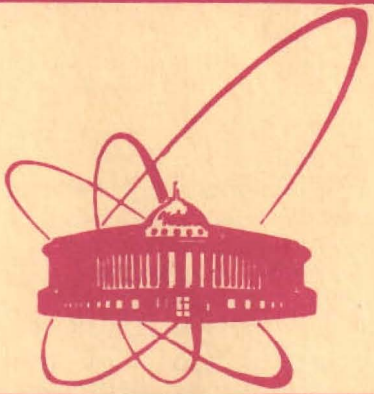

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# THE STUDY OF LOW-ENERGY PION-DEUTERON SCATTERING IN THE METHOD OF EVOLUTION WITH RESPECT TO COUPLING CONSTANT

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Низкоэнергетическое $\pi d$ -рассеяние
в эволюционном по константе связи методе

Эволюционный по константе связи метод /ЭКС/ применяется для изучения низкоэнергетического рассеяния пионов на ядрах. Рассматривается вариант ЭКС-метода с двумя разными константами связи. Получена итерационная схема для вычисления амплитуды рассеяния, в которой выполняется условие унитарности в каждом последовательном приближении. На примере низкоэнергетического $\pi d$ -рассеяния показана быстрая сходимость данного ряда для вычисления пион-дейтронной длины рассеяния к точным расчетам на основе уравнений Фаддеева. Вычисляются фазы $\pi d$ -рассеяния в статическом пределе теории. Анализируется их чувствительность к параметрам $\pi N$-взаимодействия.

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The Study of Low-Energy Pion-Deuteron Scattering
in the Method of Evolution with Respect to

1. The pion scattering by the deuteron is the simplest process of pion-nuclear interaction. The problem of $\pi d$-scattering, being an independent object of studies, can be helpful in testing approximate methods which describe the pion interaction with heavier nuclei [1].

The elastic $\pi d$-scattering was first considered by Foldy and Brueckner [2]. Later this process was treated within the nonrelativistic diagram technique [3]. In these papers the scattering length (its nonabsorptive part) was calculated. The most complete and accurate analysis of this quantity was performed on the basis of Faddeev equations [4,5]. It has been found (see also ref. [6]) that the $\pi d$-scattering length is little-dependent on the shape of the NN-potential, but very sensitive to the $\pi N$-scattering lenghts. Unfortunately the present experimental $\pi d$-scattering length is rather inaccurate (see table 2 and ref. [7]) so it is impossible now to distinguish between the known sets of $\pi N$-scattering lengths.

Recently several attempts [8-10] have been made to incorporate pion absorption in elastic or reactive $\pi$-induced reactions. A consistent solution of this problem would allow, in particular, the study of meson degrees of freedom of nuclear systems. However the inclusion of the pion absorption channel in the framework of theoretical schemes with phenomenological NN- and $\pi N$-potentials (see, e.g., ref. [10]) results in principal difficulties overcounting contributions through the pion exchange to these potentials, etc.

In this situation it would be helpful to apply to a specific method of the description of quantum systems the method of evolution in coupling constant (CCE), which is equally valid for solving the problems of nonrelativistic quantum mechanics and quantum field theory (QFT) (see review [11]). For the three-body problem this method was developed in [12,13] where a possibility was shown of constructing a convenient iteration scheme for the calculation of $nd$-elastic scattering amplitude. The rapid convergence of the iteration series comes from the exact fulfilment of unitarity condition at each stage of successive approximations.

This paper is aimed at developing the CCE-method for the problem of $\pi$-nuclear interaction. Its validity is illustrated with the simplest example of the low-energy $\pi d$-scattering without inclusion of the pion-absorption channel. We believe in the possibility of the consistent consideration of that channel in future from the experience of application of the method to QFT for which it has first been invented.

In Sec. 2 we present basic assumptions of the CCE-method for the simplest case of a one-type interaction. In Sec. 3 these results are generalized to the case of interaction of two types (for the $\pi$-nuclear interaction these are nucleon-nucleon and pion-nucleon interactions). The iteration procedure for solving corresponding equations is formulated as well. In Sec. 4 the $\pi d$-scattering phases and lengths are calculated for various known sets of $\pi N$-scattering lengths. The results are discussed in Sec. 5.

2. Basic equations of the CCE-method in the case of a one type of interaction (see [11-13]) are as follows. The Hamiltonian of the system is

$$H = H_0 + gV , \tag{1}$$

where $H_0$ is the free Hamiltonian; $gV$, the interaction; $g$, the coupling constant. The eigenfunctions of $H$ are $|\mu\rangle$, $|\nu\rangle$, etc., and the matrix elements over them $V_{\mu\nu}$, etc. The partial phases of scattering of two particles (no matter, elementary or composite) is given by the relation*)

$$\partial\delta(k)/\partial g = -\pi\rho(k)V_{\mu\nu} , \tag{2}$$

where $|\mu\rangle = |\vec{k},-\vec{k}\rangle$, $|\nu\rangle = |\vec{k}',-\vec{k}'\rangle$ $(k = k')$, $\rho(k) = m^* k/2\pi^2$ is the level density, $m^*$ the reduced mass, $\vec{k}$ and $\vec{k}'$ are momenta in the c.m.s. before and after scattering, the spherical harmonics corresponds to the angle between vectors $\vec{k}$ and $\vec{k}'$. The potential matrix element in (2) $V_{\mu\nu}$ obeys the relation

$$\partial V_{\mu\nu}/\partial g = \sum_\sigma V_{\mu\sigma} V_{\sigma\nu} \left( \frac{1}{E_\mu - E_\sigma - i\delta} + \frac{1}{E_\nu - E_\sigma + i\delta} \right), \tag{3}$$

*) Here and in Sec. 3 we do not write explicitly indices of momentum, spin,... in relations like (2), $|\vec{k}| = |\vec{k}'| = k$.



where $E$ is the exact eigenvalue of $H$ (defined by the known equation $\partial E_\mu/\partial g = V_{\mu\mu}$).

