which it follows

$$\omega_{1s} = \tilde{\omega}_s, \qquad \gamma_s = -2(\tilde{g}_s A_2(\tilde{\omega}_s) \tilde{g}_s). \tag{35}$$

Thus, the resonance width is determined by the imaginary part of *the p-h propagator*. It is seen that the eigenenergies ω_s and residues g_s can be found from the equation for Γ (or effective field), in which the imaginary part of the propagator is put to zero (as for the bound states[5]). With the accuracy of γ_s / D the imaginary part $A_\Sigma(\omega)$ affects neither the resonance energy nor residues, but gives rise to the resonance width.

It should be emphasized that γ_s in eq.(35) is the total escape-width of the resonance. This quantity can be written as a sum of partial widths that correspond to different open channels. For this purpose we present the imaginary part of the Green function (25) in the form

$$\mathrm{Im}\, G_{\ell j}(r, r'; E) = -\pi R_{E\ell j}(r) R_{E\ell j}(r'), \qquad E > 0, \tag{36}$$

where the one-particle radial functions $R_{E\ell j}$ satisfy the orthogonality condition

$$\int R_{E\ell j}(r) R_{E'\ell j}(r) r^2 dr = \delta(E - E'). \tag{37}$$

By inserting (36) and (23) into eq.(35) we get

$$\begin{aligned} \gamma_s^L &= 2\pi \sum_{i=n,p} \sum_{n\ell j} N_{n\ell j} \sum_{\ell' j'} B^L_{j\ell j'\ell'} \Big| \int R^i_{n\ell j}(r) \\ &\times g^i_s(r) R^i_{E\ell' j'}(r) r^2 dr \Big|^2 \equiv \\ &\equiv 2\pi \sum_i \sum_{n\ell j} \sum_{\ell' j'} (2L+1)^{-1} N_{n\ell j} \left| \langle n\ell j \| g^i \| E\ell' j' \rangle \right|^2, \end{aligned} \tag{38}$$

where $E \equiv \epsilon_{n\ell j} + \omega_s > 0$. Hence, the total width can be written as

$$\gamma_s \equiv \sum_i \sum_{n\ell j} N_{n\ell j} \gamma^i_{n\ell j} \equiv \sum_i \sum_{\ell' j'} \gamma^i_{\ell' j'} , \tag{39}$$

where $\gamma^i_{n\ell j}$ are partial widths for the decay to hole levels of the daughter nucleus and $\gamma^i_{\ell' j'}$ the widths for the escape of nucleons with a given ℓ', j'.

The narrow isolated resonances can be characterized by the reduced transition probability

$$B(EL) = \int_{\tilde{\omega}_s - \Delta}^{\tilde{\omega}_s + \Delta} (2L+1) S(\omega) d\omega , \tag{40}$$

where Δ is a quantity of the order γ_s. As for the bound states[5], one can define the transition density between the ground and resonance states through the relation

$$B(EL) = (2L+1) \left| \sum_i e_q^i \int V_0^i(r) \rho^i_{tr}(r) r^2 dr \right|^2 . \tag{41}$$

Using eq. (20) for $S(\omega)$ and eq. (41) we obtain

$$\rho^i_{tr}(r)_{\omega=\tilde{\omega}_s} = \frac{1}{\pi S(\tilde{\omega}_s)} \sqrt{B(EL)/(2L+1)} \tag{42}$$

$$\times \operatorname{Im} \int A^i_L(r, r_1; \tilde{\omega}_s) V^i_L(r_1, \tilde{\omega}_s) r_1^2 dr_1 ,$$

that is an extension of the transition density concept to resonance states.

