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DETERMINATION OF THE MATTER RADIUS OF ^{3}He AND ^{4}He FROM ELASTIC SCATTERING OF PIONS

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1. INTRODUCTION

A number of highly precise measuruments of elastic scattering of pions on nuclei were completed recently at the meson factories of SIN and Los Alamos [1-3]. A "justification" for these experiments was an idea about the possibility of extracting information about the parameters of neutron density of nuclei from elastic scattering of pions. In brief, the authors scheme of argument is the following:

They suppose that pion elastic scattering on nuclei has a diffraction nature and the location of the minimum in a differential cross section is only due to the zero of the Bessel function from the known amplitude for the scattering on a "black disk"

$$f_{B.D.}(\theta) = \frac{iR_{B.D.}}{2\sin(\theta/2)} J_1(2kR_{B.D.}\sin\frac{\theta}{2}). \qquad (1)$$

So, we can provide careful measurument of the position of the minimum in $d\sigma/d\Omega$ and connect it with some radius $R_{B.D.}$. Since π^--mesons in the region of Δ_{33}-resonance interact with neutrons at least 9 times stronger, than π^+, we expect that π^- will be rather sensitive to the neutron distribution in a nucleus. In case this distribution is different from the proton distribution the minimum in $d\sigma/d\Omega$ for elastic scattering of π^+-mesons must have a location other than that for π^--mesons.

Such an effect of shifting the minimum position for π^+ with respect to π^- was observed experimentally. Nevertheless, as the subsequent analysis [4] shows approximately 80% of the shift is due to Coulomb effects. The other 20% are comparable already with experimental errors and do not permit one to make any definite conclusion about the parameters of neutron density.

The failure of these experiments is quite natural. In spite of the fact that the interaction of pions with nuclei has been studied for a long time it is not clear up to now whether pions are an effective "tool" for the investigation of a nucleus and, if it is so, what characteristics of nuclear structure can be extracted from the study of pion scattering. There is a lot of sometimes totally incompatible judgments about these problems.

For example, there is a common view (see refs. [4,5]) that the elastic scattering of pions at $E_\pi < 300$ MeV proceeds mainly on the surface of a nucleus and is not sensitive to the low momenta of the nuclear density. On the other hand, it has been shown [6] that differential cross sections of the elastic π^4He, π^{16}O and π^{12}C scattering change to the greatest extent at the variation of the r.m.s. radius while a similar variation in the parameters of the nuclear shape thickness do not significantly change the results.

But suppose that we wish to measure the r.m.s. radius of a nucleus with pions. However, it is not clear, up to now, how to do that. Earlier there existed an opinion that it could be done by simply fitting the parameters of form factors, for example, in an optical model, to the experimental data. Now we known that it is wrong. The reason is the fact that there exists no theory of pion-nucleus scattering properly taking into account the real absorption of pion, but it has been assumed that the cross sections of absorption are not large and the corresponding influence on the elastic channel should not change significantly the results. However, recent experiment [7] has shown that for a number of nuclei $\sigma_{abs} \approx \sigma_{el}$ hence, we cannot neglect the influence of the coupling between absorption and elastic channels. Therefore, the values for the radius which were obtained by fitting the data with an optical model reflect, to some extent, not true nuclear sizes but the shortages of the optical model.

Finally a question can always arise: Why, strictly speaking, should one measure the radius of the nuclei by means of pions when there is an elaborate and well grounded procedure for the extraction of radius value from electron-nucleus scattering. Answering this question we can give at least two reasons. At first, as we have mentioned above, we have had a principal possibility of obtaining from pion-nucleus scattering some information about neutron density. Besides it turns out that we must be careful when using the parameters extracted from electron-nucleus scattering in pion-nucleus calculations. As has been shown by R.Mach [8] in an optical model it is impossible to take the value of the magnetic radius for magnetic form factor of ^{3}He directly from eA-scattering, a correction for the effect of meson exchange currents should be made. But this correction makes different contributions to the e^3He and π^3He scattering. It is clear that we can avoid such a model-dependent correction if we measure the magnetic form factors of ^{3}He directly in the π^3He scattering.