If the potential $V$ represents two-body forces and the number of particles in a system is larger than or equal to three, one meets with the problem of separating disconnected diagrams which correspond to the free flight of one or more particles (see [14]). Such a separation within the CCE-method has been shown in [12] and for the three-particle case it consists in the representation.

$$V_{\mu\nu} = \sum_{i=1}^{3} v^i_{\mu\nu} + V^c_{\mu\nu} , \tag{4}$$

where $v^i_{\mu\nu}$ is the exact matrix element of the two-body potential with the free flight of the third (i-th) particle, including $\delta$-functions of the conservation laws (momentum, spin, etc.) of the i-th particle; $V^c_{\mu\nu}$ is the connected part of the matrix element.

Substituting (4) in (3) gives the equation

$$\partial V^c_{\mu\nu}/\partial g = \sum_\sigma \Big[ \sum_{i\neq j} v^i_{\mu\sigma} v^j_{\sigma\nu} + \sum_i \left( v^i_{\mu\sigma} V^c_{\sigma\nu} + V^c_{\mu\sigma} v^i_{\sigma\nu} \right) + V^c_{\mu\sigma} V^c_{\sigma\nu} \Big] \left( \frac{1}{E_\mu - E_\sigma - i\delta} + \frac{1}{E_\nu - E_\sigma + i\delta} \right). \tag{5}$$

The very shape of this equation suggests an idea of its approximately solving: the construction of the iteration series in powers of matrix elements $v^i_{\mu\nu}$. With $V^c = 0$ in the r.h.s. of (5) we obtain the first term of the series. The second term arises from substituting of the first one into the term linear in $V^c$, and so on.

Each term is Hermitian and has correct analytic properties with respect to energy variables. Therefore, in contrast to, e.g., the Born series, we arrive at the scattering matrix which is unitary (and causal) at each step of successive approximations. On the other hand, the higher terms of the iteration series the further singularities of S-matrix which give a subsequently decreasing contribution. Therefore, as has been mentioned, the CCE-method provides a relatively rapid convergence of the iteration series for scattering matrix.

3. In the problem of $\pi$-nuclear interaction with the phenomenological $\pi N$-potential we have the more complicated Hamiltonian for the system "pion + nucleus"

$$H = H_0 + gV + \lambda U, \qquad U = \sum_{i=1}^{A} U^i , \tag{6}$$

where the term $gV$ describes the NN-interaction; $U^i$, the pion interaction with an i-th nucleon, and $\lambda$ plays the role of the second coupling constant.* The parameter $\lambda$ is reduced to unity at the end of some calculations.

We shall study here the low-energy elastic pion-nuclear scattering assuming the solution of the pure nuclear problem with Hamiltonian (1) to be known. So the system evolution in coupling constant $\lambda$ from $\lambda = 0$ with switched-off $\pi$-nuclear interaction to the realistic value $\lambda = 1$ must be considered. Analogously to (2), the scattering phase of the "pion-nucleus" obeys the equation

$$\partial\delta(k)/\partial\lambda = -\pi\rho(k) U_{\mu\nu}(\lambda), \tag{7}$$

where Greek indices mean eigenfunctions of Hamiltonian (6). With the mentioned above boundary conditions we have

$$\delta(k) = -\pi\rho(k) \int_0^1 d\lambda\, U_{\mu\nu}(\lambda). \tag{8}$$

The matrix elements $U_{\mu\nu}$ may be calculated through generalizing eq. (3) introducing analogous equations for derivatives of $U_{\mu\nu}$ and $V_{\mu\nu}$ with respect to $\lambda$ and $g$. However, a more simple way is to reduce the problem to the case of one type of interaction. Let us introduce eigenfunctions of the channel Hamiltonian $H_0+\lambda U$ denoting them by Latin indices and reexpand over them the quantity $U_{\mu\nu}$ :

$$U_{\mu\nu} = \sum_{m,n} \langle \mu | m \rangle U_{mn} \langle n | \nu \rangle . \tag{9}$$

*The results presented below are also valid for other problems with two types of interaction, e.g., for Coulomb effects in nuclear systems.