5. NUMERICAL RESULTS

Calculations were performed with the Saxon-Woods potential in the form given by Chepurnov [21]

$$u^0(r) = -V_0 [1 + \exp a(r - R_0)]^{-1},$$

$$u^1(r) = -u^0(r) \cdot \eta (N-Z)/A,$$

$$u_{sL} = -\frac{\lambda}{r} (\vec{\sigma} \vec{L}) \frac{d}{dr} (u^0 + u^1 r_Z),$$

Table 1

Parameters of the Saxon-Woods potential and of the effective interaction, and some characteristics of the ground states

Nucleus	^{16}O	^{48}Ca	^{58}Ni	^{90}Zr	^{208}Pb
V_0, MeV	53.35	53.35	53.35	51.0	53.3
η	—	0.63	0.63	0.90	0.66
a^{-1}, fm	0.40	0.63	0.63	0.63	0.67
$\langle r_c^2 \rangle^{1/2}$, fm theor.	2.55	3.38	3.78	4.18	5.40
exp. /22/	2.71	3.48	3.77	4.27	5.50
β_{symm}, MeV	—	20.0	27.4	30.1	26.3
κ_0, fm^2/MeV	-0.036	-0.024	-0.021	-0.017	-0.010
$\kappa_1[\eta(N-Z)/A]^2\kappa_0$	-1.0	-0.65	-1.27	-0.82	-0.70
$\kappa_c Z(Z-1)/A$, MeV	6.40	6.01	6.30	6.03	5.81

$$\lambda = 0.263(1+2(N-Z)/A)\ \text{fm}^2\ ,$$

$$V_c(r) = (Z-1)e^2 \times \begin{cases} (2R_0)^{-1}(3-(r/R_0)^2), & r \le R_0 \\ 1/r, & r > R_0 . \end{cases} \qquad (43)$$

The radius R_0 was taken to be $1.24\ A^{1/3}$ fm. The other potential parameters and the calculated strength parameters of the effective interaction are listed in table 1. The calculated r.m.s. charge radii $< r_c^2 >^{1/2}$ and the potential symmetry energy parameter

$$\beta_{symm} \equiv \frac{A}{(N-Z)^2} \int u^{-1}(r)(\rho^n - \rho^p) r^2 dr \qquad (44)$$

are given also in this table. It is seen that the values of $<r_c^2>^{1/2}$ are in good agreement with experimental data.

In the closed-shell nuclei the lowest p-h 1^- states appear in the region of the nucleon threshold energy where their mixing with the multiparticle configurations might be strong. Therefore, the calculation of such states is of methodical interest in order to estimate the effects of the one-particle continuum. Table 2 lists the calculated energies and B(E1) values for the bound states in ^{208}Pb . The results of calculation are compared with those obtained in ref. /9/, in which the one particle continuum was approximated with the quasidiscrete levels having the width $\lesssim 1$ MeV. It is seen that the correct inclusion of the continuum affects weakly the energy of states,

Table 2

Energies and B(E1) values for bound 1^- states in ^{208}Pb . The results of the descrete RPA calculations /9/ are given in parentheses

ω, MeV	6.118 (6.112)	6.135 (6.120)	6.525 (6.536)	6.825 (6.826)	6.853 (6.859)	7.135 (7.131)	7.287 (7.326)
B(E1), e^2fm^2	0.059 (0.029)	0.006 (0.028)	0.334 (0.267)	0.042 (0.100)	0.255 (0.234)	0.020 (10^{-4})	4.564 (2.089)

Table 3

Energies and escape-widths of dipole resonances (γ_n and γ_p are neutron and proton partial widths, respectively; for ^{48}Ca and ^{208}Pb $\gamma_p \approx 0$)

Nucleus	ω, MeV	γ_n, KeV	γ_p, KeV	Nucleus	ω, MeV	γ_n, KeV
^{16}O	17.86	0.4	232	^{48}Ca	10.91	91
	18.64	44	11		11.33	36
	20.60	172	768		11.83	2
	23.13	404	879		12.08	17
					12.89	3
^{58}Ni	10.98	1	1		13.37	33
	11.50	39	0.3		14.50	69
	11.80	0.007	0.08		15.71	204
	12.17	0.1	8		16.99	10
	12.53	3	11		19.03	36
	13.15	13	36			
	13.58	0.3	69	^{208}Pb	7.45	0.1
	13.96	4	0.05		7.55	0.1
	15.14	13	37		7.66	0.01
	16.30	189	264		7.87	0.1
	17.83	100	181		8.02	1.8
					8.12	0.07
^{90}Zr	9.43	0.02	0.08		8.63	4.4
	9.73	0.2	0.08		8.79	0.08
	9.96	0.7	43		9.03	4
	10.20	0.08	1		9.35	10
	10.74	21	0.3		10.00	14
	10.84	8	0.04		10.44	3
	11.58	49	2		11.22	44
	11.83	15	1		11.49	34
	12.12	70	0.5		12.12	0.7
	12.88	29	10		12.58	35
	14.11	99	4		13.46	75
	14.40	63	8			
	15.60	18	0.4			

