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J.Bang, F.A.Gareev, R.M.Jamalejev

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J.Bang,* F.A.Gareev, R.M.Jamalejev

WAVE FUNCTIONS AND PARTICLE TRANSFER FORM FACTORS OF ^{42}Ca AND ^{18}O

* The Niels Bohr Institute, Copenhagen, Denmark.

1. Introduction

A number of authors[1] have treated the problem of a few nucleons outside a closed shell. In most of these calculations, shell model wave functions were used, in a restricted sense of the words. The Hamiltonian of these nucleons was divided into a shell model part, describing, hopefully, the interactions of the particles with the closed shell core, together with the kinetic terms, and a residual part describing the interactions between the particles.

Energies and wave functions are now found by diagonalizing the total Hamiltonian in a basis of bound state eigensolutions of the shell model part.

Inclusion of core excitation terms in the above Hamiltonian presents a number of problems, but we shall here neglect these and concentrate on another type of deficiency of the above approach. What we have in mind is that the products of bound single particle state wave functions do not form a set, which is closed with respect to operation of the residual interactions.

One very consistent attempt to overcome this difficulty was made by Ibarra and Bayman[2] who calculated 0^+ states of ^{42}Ca and ^{58}Ni, including a part of the states of the continuous spectrum of the shell model Hamiltonian in the basis.

A main result of this work was, that in these cases the energies of the states were not very much shifted by including the continuum ($\sim$ 0.15 MeV), but the cross section of one- and

two-particle transfer were changed considerably. This is in agreement with the fact, that the restricted shell model basis is particularly poor in the surface region of the nucleus, whereas the main contributions to nucleon transfer amplitudes come just from that region.

The method of Ibarra and Bayman consists of dividing the continuum into energy intervals and treating the wave function of each interval as being a discrete state with energy, say, as the mean value in the interval. These, together with the bound states, are then used as basis of diagonalization.

This procedure has a number of disadvantages. First, it is very tedious. Continuum energies up to about 300 MeV and intervals of the order of magnitude 10 MeV seemed to be necessary to obtain sufficiently accurate solutions in the desired region of space. This means that already when states, where one particle is in the continuum, are included, the rank of the matrices to be diagonalized is several hundreds.

From this, also another disadvantage is clear, namely that it is practically excluded to take such state into account, where both particles are in the continuum.

In a later work[2a], Ibarra has included also such states, however in a perturbative way only.

The problem of getting a complete basis, convenient for nuclear problems, was met in other connections[3] and solved by introducing the Sturmian functions.

It seems that also in the present context these functions

represent a practical tool, which will permit us to overcome the difficulties mentioned in the description of the work of Ibarra and Bayman.

2. Theory

With the method suggested below, calculations of eigenstates for systems with a number of particles outside a closed shell seem within reach. Since our calculations are restricted to the two particle case, we shall, however, write formulae only for this, generalizations to more particles being straightforward.

We shall furthermore restrict ourselves to the case that the interactions between the particles and the closed shell are described by a potential, and the core states are pure shell model states of the same potential.

Our two-particle states are solutions of the Schrödinger equation

$$\left[-\frac{\hbar^2}{2m}(\Delta_1+\Delta_2) + V(r_1) + V(r_2) + V_{12}(|\underline{r}_1-\underline{r}_2|) - E\right]\Phi = 0 \qquad (1)$$

with the condition of being antisymmetric in particles 1,2 as well as with respect to the particles of the core states.

Neglecting for a moment the latter conditions, we try to solve (1) by an expansion

$$\Phi(r_1,r_2) = \sum_{i_1,i_2} C_{i_1,i_2}\,\varphi_{i_1}(\underline{r}_1)\,\varphi_{i_2}(\underline{r}_2) \qquad (C_{i_1,i_2} = -C_{i_2,i_1}), \qquad (2)$$

where

$$0 = \left[-\frac{\hbar^2}{2m}\Delta_{1,2} + \lambda_{i_1,i_2} V(\underline{r}_{1,2}) - E_{1,2}\right]\varphi_{i_1,i_2}(\underline{r}_{1,2}). \quad (3)$$

The indices i_1, i_2 correspond to the single-particle quantum numbers $n\ell j$.

The energies E_1, E_2 may here in principle be chosen freely, since in any case the functions ϕ_i form a set which is complete in $\mathcal{L}^2$-space. The choice of $E_{1,2}$ may then be guided by the desire for a good description of any particular component of the wave function, i.e. good convergence of (2) in some particular part of configuration space. Since in practice one always uses a truncated basis, such a convergence is important.