In the present article we suggest a simple recipe for the extraction of nuclear matter radius R_m from the data on elastic pion-nucleus scattering. We have analysed the data [8-12] on the elastic scattering of pions on ^3He and ^4He as an example of the validity of such an approach. The scattering of pions on these nuclei has a nondiffractive character and equation (1) cannot be applied even for the estimation of R in the Δ_{33}-resonance region. (For example, at E_π = 145 MeV, for ^3He $R_{B.D.}$ = 2.93 fm and for ^4He $R_{B.D.}$= 2.65 fm. These values are at least two times greater than charge radius R_{ch} for ^3He and ^4He).

Though our model is based on a rather simple local pion-nucleus potential it has some advantages over the usual optical models. We need to know the behaviour of the potential at only small momenta transfer corresponding to the forward scattering. This circumstance significantly decreases the dependence of final results upon different peculiarities of the potential. Besides as R.Landau and A.Thomas [13] have calculated the effect of true absorption does not change practically the differential cross sections of elastic scattering at small angles.

2. DESCRIPTION OF THE MODEL

Rather a long time ago in refs. [14,15] it has been mentioned that a pion-nucleus potential may be represented in the following form:

Let

$$\langle \vec{k}' | V_{\pi A} | \vec{k} \rangle = \frac{1}{(2\pi)^2} f_{\pi N}(\vec{k}' \vec{k})\, \rho_{R_m}(q^2)\,; \quad \vec{q} = \vec{k}' - \vec{k}\,, \tag{2}$$

where $\vec{k}$ and $\vec{k}'$ are the momenta of the pion in a pion-nucleus centre of mass system (ACM) and ρ_{R_m} is a form factor, which can be expanded in such a way

$$\rho_{R_m} = 1 - q^2 R_m^2/6 + \ldots\,, \tag{3}$$

where R_m is a root mean square radius of nuclear density.

Futher, let us suppose, that P_{33}-wave makes the main contribution into the scattering amplitude

$$f_{\pi N}(\vec{k}^{*\prime} \vec{k}^*) \approx b_1 \vec{k}^{*\prime} \vec{k}^*\,, \tag{4}$$

where $\vec{k}^{*\prime}$ and $\vec{k}^{*}$ are pion momenta in a pion-nucleon centre of mass system (2 CM). Then at the zero momentum transfer we can use such a transition from 2 CM to the ACM

$$f_{\pi N}(\vec{k}'\vec{k}) \approx \frac{\vec{k}}{\vec{k}^*} f_{\pi N}(\vec{k}^{*\prime}\vec{k}^*). \tag{5}$$

Substituting (4) in (2) and taking into account (5) we obtain

$$\langle\vec{k}'|V_{\pi A}|\vec{k}\rangle \approx (b_1\vec{k}\vec{k}^*)\rho_{R_m^*}(q^2), \tag{6}$$

where

$$R^{*2} = R_m^2 + 3/k^{*2}, \tag{7}$$

R^* is an effective radius of pion-nuclear interaction. It is significant that R^* is energy dependent and in our approximations we can rather simply tie together R^* and R_m. The question arises: in which way to extract the value of R^* from the data.

We propose to use a well known phenomenological expression for the pion-nucleus scattering amplitude, which has been proposed by C.Wilkin [16]

$$f_{\pi A}(\theta) = \frac{f(0)}{k}\exp\left(\frac{-R_0^2q^2}{6}\right)\prod_{i=1}^{N}\frac{(x-x_i)}{(1-x_i)}, \tag{8}$$

where $f(0)$ is the amplitude for the scattering to the zero angle, $x=\cos\theta$ and x_i are complex parameters, the number of which, N, depends on the number of the minima in a differential cross section.

This expression is rather convenient for the analysis of pion-nuclear elastic scattering because it reflects the main features of the experimental results: the exponential simulates a forward peak in $d\sigma/d\Omega$, $\mathrm{Re}\,x_i$ is connected with the position of the minimum and $\mathrm{Im}\,x_i$ is connected with the value of $d\sigma/d\Omega$ in the minimum.

It is rather easy to show that at $q^2 \to 0$

$$f_{\pi A}(\theta) \underset{q^2\to 0}{\approx} \frac{f(0)}{k}\exp(-R_{eff}^2\, q^2/6), \tag{9}$$

where

$$R^2_{eff} = R^2_0 + \frac{3}{k^2} \sum_{i=1}^{N} Re\left(\frac{1}{1-x_i}\right). \qquad (10)$$

Our main assumption is that we suppose $R^* = R_{eff}$. Then to obtain the matter radius it is necessary to approximate the experimentally found values of R_{eff} by a straight line in a variable of $1/k^{*2}$. As is seen from (7) an extrapolation to zero gives us the value of R_m.