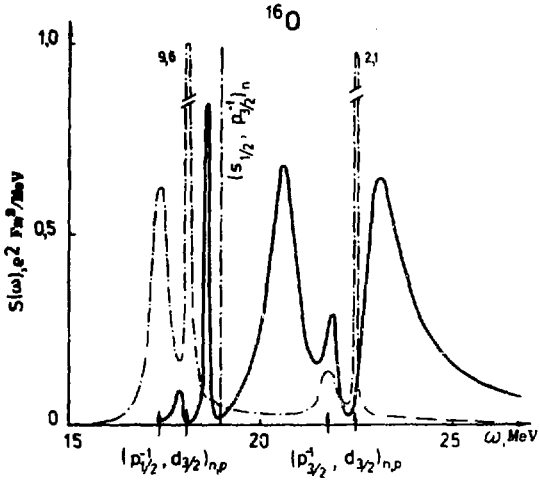

Fig.1. Strength function of the dipole excitations in ^{16}O. The dash-dotted line corresponds to the p-h strength function when the effective interaction is switched off.

while B(E1) values change noticeably. In particular, in the present calculation the bound states exhaust 5.1% of the energy-weighted sum rule, while in ref.[9] the contribution of those states amounts to 2.7%. On the whole, the predicted 1^- spectrum and B(E1) values agree qualitatively with the empirical data[23] obtained in (n,γ) and (γ,n)-reactions.

As it was mentioned, the excitation spectrum above the nucleon threshold becomes continuous and can be characterized by the strength function $S(\omega)$. Figs.1-3 show the radiative strength functions $(e_q V_0 = (N/A)e\vec{r}$ for protons and $-(Z/A)e\vec{r}$ for neutrons) for a number of nuclei. The excitations near the threshold have quite small escape-widths (see table 3) and they appear as the narrow isolated resonances. With increasing excitation energy the width increases, but in medium and heavy nuclei it does not exceed 0.1 MeV even for the high-energy resonances. It is evident that the escape-width cannot be compared with the width of the envelope curve observed experimentally. Such a comparison can be made provided the damping of p-h states and the experimental energy resolution are taken into account. However, for the excitations near the threshold the calculated escape-widths in ^{208}Pb are in qualitative agreement with experimental data[24].

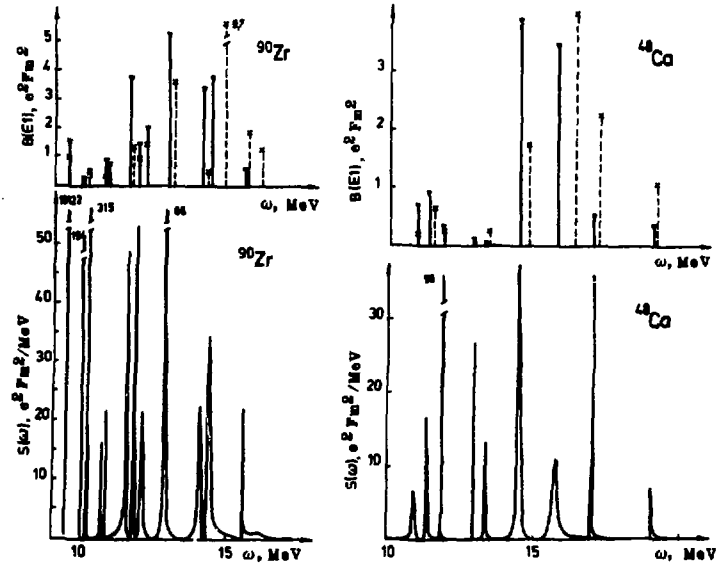

Fig.2. Bottom: The strength function of the dipole excitations in ^{48}Ca and ^{90}Zr . Top: B(E1) values for the resonances shown below (solid lines) are compared with the calculations /9/(dashed lines).