The matrix element $U_{mn}$ is well defined by the Hamiltonian $H_0 + \lambda U$ and is calculated like $V_{\mu\nu}$ in Sec. 2. Thus, we suppose

$$U_{mn} = \sum_{i=1}^{A} u^i_{mn} + U^c_{mn} , \tag{10}$$

where $u^i_{mn}$ is the exact two-particle matrix element of the pion interaction with an i-th nucleon, $U^c_{mn}$ is its connected part. Substitution of (10) in the equation (similar to (3)):

$$\partial U_{mn}/\partial\lambda = \sum_s U_{ms} U_{sn} \left( \frac{1}{E_m - E_s - i\delta} + \frac{1}{E_n - E_s + i\delta} \right), \tag{11}$$

results in the equation analogous to (5)

$$\partial U^c_{mn}/\partial\lambda = \sum_s \Big[ \sum_{i\neq j} u^i_{ms} u^j_{sn} + \sum_i \left( u^i_{ms} U^c_{sn} + \right. \tag{12}$$

$$\left. + U^c_{ms} u^i_{sn} \right) + U^c_{ms} U^c_{sn} \Big] \left( \frac{1}{E_m - E_s - i\delta} + \frac{1}{E_n - E_s + i\delta} \right),$$

which can now be solved by the iteration method. We write out only two first terms *

$$U_{mn} = \sum_i u^i_{mn} + \sum_{i\neq j} \sum_s \int_0^1 d\lambda\, u^i_{ms} u^j_{sn} \left( \frac{1}{E_m - E_s - i\delta} + \frac{1}{E_n - E_s + i\delta} \right) \tag{13}$$

We are now to find the iteration series for $\langle n|\nu\rangle$ in (9) which for the switched-off interaction $U$ is simply the nucleus wave function in the representation of free particles denoted further by $\psi_\nu(n)$. With the corresponding equations for state vectors[11-13] :

$$\partial|\nu\rangle/\partial\lambda = \sum_\sigma \frac{|\sigma\rangle U_{\sigma\nu}}{E_\nu - E_\sigma - i\delta} ; \quad \partial|n\rangle/\partial\lambda = \sum_s \frac{|s\rangle U_{sn}}{E_n - E_s - i\delta}$$

*If the pion-nucleon "bound-states" (isobar) are not taken into account, the energy variables in (11)-(13) may be considered independent of $\lambda$.

and (9) the equation for $\langle n|\nu\rangle$ is:

$$\frac{\partial}{\partial\lambda}\langle n|\nu\rangle = \sum_s \frac{U_{sn}\langle s|\nu\rangle}{E_n - E_s + i\delta} + \sum_{\sigma,s,t} \frac{\langle n|\sigma\rangle\langle\sigma|s\rangle U_{st}\langle t|\nu\rangle}{E_\nu - E_\sigma - i\delta}. \quad (14)$$

This nonlinear equation can also be solved by the iteration method taking $\psi_\nu(n)$ to be a zero-order approximation. First two terms of the series are

$$\langle n|\nu\rangle = \psi_\nu(n) + \sum_s \int_0^\lambda d\lambda \frac{U_{ns}\psi_\nu(s)}{E_n - E_s + i\delta} + \quad (15)$$

$$+ \sum_{\sigma,s,t} \int_0^\lambda d\lambda \frac{\psi_\sigma(n)\bar{\psi}_\sigma(s)U_{st}\psi_\nu(t)}{E_\nu - E_\sigma - i\delta}.$$

And finally, combining (15) with (13), allowing for (9), and introducing the Green functions of the nucleus $G^{(-)}$ and the system of free nucleons $G_0^{(-)}$ :

$$G^{(-)}_{mn}(E) = \sum_\sigma \frac{\psi_\sigma(m)\bar{\psi}_\sigma(n)}{E - E_\sigma - i\delta}, \quad G^{(-)}_{0\,mn}(E) = \frac{\delta_{mn}}{E - E_m - i\delta},$$

we arrive at the matrix element $U_{\mu\nu}$ :

$$U_{\mu\nu} = \sum_{m,n} \bar{\psi}_\mu(m) O_{mn}(\lambda)\psi_\nu(n), \quad (16)$$

where

$$O_{mn}(\lambda) = \sum_i u^i_{mn}(\lambda) + \sum_{i\neq j}\sum_s \int_0^\lambda d\lambda_1 u^i_{ms}(\lambda_1)u^j_{sn}(\lambda_1) \times$$

$$\times\left(\frac{1}{E_m - E_s - i\delta} + \frac{1}{E_n - E_s + i\delta}\right) + \sum_{i,j}\sum_{s,t}\int_0^\lambda d\lambda_1 \{u^i_{ms}(\lambda_1)\times \quad (17)$$

$$\times[G^{(-)}_{st}(E_\mu) - G^{(-)}_{0\,st}(E_m)]u^j_{tn}(\lambda) + \text{h.c.}\,(m \rightleftarrows n)\},$$

and the equality $E_\mu = E_\nu$ is included.



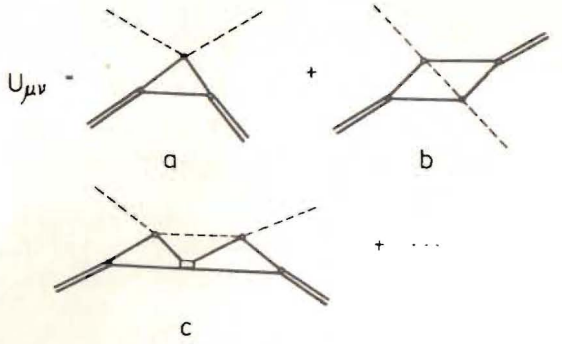


Fig. 1. The graphical image of the iteration series (17) for $\pi$d -scattering.

