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COULOMB EFFECTS IN THE $^{3}He \rightarrow D+P$ VERTEX

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In nuclear physics it has been attempted to extract the strength of the nearest singularity to the physical region in the $\cos\theta$ plane at fixed energy from differential cross section data for various transfer processes [1-6]. In ref. [5] the Coulomb interaction in the vertices was neglected, though it changes even the character of the transfer pole singularity [9]. In this paper we determine the $h \to d + p$ vertex constant ($h = {}^3He$) taking into account the Coulomb interaction in the vertex. We also present new results concerning the $t \to d + n$ vertex constant.

As this paper is a direct continuation of our earlier paper [6], the reader is referred to it as concerning the general background of the subject. Here we give only a grossly simplified description of the singularity subtraction method.

Usually in the $z = \cos\theta$ plane the pole singularity is nearer to the physical region than the other singularities are. Therefore the pole determines the asymptotics of the expansion coefficients of the differential cross section according to a set of regular functions. Using this fact one can determine the strength of the pole singularity as follows. First of all, one should remove the pole from the differential cross section by a suitably chosen factor and by the least squares procedure fit it according to some set of polynomials. These polynomials might be the well known orthogonal polynomials $B_n(z)$, which are orthogonal with respect to the weights of the least squares procedure. Then one has:

$$(z_p - z)^2 \frac{d\sigma}{d\Omega} = \sum_{n=1}^{N+1} a_n B_n(z), \qquad (1)$$

where only the significant terms are included into the sum. If one removes only the interference term of the pole with the background, then one has:

$$(z_p - z)\frac{d\sigma}{d\Omega} = \frac{P}{z_p - z} + \sum_{n=1}^{N} b_n B_n(z) = \sum_n A_n B_n(z). \qquad (2)$$

Here the $B_n(z)$ polynomials differ from that in (1), but this is of no importance. From (2) it follows that if one analyzes $(z_p - z)d\sigma/d\Omega - P/(z_p - z)$ with a guess strength P , then at the correct strength the A_n coefficients with $n > N$ become insignificant. To increase the effectiveness of the method all these are carried out not in the $z = \cos\theta$ plane, but in a new variable received by the so-called optimal conformal mapping.

The problem of the Coulomb interaction in the vertices was studied in refs.[7-9] (see also[14]). In the presence of the Coulomb interaction the vertex function has a branch point singularity at the relative momentum of the decay products which corresponds to their binding energy. As a consequence, the "pole graph" which describes the transfer mechanism, ceases to have a pole singularity as its proper one. Using the formulae of ref. [9] it is easy to determine the behaviour of the differential cross section around the singularity:

$$(z_p - z)^2 \frac{d\sigma}{d\Omega} \to \frac{5}{8\pi^2} \cdot \frac{m_a^2 c^4}{E_i \cdot E_f} \frac{k_f}{k_i} \frac{2J_B + 1}{(2J_A + 1)(2J_a + 1)} G_x^2 G_B^2 f(z), \qquad (3)$$

where we consider the $A + x \to B + y$ $(x = a + y, B = A + a)$ transfer process with zero orbital momentum in both vertices. $E_i(E_f)$ and $k_i(k_f)$ are the CM relative kinetic energy and momentum in the initial (final) state; m and J denote the mass and spin of the corresponding particle; G_x^2 and G_B^2 are the vertex constants containing information on the structure of particles; and $f(z)$ contains all the corrections due to the Coulomb interaction in the vertices:

$$f(z) = \Gamma^2(1-\eta_x)\left(\frac{m_y}{2m_x\kappa_x}\frac{k_i k_f}{2}(z_p - z)\right)^{\eta_x} \Gamma^2(1-\eta_B)\left(\frac{m_A k_i k_f}{2m_B\kappa_B^2}(z_p - z)\right)^{\eta_B}. \qquad (4)$$

Here i κ is the wave number corresponding to the binding energy, η is the well known Coulomb parameter ($\eta > 0$). The G vertex constant has the same connection with the asymptotic behaviour of the bound state wave function as earlier, but the undisturbed Hankel function ($e^{-\kappa r}/r$) should be replaced by the corresponding Coulomb distorted wave function /9/.

At $z = z_p$ one has a branch point singularity instead of a pole. This effect can easily be accounted for if in formulae (2) and (1) one multiplies by $(z_p - z)^{1-(\eta_x + \eta_B)/2}$ instead of $z_p - z$ (or by some other expression corresponding to it).