3. RESULTS AND DISCUSSIONS

We have been fitting the experimental total and differential cross sections for $\pi^{\pm}\,{}^3He$ and $\pi^{\pm}\,{}^4He$ scattering [8-12] by equation (8). The free parameters are $f(0)$, R_0 and x_i. We take $N = 1$ because there is only one minimum in the $\pi\,{}^3He$ differential cross section.

Table 1 contains the results of the fit for the values of R_0, R_{eff} and x_i. For the transition from ACM to 2 CM we have used the method proposed by R.Landau et al. [17] and have chosen the energy ω^2_{2CM} in such a form:

$$\omega^2_{2CM} = [E_\pi(k) + E_N(\frac{k}{A})]^2 - k^2(1 - \frac{1}{A})^2 ,$$

where $E_\pi(k)$ and $E_N(k)$ are total energies of pion and nucleon, k is the momentum of pion in the ACM.

Figure 1 shows the dependence R^2_{eff} from $1/k^{*2}$. Table 2 contains the results of the fit for R^2_{eff} by the equation

$$R^2_{eff} = R^2_m + C/k^{*2}. \qquad (11)$$

It is seen that χ^2 is small and we may draw a straight line with confidence. However we obtain too big values for R_m. Even a charge radius of ${}^3He$ equals $R_{ch} = 1.87$ fm.

Maybe the reason is in the fact that equation (7), which connects R^* with R_{eff}, was obtained on the basis of a rather crude approximation on the dominancy of the P_{33}-wave in the πN-scattering (though, the approximation is valid in the Δ_{33}-region). If we take into account in (4) the S-wave and higher partial waves than though their contribution into the πN-scattering is small, the dependence between R^* and R_{eff} will be

Table 1

The results of the fit for parameters in formula (8) for $\pi^{\pm}\,{}^3He$ scattering.

Sign of pion	E MeV	R fm	Re x_1	Im x_1	R^2_{eff} fm^2
+	68	1.68 ± 0.23	0.35 ± 0.05	0.27 ± 0.08	10.23±1.4
-	68	2.17 ± 0.15	0.16 ± 0.06	0.28 ± 0.05	10.82±1.0
+	98	1.77 ± 0.04	0.27 ± 0.02	0.20 ± 0.02	7.86±0.23
-	98	1.80 ± 0.07	0.22 ± 0.04	0.35 ± 0.03	7.20±0.41
+	120	1.84 ± 0.05	0.27 ± 0.02	0.22 ± 0.03	7.03±0.33
-	120	1.83 ± 0.12	0.23 ± 0.08	0.34 ± 0.07	6.49±0.62
+	135	1.93 ± 0.08	0.35 ± 0.06	0.26 ± 0.05	7.05±0.40
-	135	1.87 ± 0.10	0.27 ± 0.08	0.37 ± 0.05	6.24±0.60
+	145	1.93 ± 0.08	0.35 ± 0.03	0.21 ± 0.04	6.89±0.42
-	145	1.93 ± 0.13	0.32 ± 0.05	0.44 ± 0.10	6.10±0.81
+	156	1.88 ± 0.06	0.37 ± 0.03	0.23 ± 0.04	6.46±0.30
-	156	1.90 ± 0.08	0.33 ± 0.04	0.31 ± 0.05	6.17±0.46
-	180	1.89 ± 0.06	0.33 ± 0.03	0.22 ± 0.03	5.92±0.26
-	195	1.88 ± 0.08	0.34 ± 0.03	0.27 ± 0.04	5.58±0.34
-	208	1.89 ± 0.11	0.37 ± 0.07	0.26 ± 0.08	5.50±0.51

Table 2

The values for matter radius R_m are obtained from the analysis of $\pi^{\pm}\,{}^3He$ and $\pi^{\pm}\,{}^4He$ scattering.