The series obtained is the functional expansion over the exact two-particle matrix element $u^i_{mn}$ of the pion interaction with a separate nuclear nucleon. In the simplest case of $\pi$d -scattering this expansion is shown by diagrams in Fig. 1, where the double line represents the deuteron; solid line, nucleon; dashed line, pion; black vertex, $\pi$N -interaction; white vertex, deuteron wave function and square-nucleon rescattering. The diagram of Fig. 1a represents the first term of (17); Fig. 1b, the second term. The diagram of Fig. 1c, the third term, describes the nucleon rescattering (the difference of $G^{(-)}$ from $G_0^{(-)}$ ). It should be noted that the considered diagrams are not the Feynman diagrams but some auxiliary graphical image (for details see refs./11-13/ ). In particular, their vertices represent the potential matrix elements rather than scattering amplitudes.

4. Consider now the low-energy $\pi$d -scattering. The pion (particle 3) interaction with nuclear nucleons (particles 1 and 2) can be assumed /3-5/ to be s -wave. The partial phase of $\pi$d -scattering (8) has the form

$$\delta_\ell(k) = -(\mu_{\pi d}\cdot k/2\pi)\int_0^1 d\lambda\, U_{\pi d,\ell}(k,k;\lambda), \quad (18)$$

where $\mu_{\pi d} = 2\mu m/(\mu+2m)$ ; $\mu$ is the pion mass; m is the nucleon mass ($m_p = m_n = m$) ; $U_{\pi d}$, $\ell$, the partial component of the matrix element $U_{\pi d}(\vec{k},\vec{k}';\lambda) = \langle \psi^{(+)}_{\pi d}(\vec{k},\lambda)|U|\psi^{(+)}_{\pi d}(\vec{k}',\lambda)\rangle$ at $k=k'$ ; $U = \sum_{i=1}^{2} U^i$ , the potential (see (6)); and $|\psi^{(+)}_{\pi d}\rangle$ , eigenfunctions of the Hamiltonian (6) with the usual asymptotics of states at large distances between the pion and deuteron.

We shall calculate $\delta_\ell(k)$ within the iteration scheme (17). In the first approximation (diagram Fig. 1a) for the matrix element $U_{\pi d}$ and with the identity of nucleons we have

$$U^{(1)}_{\pi d}(\vec{k},\vec{k}';\lambda) = 2\sum_{m,n} \bar{\psi}_{\pi d}(\vec{k},m)\, u^1_{m,n}(\lambda)\, \psi_{\pi d}(\vec{k}',n), \qquad (19)$$

where $\psi_{\pi d}(\vec{k},m)$ and $\psi_{\pi d}(\vec{k}',n)$ are asymptotical initial and final states of the $\pi d$ -system. For instance, in the momentum representation $\psi_{\pi d}(\vec{k},m) = (2\pi)^3\delta(\vec{k}-\vec{p}^{(m)}_3)\phi_d(\vec{k}^{(m)}_{12})\eta^a$ , $\phi_d$ is the deuteron wave function, $\eta^a$ is the isotopic part of the $\pi d$ -wave function

$$\eta^a = \frac{1}{\sqrt{2}}\eta_{1M}(3)[\eta_+(1)\eta_-(2) - \eta_+(2)\eta_-(1)],$$

where $\eta_{1M}$ is the pion isospin function and the brackets is the deuteron isospin function. In view of the footnote to page 5, the sum in (19) means integration over the momenta of particles in intermediate states: $\sum_m = (1/(2\pi)^6)\int d\vec{p}^{(m)}_3 \times \int d\vec{k}^{(m)}_{12}$ .

The matrix element $u^1_{mn}$ in the Jacobi coordinates corresponding to scheme 1 (23) is

$$u^1_{mn}(\lambda) = (2\pi)^3\delta(\vec{p}^{(m)}_1 - \vec{p}^{(n)}_1)\sum_{\tau=1,3} P^\tau(23)\, U^\tau_{\pi N}(\vec{k}^{(m)}_{23}, \vec{k}^{(n)}_{23}, \lambda), \qquad (20)$$

where $P^\tau$ is the projection operator into the $\pi N$-states with isospin $T=\tau/2$ , and $U^\tau_{\pi N}$ is the two-particle matrix element of the $\pi N$-interaction potential in the given isotopic state.

Substituting (20) into (19), averaging over the isospin, and allowing for (18), we obtain the phase $\delta_\ell$ in the first approximation:



$$\delta^{(1)}_\ell(k) = -\frac{\mu_{\pi d}\cdot k}{3\pi}\sum_{\tau=1,3} c_\tau \Big[\int \frac{d\vec{p}}{(2\pi)^3}\phi_d(\vec{q})\phi_d(\vec{q}')\int_0^1 d\lambda\, U^\tau_{\pi N}(\vec{q}_1,\vec{q}'_1;\lambda)\Big]_\ell , \qquad (21)$$

where $\vec{q} = \vec{p} - \frac{1}{2}\vec{k}$, $\vec{q}' = \vec{p} - \frac{1}{2}\vec{k}'$, $\vec{q}_1 = \vec{k} - \omega\vec{p}$, $\vec{q}'_1 = \vec{k}' - \omega\vec{p}$, $k = k'$ , the parameter $\omega = \mu/(m+\mu)$ , and coefficients $c_1 = 1$, $c_3 = 2$. In this approximation the scattering length (the scattering amplitude at the threshold) is

$$a^{(1)}_{\pi d} = \frac{2}{3}\gamma_{\mu m}\int \frac{d\vec{p}}{(2\pi)^3}\phi^2_d(p)\sum_\tau c_\tau \frac{\delta^\tau_{\pi N}(\omega p)}{\omega p}, \qquad (22)$$

where $\gamma_{\mu m} = \mu_{\pi d}/\mu_{\pi N} = (1+\mu/m)/(1+\mu/2m)$, $\mu_{\pi N}$ is the reduced mass of the $\pi N$ -system. Expression (22) was derived with the use of eq. (8) for the $\pi N$-scattering s-phase, $\delta^\tau_{\pi N}$ .