In the present case, we are looking for comparatively small deviations from the simple bound state shell model wave functions. This seems clear from the results of ref.2) and a possible choice is therefore that for each value of ℓ, j

$$E_1 = E_2 = E_{\ell j}\,, \quad (4a)$$

where $E_{\ell j}$ is determined in such a way that when V in (3) is the shell model potential V_s which gives the best fit to the energies of ^{42}Ca, $\lambda_{n\ell j} = 1$, where $n\ell j$ are the quantum numbers of the four unoccupied states in ^{41}Ca. This means that some components in (2), which can be assumed to be the main components, are completely identical to shell model components.

Such a choice will in general be possible in spherical nuclei, where unoccupied bound single particle states, differing only in n, do not occur. The choice of $E_{1,2}$ for such states, where ℓ, j is not represented among the shell model states, is of course still left open, it was here chosen according to the next suggestion.

Another possible choice of energies, which is commonly used when two-particle form factors are calculated, is

$$E_1 = E_2 = \frac{E}{2}\,, \quad (4b)$$

where E is the two-particle binding energy in ^{42}Ca.

The third possibility is to take E_1 as the binding energy of some state in ^{41}Ca

$$E_2 = E - E_1\,. \quad (4c)$$

This choice gives the most realistic form factors for one-particle transfer. For two-particle transfer, calculations were made with all three choices. With the present truncation, they lead to cross sections, which differ by less than 5%.

When nothing else is mentioned, the results given in tables and figures correspond to the choice (4c).

For the functions ϕ_i we have the orthogonality, which can also be used for normalization

$$\int \varphi_i V \varphi_j \, d\tau = \delta_{ij}\,. \quad (5)$$

Inserting (2) in (1), multiplying from left with $\phi^*_{i_1'}(r_1)\phi^*_{i_2'}(r_2)V(r_1)V(r_2)$ and using (3), (4) and (5), we obtain, after integration over $\underline{r}_1$ and $\underline{r}_2$

$$\sum_{i_1 i_2} c_{i_1 i_2} \, U_{i_1 i_2, i_1' i_2'} - c_{i_1' i_2'}(E - E_{i_1'} - E_{i_2'}) = 0, \quad (6)$$

where

$$U_{i_1 i_2 i_1' i_2'} = \int\int \varphi^*_{i_1'} \varphi^*_{i_2'} V_1 V_2 \left(V_1 (1 - \lambda_{m_1 \ell_1 j_1}) V_2 (1 - \lambda_{m_2 \ell_2 j_2}) + V_{12} \right) \varphi_{i_1} \varphi_{i_2} \, d\tau_1 \, d\tau_2 \quad (7)$$

The index i, still stands for $n\,\ell\,j\,m$ but we are in the following restricting ourselves to the interesting case of 0^+ states. Here, the expansion (2) reduces to

$$\Phi(1,2) = \sum_{\ell j n_1 n_2} A_{\ell j n_1 n_2} \left[\varphi_{n_1 \ell j}(\underline{r}_1) \varphi_{n_2 \ell j}(\underline{r}_2) \right]_{0^+} \quad (8)$$

with the antisymmetry requiring $A_{\ell j n_1 n_2} = A_{\ell j n_2 n_1}$.

From equations (6), (7) E and $\underline{A}$ are found by diagonalizing the unsymmetric matrix

$$U^{+}_{i_1 i_2 i_1' i_2'} (E_{i_1} + E_{i_2}) \, \delta_{i_1 i_1'} \, \delta_{i_2 i_2'} .$$

Still, however, we are faced with the problem of obtaining a wave function which is antisymmetric in the coordinates of all particles.

In conventional shell model calculations, this problem is solved by assuming that the particles in closed shells are

occupying single particle states corresponding to the same potential as that of the unfilled shell. Then the space in which the two-particle wave function is expanded is easily chosen to be orthogonal to that of the closed shells, and the Pauli principle is automatically taken into account.

In such a procedure the basic symmetry of the Hamiltonian for exchange of all particles is obviously neglected. However, just for the closed shell particles, the neglect of residual interactions may be a good approximation in this connection, and we shall do the same here.

So in addition to equations (6), we shall require that our wave functions are orthogonal to the occupied shell model states, again to be calculated with the potential V.