Sign of pion	Target	χ^2/NDF	R_m fm	C	Reference
-	^3He	0.5	1.99 ± 0.11	2.22 ± 0.41	/12/
+	^3He	0.2	2.14 ± 0.15	2.06 ± 0.47	/12/
-	^4He	11.	1.92 ± 0.02	2.32 ± 0.07	/8,10,11/
+	^4He	9.	1.94 ± 0.02	2.30 ± 0.09	/8,10,11/
-	^4He	3.4	1.58 ± 0.04	2.87 ± 0.06	/9/

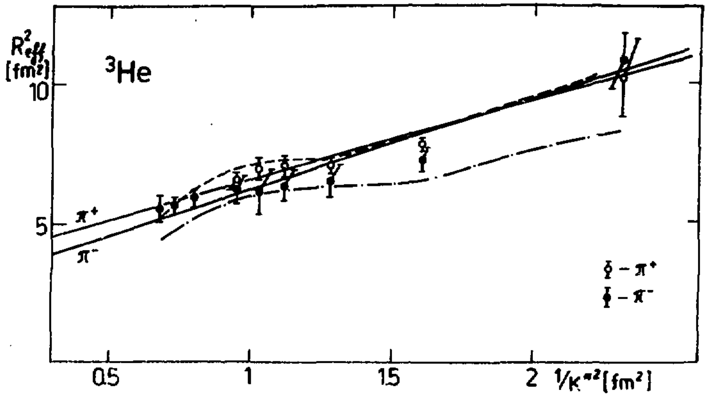

Fig.1. The dependence of R_{eff} on the $1/k^{*2}$ for the scattering of $\pi^{\pm}$-mesons on ^3He. Straight lines are the result of a fit by the formula (11). Dashed (π^+) and dot-dashed (π^-) lines show the R_{eff} from the analysis of theoretical distributions obtained by the optical model.

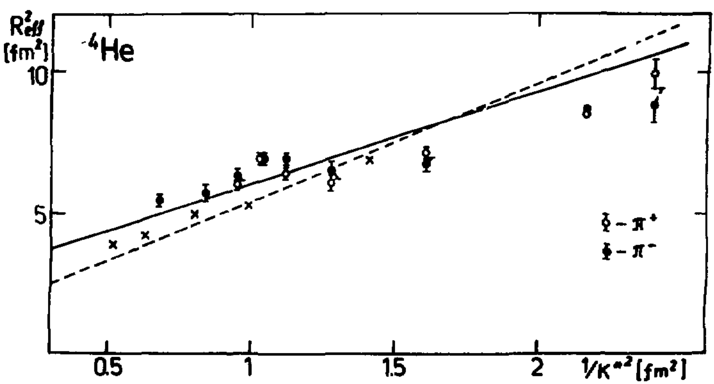

Fig.2. The dependence of R_{eff} on the $1/k^{*2}$ for the scattering of $\pi^{\pm}$-mesons on ^4He. Full line is the result of the fit by only the data of /8,10,11/ . Dashed line is the fit by the data of /9/ (crosses).

changed and, probably, the accuracy of the extraction of the R_m will be improved, too. Nevertheless it must be recognized that the value of R_m makes sense though the forementioned effect have not been taken into account. It must be also kept in mind that the accuracy of measurements of pion-^3He differential cross sections is up to now rather less than the accuracy in electron-^3He scattering.

An interesting situation arises in the analysis of the energy dependence of R_{eff} for π^4He scattering. Figure 2 shows the results obtained from the data of Shcherbakov et al.[8,10,11], Dubna (circles) and Binon et al.[9], CERN (crosses).

It is clearly seen that there is a discrepancy between the results of these experiments in the region of $E_\pi > 100$ MeV ($1/k^{*2} < 1.6$). The behaviour of R_{eff} calculated according to the Dubna data is not linear, it is clear from Figure 2 and from the results in Table 2. If we take only the data from CERN[9], then the value of χ^2 is quite acceptable and we obtain for $R_m = 1.57$ fm that is quite close to the value of matter radius of ^4He measured in $e\,^4He$ scattering - $R_m =$ $= 1.47$ fm.

The reason for the discrepancy may be in different accuracy of these experiments. When we analysed data[9], we took N in equation (10) equal to 2 at $T_\pi > 110$ MeV. In case of the Dubna data we cannot introduce in the same manner two additional parameters because of the scarcity of experimental points. Besides, some systematical errors can exist, too.