Expressions (21) and (22) contain the small parameter $\omega \simeq 0.13$. Let the static limit of the theory be $\omega \to 0$. In this limit $\gamma_{\mu m} = 1$, and (21) and (22) become

$$\delta^{(1)}_\ell(k,\mathrm{stat}) = \frac{2}{3}\sum_\tau c_\tau \delta^\tau_{\pi N}(k)\Big[\int \frac{d\vec{p}}{(2\pi)^3}\phi_d(\vec{q})\phi_d(\vec{q}')\Big]_\ell ; \qquad (23)$$

$$a^{(1)}_{\pi d}(\mathrm{stat}) = \frac{2}{3}(a_1 + 2a_3), \qquad (24)$$

where $a_\tau$ ($\tau = 1,3$) are s-wave lengths of $\pi N$ -scattering. Formula (24) coincides with the first term of the series of the multiple scattering theory (MST) in the impulse approximation (see [1,15] ).

Table 1 shows values of $a^{(1)}_{\pi d}$ (stat.) for various sets [16-21] of $\pi N$-scattering s-wave lengths and those calculated directly by formula (22) with the separable potential

$$U^\tau(k,k') = \eta_\tau g_\tau(k) g_\tau(k'), \qquad g_\tau(k) = (k^2+\beta^2_\tau)^{-1} \qquad (25)$$

for all isospin states of the two-particle system: $T = \frac{1}{2}\tau = 0, \frac{1}{2}, \frac{3}{2}$. The parameters $\eta_\tau$ and $\beta_\tau$ are defined by the data on particle interactions at low energies and are taken the same as in ref. [5] . For the isosinglet state of the NN -system: $\beta_0 = 1.44\ \mathrm{fm}^{-1}$ , $\eta_0 = -8\pi\beta_0^3 g/m$ , $g = 1+(\kappa_0/\beta_0)^2 \simeq 1$ , $\kappa_0 = \sqrt{m|\epsilon_d|}$ , $\epsilon_d = -2.23\ \mathrm{MeV}$ is the deuteron binding energy.

Table 1.

Two first iterations for the $\pi d$ -scattering length for various sets of $\pi N$-scattering lengths [16-21].

| $\pi N$ -scattering length, fm | | | $\pi d$ -scattering length, fm | | | |
|---|---|---|---|---|---|---|
| $a_1$ | $a_3$ | Reference | $a^{(1)}_{\pi d}$ (22) | $a^{(2)}_{\pi d}$ (stat) (24) | $a^{(2)}_{\pi d}$ (30) | $a^{(2)}_{\pi d}$ (32) | $a^{(2)}_{\pi d}$ (MST) (33) |
| 0.2418 | -0.1244 | [16] | -0.0052 | -0.0047 | 0.0291 | -0.0336 | -0.0361 |
| 0.2573 | -0.1541 | [17] | -0.0365 | -0.0339 | -0.0360 | -0.0426 | -0.0452 |
| 0.2620 | -0.1452 | [18] | -0.0204 | -0.0189 | -0.0354 | -0.0416 | -0.0445 |
| 0.2441 | -0.1263 | [19] | -0.0062 | -0.0057 | -0.0297 | -0.0344 | -0.0369 |
| 0.2566 | -0.1260 | [20] | 0.0032 | -0.0030 | -0.0314 | -0.0365 | -0.0394 |
| 0.2404 | -0.1300 | [21] | -0.0141 | -0.0131 | -0.0297 | -0.0345 | -0.0369 |



For the isospin states $T=\frac{1}{2}, \frac{3}{2}$ of $\pi N$-system $\beta_1=\beta_3=\beta=3.5\text{fm}^{-1}$, $\eta_\tau=-(4\pi\beta^3/\mu_{\pi N})f_\tau$ , where the parameters

$$f_\tau = a_\tau\beta/(2+a_\tau\beta) \tag{26}$$

are fitted to the $\pi N$-scattering lengths, $a_\tau$ . The typical value $|\ f_\tau\ | \simeq 0.3$ . The matrix elements for potential (25) are as follows

$$U^\tau_{\pi N}(q,q';\lambda) = \eta_\tau\, a^{\tau *}_{\pi N}(q,\lambda)\, a^\tau_{\pi N}(q',\lambda),$$

$$a^\tau_{\pi N}(q,\lambda)=(q-i\beta)/(q+i\beta)[(q-i\beta)^2-\lambda f_\tau\beta^2]. \tag{27}$$

As is seen from Table 1, the values of $a^{(1)}_{\pi d}$ (22) and $a^{(1)}_{\pi d}$ (stat) are practically the same and

$$a^{(1)}_{\pi d} \simeq \gamma_{\mu m}\cdot a^{(1)}_{\pi d}\ (\text{stat.}), \tag{28}$$

where $\gamma_{\mu m}=\mu_{\pi d}/\mu_{\pi N}\simeq 1.07$ . Hence it may be concluded that the small parameter $\omega$ validates the static limit. The $\pi d$ -scattering lengths calculated in the first approximation are highly sensitive to the $\pi N$-scattering lengths and differ from the exact results obtained in [5] (see Table 2) by using the Faddeev equations. Therefore, we must consider subsequent terms (diagrams in Figs. 1b, 1c) of the iteration series (17). In their analysis we restrict ourselves to the static limit that seems to be (see eq. (28)) quite reasonable.