Denoting the occupied states $\psi_1, \psi_2 \ldots\ldots \psi_N$, the orthogonality in question means $2N$ equations -

$$\int \Phi(\underline{r}_1, \underline{r}_2) \psi_m(\underline{r}_1) \, d\tau_1 = \int \Phi(\underline{r}_1, \underline{r}_2) \psi_m(\underline{r}_2) \, d\tau_2 = 0 . \quad (9)$$

When symmetries are taken into account, the number of equations reduces; for the general case we have, however, introducing (2) in (9)

$$\sum_{i_1 i_2} c_{i_1 i_2} \langle \varphi_{i_1}, \psi_m \rangle \varphi_{i_2}(\underline{r}_2) = 0 , \quad (10a)$$

(N equations)

$$\sum_{i_1 i_2} c_{i_1 i_2} \langle \varphi_{i_2}, \psi_m \rangle \varphi_{i_1}(\underline{r}_1) = 0 . \quad (10b)$$

(N equations)

Multiplying (10a) with $\phi_i(\underline{r}_2)V(\underline{r}_2)$ and integrating over $\underline{r}_2$ we obtain the equations

$$\sum_{i_1} c_{i_1,i} \langle \varphi_{i_1}, \psi_m \rangle = 0 \qquad (11a)$$

correspondingly we get from (10b)

$$\sum_{i_2} c_{i,i_2} \langle \varphi_{i_2}, \psi_m \rangle = 0 \, . \qquad (11b)$$

Now, in practical calculations, the basis $\{\phi_i\}$ must of course always be truncated. If we include M values of i_1 and i_2, (6) represents M^2 equations, which are to be solved with the 2 x N x M subsidiary conditions (11). (But both contain fewer equations, when symmetries are taken into account).

A very convenient tool for this is the following 4). Let us, for clarity, introduce a briefer notation in (6)

$$\sum_q H_{pq} c_q - E c_p = 0 \, . \qquad (6^+)$$

Correspondingly any of the equations (11) may be written

$$\sum_q h^s_q c_q = 0 \, . \qquad (11^+)$$

Now (6^+) is replaced by

$$\sum_q (H_{pq} + t H'_{pq}) c_q = E c_p \, , \qquad (6^+a)$$

where

$$H'_{pq} \equiv h^s_p h^s_q \qquad (12)$$

and we find the eigenvalues and eigenvectors of (6^+a) for $t \to \infty$. One of these eigenvalues corresponds to tH' and will go to infinity with t. The other eigenvalues and eigenvectors in this limit are solutions of our problem (6^+) with the conditions (11^+).

If a number of such conditions are introduced, M' must of course be replaced by

$$H'_{pq} = \sum_s h^s_p h^s_q \, . \qquad (13)$$

Returning to our previous notation, we easily see that

$$H'_{i_1,i_2,i'_1,i'_2} = \sum_m \left(\langle \varphi_{i_1}, \psi_m \rangle \langle \psi_m, \varphi_{i'_1} \rangle \delta_{i_2 i'_2} + \langle \varphi_{i_2}, \psi_m \rangle \langle \psi_m, \varphi_{i'_2} \rangle \delta_{i_1 i'_1} \right) \, . \qquad (14)$$

Table 1 shows the dependence of the A^is on t for the case treated below (^{42}Ca). In this case, the occupied states in question are $1p_{1/2}$ and $1p_{3/2}$. Apart from the convergence with t the table shows that the coefficients of the corresponding Sturmian wave functions ($1p_{1/2}$ and $1p_{3/2}$) become very small in the orthogonalized state. This is of course due to the great similarity between shell model and Sturmian wave functions with the same quantum numbers, and it shows that a very good approximation to the orthogonalization is obtained by simply leaving out the Sturmian states corresponding to the occupied ones in the expansion (2).

Table 1

Energy and configuration mixing coefficients as functions of the parameter t(see text).

	t = 1	t = 10	$t = 10^2$	$t = 10^3$	$t = 10^4$	without $1p_{1/2}$ and $1p_{3/2}$
E	-19.200	-19.942	-20.367	-20.199	-20.198	-20.194
configuration						
$(1p_{1/2}^2)^{0+}$	-.0262	-.0359	.0238	.0019	.0019	
$(1p_{1/2}2p_{1/2})^{0+}$	-.0170	-.0461	.000	-.0014	-.0014	
$(2p_{1/2}1p_{1/2})^{0+}$	-.0159	-.0142	-.0091	-.0129	-.0129	
$(2p_{1/2}^2)^{0+}$	.0264	.0265	.0343	.0305	.0305	.0309
$(1p_{3/2}^2)^{0+}$	-.0570	-.0696	.0558	.0030	.0030	
$(1p_{3/2}2p_{3/2})^{0+}$	-.0091	-.0106	-.0032	-.0034	-.0034	
$(2p_{3/2}1p_{3/2})^{0+}$	-.0089	-.0098	.0072	-.0008	-.0008	
$(2p_{3/2}^2)^{0+}$	.0707	.0699	.0884	.0799	.0798	.0794
$(1f_{5/2}^2)^{0+}$	.0689	.0626	.0843	.0764	.0764	.0762
$(1f_{7/2}^2)^{0+}$	.9935	.9928	.9896	.9930	.993	.993