In light nuclei the sum of calculated escape-widths is comparable with the total observed one (see, e.g., for ^{16}O fig.1 and table 3).

To demonstrate the role of the effective interaction we present in fig.1 for ^{16}O the p-h strength function when the interaction (13) is switched off (broken line). In this case the maxima of $S(\omega)$ correspond to the p-h transitions $(p^{-1}_{3/2}, d_{3/2})_{n,p}$, $(p^{-1}_{1/2}, d_{3/2})_{n,p}$, and $(p^{-1}_{3/2}, s_{1/2})_n$. The effective interaction shifts up those maxima, mixes the configurations, changes the distribution of the radiative strength, and increases the escape-widths up to approximately 1.5 MeV.

In figs.2,3 the results obtained are compared with the discrete RPA calculations /9/. The following effects are seen:

i) all the high-energy resonances are shifted down by $\lesssim$0.5 MeV;

ii) E1-transitions in the low-energy part are enhanced, so

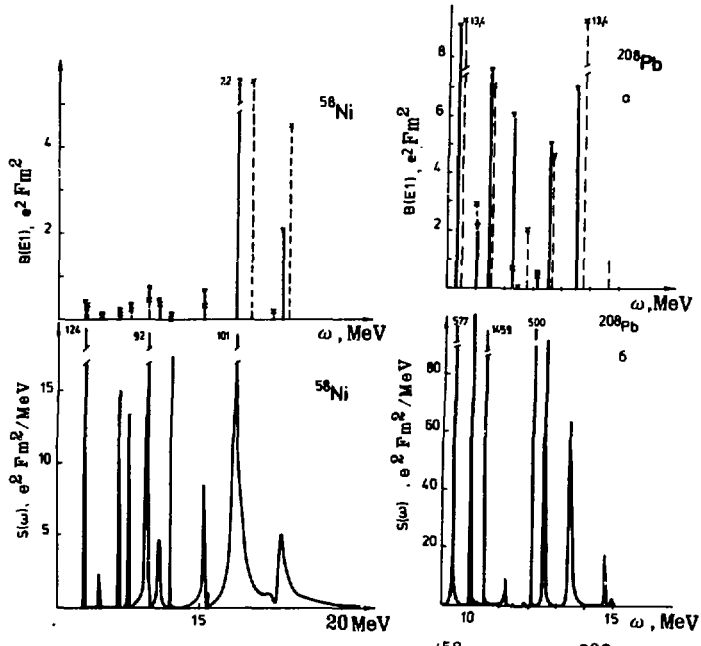

Fig.3. The same, as in fig.2, for ^{58}Ni and ^{208}Pb.

that their contribution to the energy-wieghted sum rule increases by as much as 10% as compared with the discrete RPA calculations.

It should be mentioned that in the present calculations, as in ref.[9], the predicted position of the E1-resonance maximum is by approximately 10% lower than the experimental one (except for ^{16}O where the resonance position was fitted by an appropriate choice both of the average field parameters and the strength of the isovector interaction).

The correct inclusion of the one-particle continuum affects slightly the shape of the transition density $\rho_{tr}(r)$ as compared with previous calculations[9] (see fig.4). It is important that the present calculations confirm the change of ρ_{tr} with increasing the excitation energy, as it was pointed out in ref.[9]. For the low-energy states ρ_{tr} have the large volume oscillations, while in the region of the E1-resonance maximum the transition density is surface-peaked. This feature is demonstrated in fig.4 by comparison of calculated ρ_{tr} with the