Let us show that the third term in (17) (diagram Fig. 1c) in the static limit is strictly zero owing to the cancellation of $G^{(-)}_{st}$ and $G^{(-)}_{0\,st}$ . Indeed, in that limit $G^{(-)}_{st}$ in terms of variables $(\vec{p}_3, \vec{k}_{12})$ has the form

$$G^{(-)}_{st}(E_\mu)=2\mu_{\pi d}(2\pi)^3\delta(\vec{p}^{\,(s)}_3-\vec{p}^{\,(t)}_3)\{\int\frac{d\vec{q}}{(2\pi)^3}\times$$

$$\times\frac{\psi^{(+)}_{NN}(\vec{q},\vec{k}^{\,(s)}_{12})\bar{\psi}^{(+)}_{NN}(\vec{q},\vec{k}^{\,(t)}_{12})}{k^2-p^{(t)2}_3-\epsilon_\mu(\kappa_0^2+q^2)-i\delta}+\frac{\phi_d(\vec{k}^{\,(s)}_{12})\phi_d(\vec{k}^{\,(t)}_{12})}{k^2-p^{(t)2}_3-i\delta}\},$$

Table 2.

The $\pi$d -scattering length calculated for various sets of $\pi$N-scattering lengths[16-21]; Quantities $a_{\pi d}$[5] are from ref.[5]; the experimental $\pi$d-scattering length equals $-(0.073\ ^{+0.031}_{-0.024})$ fm [7]

| $\pi N$-scattering length | $\pi d$-scattering length, fm | | | |
|---|---|---|---|---|
| Reference | $a_{\pi d}(CCE)$ | $\tilde{a}_{\pi d}$ | $a_{\pi d}(MST)$ | $a_{\pi d}$ [5] |
| [16] | -0.0343 | -0.0388 | -0.0411 | -0.0365 |
| [17] | -0.0725 | -0.0789 | -0.0815 | -0.0746 |
| [18] | -0.0558 | -0.0619 | -0.0647 | -0.0582 |
| [19] | -0.0359 | -0.0405 | -0.0430 | -0.0381 |
| [20] | -0.0284 | -0.0333 | -0.0362 | -0.0309 |
| [21] | -0.0438 | -0.0485 | -0.0510 | -0.0469 |



where $\epsilon_\mu = 2\mu/(\mu+2m)$, $E_\mu = k^2/2\mu_{\pi d} + \epsilon_d$, $\epsilon_d = -\kappa_0^2/m$ is the deuteron binding energy. As $\omega \to 0$ , $\epsilon_\mu = 0$ and because of the completeness of states of the NN-system we obtain that

$$G^{(-)}_{st}(E_\mu) = 2\delta_{st}/(k^2 - p_3^{(t)2} - i\delta),$$

where $\delta_{st} = (2\pi)^6 \delta(\vec{p}_3^{(s)} - \vec{p}_3^{(t)}) \delta(\vec{k}_{12}^{(s)} - \vec{k}_{12}^{(t)})$ . This expression coincides with $G^{(-)}_{0\,st}(E_m)$ as $\omega \to 0$ if one takes into account that the momentum $\vec{p}_3^{(m)} = \vec{k}$ owing to the function $\psi_\mu(m) \sim \delta(\vec{p}_3^{(m)} - \vec{k})$ in (16).

Therefore, the second approximation requires the calculation of the diagram of Fig. 1b in the static limit. In full analogy with the derivation of (21) we obtain for the phase $\delta_\ell^{(2)}$ the following expression

$$\delta_\ell^{(2)}(k) = -\gamma_{\mu m} \frac{4\mu^2 k}{9\pi} \sum_{r_1, r_2 = 1,3} b_{r_1 r_2} \Big[ \int \frac{d\vec{q}}{(2\pi)^3} P \frac{1}{k^2 - q^2} \int_0^1 d\lambda\, (1-\lambda) \times$$

$$\times U^{r_1}_{\pi N}(k, q; \lambda) U^{r_2}_{\pi N}(q, k; \lambda) \int \frac{d\vec{Q}}{(2\pi)^3} \phi_d(\vec{Q}_1) \phi_d(\vec{Q}_2) \Big]_\ell , \qquad (29)$$

where $\vec{Q}_1 = \vec{Q} + \frac{1}{2}(\vec{q} - \vec{k})$, $\vec{Q}_2 = \vec{Q} - \frac{1}{2}(\vec{q} - \vec{k}')$. The sign P means the principal value integration, the coefficients $b_{11} = -1$, $b_{13} = b_{31} = 4$ , $b_{33} = 2$ , and the factor $\gamma_{\mu m}$ in (29) is due to (28). The second iteration of the scattering length is

$$a^{(2)}_{\pi d} = \gamma_{\mu m} \frac{4\mu^2}{9\pi} \sum_{r_1, r_2} b_{r_1 r_2} \int \frac{d\vec{q}}{(2\pi)^3} P \frac{1}{q^2} \int_0^1 d\lambda\, (1-\lambda) \times$$

$$\times U^{r_1}_{\pi N}(0, q; \lambda) U^{r_2}_{\pi N}(q, 0; \lambda) \int \frac{d\vec{Q}}{(2\pi)^3} \phi_d(\vec{Q} + \tfrac{1}{2}\vec{q}) \phi_d(\vec{Q} - \tfrac{1}{2}\vec{q}). \qquad (30)$$