However it must be noted that in ref.[9] there is some inaccuracy in the analysis of the energy dependence of R_{eff}. In equation (11) they used the momentum k in a pion-nuclear system. But then it is impossible to understand the sense of R_m in (11), because under such condition it does not connect this value with the matter radius of a nucleus and it is impossible to make references to the theory of an effective radius[14,15] like it has been done in[9]. According to this theory the effective radius is connected with R_m just through k^*, the momentum in the 2 CM system.

It is interesting to compare our data with results of other experiments and theoretical predictions.

Figure 3 shows the values of mean radius $\bar{R} = (R_m^{\pi^+} + R_m^{\pi^-})/2$ from the elastic scattering of pions on different nuclei. Our data is compared with the results of Corfu et al.[18], who have used eguation (1) for the extraction of the radius. As has been expected R_m increases as $A^{1/3}$. Our data are in

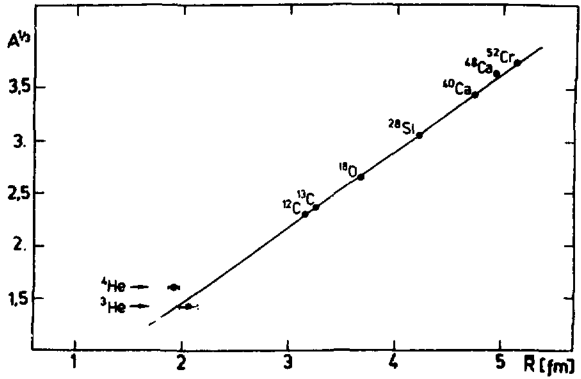

Fig.3. The dependence of the mean radius $\bar{R}$ for different nuclei, measured in the elastic πA-scattering. Data for $A > 4$ are cited from ref. /18/.

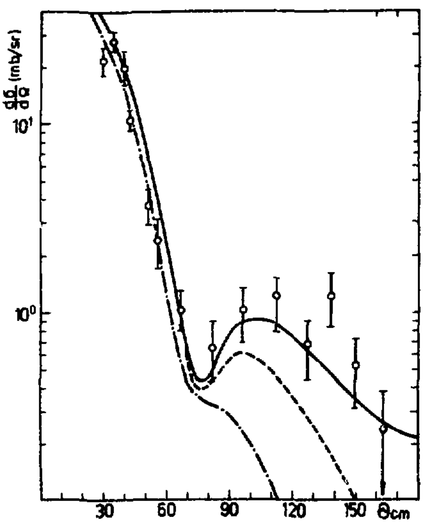

Fig.4. Differential cross sections of elastic π^+ scattering on ^{3}He at E_π = = 145 MeV for different charge radii. Full line - R_{ch} = 1.87 fm, dashed line - R_{ch} = 2.02 fm, dot-dashed line - R_{ch} = 2.13 fm.

agreement with the dependence, expect for the radius for ^{4}He but this radius does not obey this dependence.

To compare the behaviour of R_{eff} from the experiment with the theoretical predictions we have calculated the differential cross section for π^3He scattering in the frame of the optical model with a first order potential in momentum space. The program for calculations has been done by R.Mach and M.Gmitro [8,19]. The theoretical values for $d\sigma/d\Omega$ have been fitted by equation (8) and the values of R_{eff} were calculated, too. Figure 1 shows the corresponding curves. Dashed line corresponds to π^+ and dot-dashed line corresponds to π^- ^{3}He scattering. As it is seen from the behaviour of the theoretical R_{eff} there is really a linear dependence in some energy region. Therefore, simple formula (7) reflects to a large extent the characteristic features of behaviour of R_{eff} for π ^{3}He scattering.

Along with this we have checked the dependence $d\sigma/d\Omega$ on the radius of the nucleus used in calculations. Figure 4 shows the differential cross sections of the π^+ ^{3}He scattering at 145 MeV for different values of charge radius. It is seen that when R_{ch} increases the backward cross section decreases. Even the 10% variation of the radius changes significantly the character of the differential cross section. This is in accordance with the conclusion of ref. [6] that the elastic pion scattering on nuclei is largely sensitive to the changing of the root mean square radius of a nucleus.

4. CONCLUSION

Thus the method which we propose for connecting a matter radius of a nucleus with experimental values measured in the elastic pion-nucleus scattering leads to quite reasonable results. As it has turned out from the comparison with the optical model, a simple linear dependence (11) reflects the essential features of energy behaviour of R_{eff}. Therefore, in our opinion, there is every reason to develop such an approach further.

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