There is no other modification needed. At $\eta_x = \eta_B = 0$ one gets the formulae of ref. /6/ back. As finite order expansion coefficients of the differential cross section are stable against small changes in η_x and η_B, therefore at "small" Coulomb parameters it is not crucial to include vertex Coulomb effects into the subtraction method. On the other hand, in the continuation method, in which one explicitly extrapolates up to the "pole", where the differntial cross section is unstable, it is of vital importance to include this effect, at least in principle. In practice the numerical results are not altered strongly, because one necessarily works with finite order expansion coefficients, which are stable. Taking into account Coulomb effects in the way described above we have analysed the experimental data of ref./12/ on the d(d,n)h reaction with the subtraction method /6/. The A_n coefficients with $n = 4,5$ at E_d=13.6, 12.2, 10.4 MeV and with $n = 4$ at E_d = 8.15, 5.8 MeV could be used for the determination of $G_d^2 G_h^2$. The results are presented in Table 1, the errors contain an estimated normalization error of 3%. The weighted average is $G_d^2 G_h^2 = 0.429 \pm 0.013\ f^2$. The scattering of the results about their average was slightly less than it follows from their errors. Neglecting the Coulomb effects in the h vertex one gets 2.5-5% higher results depending on E_d.

We have also analysed the experimental data of ref./12/ on the d(d,p)t reaction. In this case at all energies (E_d = 13.8, 12.15, 8.1 and 6.1 MeV) the A_n coefficients with $n = 4,5$ could be used for the determination of

$G_d^2 G_t^2$. These results are also presented in Table 1. The errors contain a normalization error of 3%. The weighted average (together with our earlier $E_d = 25$ MeV results /6/) is $G_d^2 G_t^2 = 0.530 \pm 0.015 \, f^2$. The scattering of the results about their average is two times larger than it follows from their errors, therefore we adopted a two times larger error for the average too. As this large scattering is not present in the h vertex results, the reason of it might be that in the experiment it was more difficult to detect protons with emulsion technique (!) than helions. In addition, the protons were detected at larger and smaller angles than helions and this also might have introduce other systematical errors. It is possible that only the lower energy results are good, as it follows from the comparison with the $E_d = 25$ MeV result. In spite of it, one should think of this pioneer measurement in the field of nuclear reaction with great respect. We notice that all these are fine effects, the final error is less than 3%, and due to the normalization error, one could not get better results. The continuation method gave similar results, though there were larger deviations than in the $E_d = 25$ MeV case.

We do not maintain our statement that the results of the subtraction method are necessarily free of any systematical error /6/. The point is that when one determines the value of N from formula (1), the expansion series have no necessarily N+1 significant terms, because "by chance" the expansion coefficients of the background contribution in formula (2) (i.e., the b_n coefficients) might obey the same recurrence relation (within their error, of course) as the pole contribution coefficients do *. It means that some model assumption enters the method.

One should especially be aware of it if the background singularities lie near the pole and if one is able to extract only the lowest order coefficients from the

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* Actually this recurrence relation assures that in formula (1) the pole contribution is present only in the constant and linear terms of the expansion.

Table 1
Results for the strength of the transfer pole singularity

E_d	$G_d^2 G_h^2$		$G_d^2 G_t^2$	
	n = 4	n = 5	n = 4	n = 5
25.3			0.494±0.014	0.501±0.041
13.7	0.438±0.025	0.473±0.163	0.549±0.019	0.665±0.060
12.2	0.417±0.024	0.365±0.174	0.564±0.020	0.559±0.066
10.4	0.436±0.029	0.533±0.215		
8.1	0.436±0.034	-	0.533±0.21	0.706±0.128
6.1			0.492±0.022	0.433±0.173
5.8	0.408±0.044	-		

experimental data due to their large errors. These circumstances "help" the background coefficients to obey the recurrence relation of the pole coefficients. We do not think that the large scattering of the vertex results is caused by this effect, because i) neither of these unfavourable circumstances is present in our case and ii) the large scattering is not present in the h vertex results, where similar physics is involved. On the whole, the errors given by us seem to be reliable, though some caution is needed.

If one assumes $G_d^2 = 0.43 \pm 0.01f$, which follows from the effective range formula as well as from the asymptotic behaviour of the calculated deuteron wave functions for various potentials, then our results are: $G_t^2 = 1.23 \pm 0.04 f$ for the $t \to d+n$ vertex constant and $G_h^2 = 1.00 \pm 0.04 f$ for the $h \to d+p$ vertex constant. The ratio of the two constants is $G_t^2/G_h^2 = 1.23 \pm 0.05$.

Our results are in full agreement with our determination of these constants /10/ with the peripheral model and are in good agreement with the later results of other methods (for a review see refs. /11,13/). By similar methods from $n-t$ scattering data the triton vertex constant was determined in ref. /2/ , while the helion vertex constant was extracted in ref. /5/ from $p-h$ scattering data (Coulomb effects were neglected), but as we have a far more favourable physical situation, our results have smaller errors. We also notice that in ref. /14/ a similar difference was found between the $\alpha \to t+p$ and the $\alpha \to h+n$ vertex constants by the extrapolation of the α -nucleon elastic scattering amplitude.

We have demonstrated the applicability of the singularity subtraction method (and that of the continuation method too) for nuclear reactions (the other applications /1-5/ were to elastic scattering processes). There are only minor physical differences between the $d(d,p)t$, $d(d,n)h$ reactions and reactions like (d,p) , (h,d) on light and heavy nuclei, therefore such methods could be usefull in this field too. Such calculations are in progress.

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