Formulae (29) and (30) represent the second term of the iteration series (17) for $\pi$d-scattering phases and lengths in the static limit. They are considerably simplified by taking the matrix elements $U^r_{\pi N}(k, q; \lambda)$ at $q = k$ out of the integral over q . Then, the phase $\delta_\ell^{(2)}$ becomes

$$\delta_\ell^{(2)}(k) = \gamma_{\mu m}\frac{2}{9k^2}\sum_{\tau_1,\tau_2} b_{\tau_1\tau_2}\delta_{\pi N}^{\tau_1}(k)\delta_{\pi N}^{\tau_2}(k)d_\ell(k)[\,1 - \qquad (31)$$

$$-(f_{\tau_1}+f_{\tau_2})/3\,],\quad d_\ell(k) = [\int d\vec{r}\,\phi_d^2(r)\exp(i\vec{\Delta}\vec{r})\cos(kr)/r]_\ell\,,$$

where $\vec{\Delta} = \vec{k} - \vec{k}'$, $\delta_{\pi N}^{\tau}$ are the s-phases of $\pi N$-scattering resulting from the integration over $\lambda$ in (30) with the use of (8); parameters $f_\tau$ are defined by (26). The corresponding scattering length $a_{\pi d}^{(2)}$ has the form

$$a_{\pi d}^{(2)} = \gamma_{\mu m}\frac{2}{9}\sum_{\tau_1,\tau_2} b_{\tau_1\tau_2}a_{\tau_1}a_{\tau_2}[1-(f_{\tau_1}+f_{\tau_2})/3]\langle\frac{1}{r}\rangle_d\,, \qquad (32)$$

where $\langle\frac{1}{r}\rangle_d = 0.566\,\mathrm{fm}^{-1}$ for the deuteron wave function, defined by the potential (25). Arguments for (31) and (32) are as follows: The dominant contribution to integrals (29) and (30) comes from the integrand singularities in q-plane nearest to the coordinate origin. This is the point $q = k$ ($k \approx 0.2\,\mathrm{fm}^{-1}$ for the pion energy 5 MeV). Other poles (see (27)) are at the points $q \approx i\beta$, i.e., far from $q=0$, because $\beta = 3.5\,\mathrm{fm}^{-1}$. Therefore, their contribution to the integrals should be expected to be small. Note that (32) without the term $\sim(f_{\tau_1}+f_{\tau_2})/3$ coincides with the second term of the series of MST for the $\pi d$-scattering length in the impulse approximation (see, e.g., ref.[1]):

$$a_{\pi d}^{(2)}(\mathrm{MST}) = \gamma_{\mu m}\frac{2}{9}\sum_{\tau_1,\tau_2} b_{\tau_1\tau_2}a_{\tau_1}a_{\tau_2}\langle\frac{1}{r}\rangle_d\,. \qquad (33)$$

The typical value of the quantity $|f_{\tau_1}+f_{\tau_2}|/3$ is $\approx 0.2$. The difference of (32) comes from the exact fulfilment of the unitarity condition in the given iteration scheme in contrast to the series of MST.

Table 1 shows results of calculations of $a_{\pi d}^{(2)}$ by formulae (30) and (32). Comparison testifies to the small difference ($\approx 0.006\,\mathrm{fm}$) of those calculations for all sets of $\pi N$-scattering lengths.

This supports our arguments in deriving (31) and (32). Table 1 shows also values of $a_{\pi d}^{(2)}$ (MST). As is seen the second term is less sensitive to the $\pi N$-scattering lengths



than the first one. Table 2 shows the values of sums of the first two iterations *

$$a_{\pi d}(\mathrm{CCE}) = a_{\pi d}^{(1)}(22) + a_{\pi d}^{(2)}(30),\qquad \tilde{a}_{\pi d} = a_{\pi d}^{(1)}(28) + a_{\pi d}^{(2)}(32),$$

$$a_{\pi d}(\mathrm{MST}) = a_{\pi d}^{(1)}(28) + a_{\pi d}^{(2)}(33)$$

and the exact results calculated by the Faddeev equations in ref.[5] (s-wave $\pi N$-interaction). The results indicate the fast convergence of the considered series to the exact values of $a_{\pi d}$[5]. The best agreement is for $a_{\pi d}$ (CCE) where $a_{\pi d}$[5] $- a_{\pi d}$ (CCE) $= -0.002$ fm. The analogous difference for $\tilde{a}_{\pi d}$ is within (0.002 - 0.004) fm, for $a_{\pi d}$(MST) is larger: (0.005 - 0.007) fm. It should be noted that the contribution of the p-wave $\pi N$-interaction, as is shown in[5], amounts approximately to -0.006 fm.