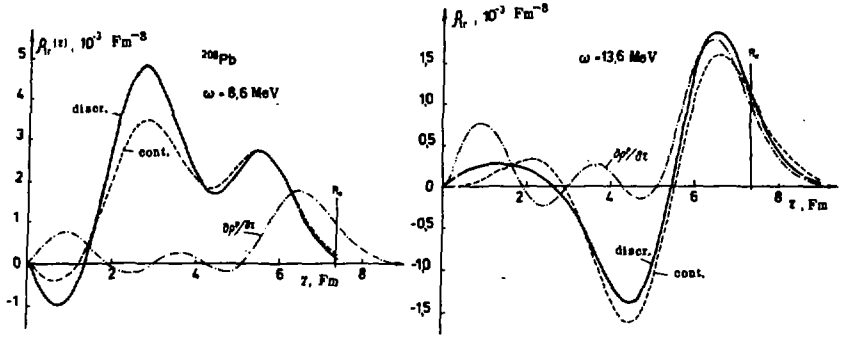

Fig.4. Transition densities for two resonances in ^{208}Pb. Calculations /9/ (solid line) are compared with ours (dashed line). The dash-dotted line shows the derivative of the ground state proton density. All the curves are subject to normalization $\int \rho_{tr}\, r^3\, dr = 1$.

derivative of the ground state charge distribution $\partial\rho^p/\partial r$, that represents the Tassie-model transition density. This result seems to be important for analysing the inelastic electron scattering data in order to separate the giant multipole resonances.

6. CONCLUSIONS

From the present consideration the conclusion can be drawn that the simple translationally invariant model with the separable interaction gives practically the same qualitative description of nuclear dipole states as many other microscopic approaches. The specific feature of our calculations is the correct inclusion of the one-particle continuum. As a result, the excited states above the nucleon threshold acquire the escape-width. The calculated widths for low-energy states (near the threshold) agree well with the observed values. In light nuclei the sum of the escape-widths is quite comparable with the total measured giant resonance width.

We should like to emphasize that in the model employed the effects on the nuclear surface play the dominant role because

for all the dipole states the collective amplitudes are assumed to be proportional to the average field derivative ($g_s \sim \partial u/\partial r$) . Such a form of the collective amplitude is true for the case of the spurious 1^- state ($\omega = 0$), that corresponds to the centre-of-mass motion. Note, that the nucleon interaction amplitude for ω in the vicinity of the eigenergies ω_s of the nuclear system becomes separable. Therefore, the separabilization of the effective interaction is not devoided of a certain sense by itself.

In the general case for excited states there exist the quantum volume corrections to the effective nuclear field which are associated with the volume components of the effective interaction. These corrections are neglected in the present consideration. Another part of the volume oscillations in our model may appear naturally, if a more general self-consistent potential (e.g., found from Hartree-Fock calculations or from self-consistency eqs.) is used instead of the smooth Saxon-Woods function.

The above-mentioned corrections are small for the low-energy bound states, as it was seen from direct calculations carried out in the self-consistent version of the theory of finite Fermi-systems[5] . It has been shown that the shape of ρ_{tr} practically coincides with $\partial\rho^p/\partial r$ only for the lowest-energy collective 3^- excitations ("pure capons"). The transition densities for higher-lying states may acquire the noticeable volume components due to the dominant contribution of some p-h configurations in the vicinity of the state considered.

There is a number of questions that cannot be answered at the present stage of investigation. Up to what excitation energy is the surface-peaked character of the effective field (the collective amplitude g_s) preserved? In particular, is the collective amplitude for the giant resonance peaked on the surface or has it large volume oscillations? Up to now there is not enough experimental data from which the information about g_s can be gained. Of particular interest are the inelastic hadron scattering reactions, but the results are usually analysed in terms of parameters of the dynamic deformations β_s , i.e., it is supposed that $g_s \sim \beta_s (\partial u/\partial r)$. The model considered is in line with this suggestion and it allows one to calculate the parameters β_s microscopically.

It is difficult to carry out the model-independent analysis of such reactions, from which the information concerning g_s can be obtained, because there exist the nucleon distortion effects in the optical potential. The change of the radial shape of g_s with increasing excitation energy can be traced

by employing the "realistic" interaction that includes the volume components. The work along this line is in progress.

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