3. Calculation of wave functions

Since we were interested in comparing our results to those of Ibarra and Bayman[2] we have been sticking to the potential and interaction used in that reference, that is

$$V = V_0\left(\frac{1}{1+\exp\frac{r-R}{a}} + \text{spin-orbit term}\right), \qquad (15)$$

$$V_{12} = V_{12}^0 \exp\left(-|r_1 - r_2|^2/\sigma^2\right),$$

$R = 1.25\,A^{\frac{1}{3}}$ fm, $a = 0.65$ fm.

The integrals in (7) fall in two parts. The first, containing only the spherical potentials $V(r_1)$ and $V(r_2)$ reduces to a simple product of integrals over radial wave functions. The second part, containing V_{12} was calculated by the method of Bayman and Kallio[5], modified by the presence of the factor $V_l V_L$[6], and taking into account not only states, but also other components of the relative motion.

3a $^{40}Ca(t,p)^{42}Ca(0_1^+$ (g.s.) and $0_3^+)$

The wave functions of ^4Ca(g.s) were calculated with $E_1 = E_2 = -9.9$ MeV for the basis functions. This is half the separation energy of two neutrons in ^{42}Ca, so when we limit our basis to the quantum numbers of the shell model states $(1f_{1/2}, 1f_{5/2}, 2p_{3/2}, 2p_{1/2})$ we obtain the usual WDP form factor, however with the modification that, in the usual WDP, the coefficients of the different components are taken from shell model calculations, whereas here they result from a consistent diagonalization in the Sturmian basis.

These two sets of coefficients which therefore in principle are different are compared in table 2. It is seen that the difference is rather small, except for the smallest components. This is, however, a somewhat special feature of the type of state under consideration and should not be expected for other nuclei or at other energies.

Eigenvalues and coefficients in a more complete Sturmian basis for ^{42}Ca are given in table 3. The parameters were chosen as (2) $V_o = -54.63$ MeV, $V_{12}^o = -43.5$ MeV, $\sigma = 1.5$ fm. In the table only the larger A- coefficients are given.

It is seen that, as in the previous calculations 2), 7) each state contains a dominating shell model component, and small admixtures of other components. Among those admixtures, again, the bound states are dominating, but other components are not negligible.

The components corresponding to two particles in the continuum are seen to be of comparable size as those where one particle is occupying a bound state. This is not unexpected from the structure of U, and the neglect of such components in ref 2) seems to represent a relatively crude approximation.

The smallness of <u>all</u> admixtures to the shell model states, seen in these calculations, is of course a special feature of the chosen configuration and residual interaction, and could not be expected in other nuclei or in excited states.

The present method avoids the problems of ref. 2), concern-

Table 2

Configuration - mixing amplitudes in ^{42}Ca(g.s.)

configuration	$(1f_{7/2}^2)^{0+}$	$(1f_{5/2}^2)^{0+}$	$(2p_{3/2}^2)^{0+}$	$(2p_{1/2}^2)^{0+}$
ordinary shell model	0.989	0.083	0.102	0.041
WDP in Sturmian method	0.993	0.076	0.079	0.031

ing the energy step length and total energy interval. There is then, however, a question of the number of states in the Sturmian basis.

It was found, as seen from table 3 that a good convergence was obtained for eigenvalues as well as eigenfunctions already when the basis included functions up to $n = 2 - 3$.

The following calculations are made with the 3. variant for the choice of the energies, i.e. $E_1 = E_{Bind}(^{41}Ca)$, $E_2 = E - E_{Bind}(^{41}Ca)$.

Form factors of two-nucleon transfer can be calculated only when definite assumptions concerning reaction mechanisms and structure of the projectile are made.