As foolows from table 1, the values of $a_{\pi d}^{(1)}$ are always smaller than of $a_{\pi d}^{(2)}$. Nevertheless, the numerical convergence of the iteration scheme is not accidental. Smallness of $a_{\pi d}^{(1)}$ as compared to $a_{\pi d}^{(2)}$ is due to the specific isotopic structure of the problem resulting in the cancellation of two "large" quantities in the first approximation (see (24)). Indeed, consider a simple model in which the isoscalar pion is scattered on the deuteron. As the s-wave $\pi N$-scattering length we take the maximum value $|a_{1,3}|$ from table 1, i.e., $a_{\pi N} = 0.26$ fm. In this model for the $\pi d$-scattering length the sum of first two iterations is

$$a_{\pi d} = 2a_{\pi N}\,(1 + a_{\pi N}\langle\frac{1}{r}\rangle_d).$$

Since $\langle 1/r\rangle = 0.57\ \mathrm{fm}^{-1}$, $\lambda = a_{\pi N}\langle 1/r\rangle_d = 0{,}15$. The parameter $\lambda$ indicates smallness of the second term as compared to the first one and reflects convergence of the method.

The $\pi d$-scattering phases can be easily calculated within first two iterations by simple expressions (23) and (31) in the static limit. Like for the scattering length, the calculated scattering phases at low energies are naturally believed to coincide practically with exact values. Unfortunately, at present there are no calculations by Faddeev equations for the $\pi d$-scattering phases in that energy region.

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* In the round brackets there are presented the numbers of formulae.

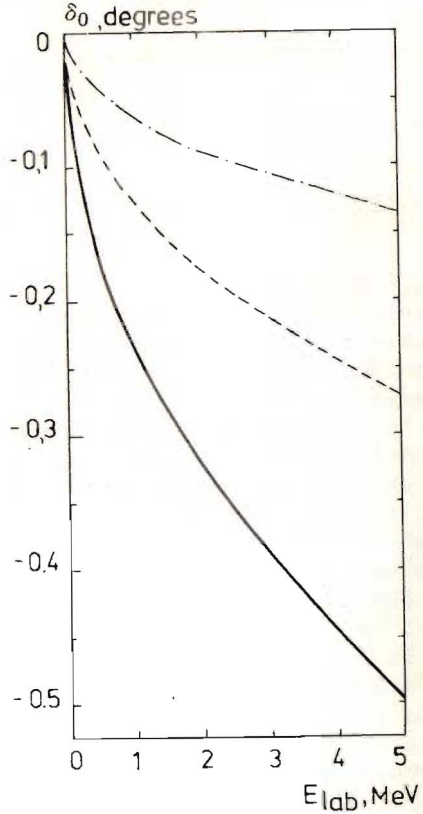


Fig. 2. The s-phase of $\pi$d -scattering calculated for three sets of $\pi$N -scattering lengths: solid line - /17/, dotted line - /21/, dash-dotted line - /20/.

Figure 2 shows the s -phases for three sets of $\pi$N-scattering lengths: the solid line corresponds to the $\pi$N-scattering lengths from ref./17/ ; dotted line,/21/ ; dash-dotted line,/20/ . The s -phases of $\pi$d-scattering are rather sensitive to the parameters of $\pi$N-interaction. Note that the corresponding p -phases are very small and amount to 0.01 of the s -phases.

5. In this paper the CCE-method has been developed for the problem of pion-nucleon scattering. The new iteration scheme (17) is proposed for calculation of the scattering amplitude, which allows a simple diagram interpretation (see, e.g., Fig. 1). This series is an expansion over the exact two-particle matrix element of interaction of a pion



with an individual nuclear nucleon. In contrast to the multiple scattering theory /15/ the unitarity condition for amplitude holds in each step of the successive approximation.

The analysis of $\pi$d -scattering was aimed at studying convergence of the given iteration scheme for the scattering length to its exact value calculated in/5/ by the Faddeev equations. From the careful study of the first iteration (diagram in Fig. 1a) it was established applicability of the theory static limit $\omega = \mu/(m+\mu) \to 0$ . It turns out that unlike the case of nd -scattering considered earlier/12,13/ the first approximation of the $\pi$d -scattering length is essentially smaller than the exact value of $a_{\pi d}$ /5/ because of the specific isospin structure of this problem, as indicated at the end of Sec. 4. The accidental smallness of the first approximation is also verified by the practical coincidence of the sum of two first iterations (see table 2) with the exact value $a_{\pi d}$ . From table 2 it follows that the considered iteration series converges to the exact $a_{\pi d}$ /5/ better than the MST series. This can be regarded as a consequence of the fulfilment of unitarity at each iteration step. More definite conclusions require, of course, a more careful analysis of applicability of the static limit, in particular, exact calculation of the diagram of Fig. 1c, and estimation of the higher-iteration contribution. The results obtained for the $\pi$d -scattering phases (see Fig. 2) indicate their high sensitivity to $\pi$N -interaction parameters. Note that all basic relations of Sec. 4 corrected to the spin-isospin dependence can be applied for the pion scattering on heavier nuclei, e.g., $^6$Li /22/, which admit a two-cluster representation.

An interesting problem is to generalize this formalism to the channels with nonconservation of pions in intermediate states and to the general case of multichannels reactions. The latter in terms of the Jost matrices is discussed in ref./13/ .

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