We shall here limit ourselves to the model of a cluster

Table 3

Configuration mixing amplitudes in Ca^{42} (g.s.) in Sturmian basis

Configuration	$(3s^2)^{0+}$	$(2p_{\frac{1}{2}}^2)^{0+}$	$(2p_{\frac{1}{2}}3p_{\frac{1}{2}})^{0+}$	$(3p_{\frac{1}{2}}^2)^{0+}$
A jn_1n_2	0.004	.0.491	.0132	.0062
	$(2p_{\frac{1}{2}}^2)^{0+}$	$(2p_{3/2}3p_{3/2})^{0+}$	$(3p_{3/2}^2)^{0+}$	$(2d_{3/2}^2)^{0+}$
	.1170	.0286	.0123	.0058
	$(2d_{5/2}^2)^{0+}$	$(1f_{5/2}^2)^{0+}$	$(1f_{5/2}2f_{5/2})^{0+}$	$(2f_{5/2}^2)^{0+}$
	.0041	.0846	.0423	.0121
	$(1f_{7/2}^2)^{0+}$	$(1f_{7/2}2f_{7/2})^{0+}$	$(1f_{7/2}3f_{7/2})^{0+}$	$(2f_{7/2}^2)^{0+}$
	.9414	.2454	.1515	.0806
	$(2f_{7/2}3f_{7/2})^{0+}$	$(3f_{7/2}^2)^{0+}$	$(1g_{7/2}^2)^{0+}$	$(1g_{9/2}^2)^{0+}$
	.0441	.0289	.0441	.0282
	$(1h_{9/2}^2)^{0+}$	$(1h_{11/2}^2)^{0+}$		
	.0101	.0481		

transfer brought about by a zero range force between the proton and the center of mass of the two transferred neutrons. It was shown by Bayman[8] that, at least as angular distributions are concerned, there is little difference between the results obtained from this model, and those of much more elaborate calculations, using realistic interactions (but still considering the transfer as being a one step process).

In this model the form factor is now obtained as the overlap of the two-particle wave function $\phi(\underline{r}_1, \underline{r}_2)$ with the function describing the relative motion of the two neutrons in the triton.

For the internal triton wave function we used the conventional Gaussian form

$$\varphi_t = \left(\frac{216\,\alpha^6}{\pi^3}\right)^{\frac{1}{4}} \exp\left(-\tfrac{3}{8}\alpha\left(|\underline{r}_1-\underline{r}_2|^2+|\underline{r}_2-\underline{r}_3|^2+|\underline{r}_p-\underline{r}_1|^2\right)\right) \qquad (16)$$

with $\alpha = 0.24$ fm^{-2} like in ref. 2).

The overlap integral was calculated by the modified method of Bayman and Kallio[6]. The two-particle form factor for ^{42}Ca (p,t) is shown in fig. 1, where, for comparison, also the results of WDP and those obtained by using usual shell model wave function with admixture coefficients from table 1 are shown.

It is seen that the extra node, obtained in the calculations of Ibarra and Bayman is absent in the present form factor. It was explained by Ibarra and Bayman as an effect of admixtures of the type $(2p_{3/2}\ 3p_{3/2})_{0^+}$ in the wave function. It

was also argued by these authors that admixtures of the type $(3p_{3/2}\ 3p_{3/2})^{0^+}$, omitted in their calculations, would likely be too small to wipe out this structure again. The conclusion of the present calculation seems to be that these latter admixtures are nevertheless large enough to give such an effect.

The cross sections for ^{40}Ca (t,p) leading to the ground state and the 0_3^+ state at 5.85 MeV of ^{42}Ca calculated with the different form factors are shown in figs.2, 3 +. It is seen again that angular distributions are not very sensitive to the admixtures in question, but that the absolute cross sections are sensitive.

From the experimental data the ratio $\frac{\sigma(0_3^+)}{\sigma(0_1^+)} \sim 1$, whereas in our calculation we obtain ~ 9, compared to ~ 7 in WDP and 5.3 in a pure shell model. It is therefore likely that the (0_3^+) state and maybe also (0_1^+) are of a more complicated nature than the one calculated here, presumably as a result of coupling to core excitations of the deformed type 9). The amplitude of the configuration we have used here (spectroscopic amplitude) could only be ~ 0.3.

Another choice of the residual interaction might lead to larger amplitudes of $(1f_{7/2}{}^n\ f_{7/2})$ which in our calculation interferes destructively in the (0_3^+) cross section. In this case the spectroscopic amplitude might be larger.

+) The optical parameters were taken as in ref. 2).

3b $^{16}O(t,p)^{18}O(0_1^+$ (g.s.) and $0_2^+)$

In figs. 4,5 are shown the cross sections of $^{16}O(t,p)$ $^{18}O(0_1^+)$(g.s) and 0_2^+(3.6 MeV), calculated with different approximations.

The parameters of the optical potential were taken from 11), and the found state parameters were $V_o = -53.63$ MeV, $V_{12}^o = -32$ MeV, $\sigma = 1.8$ and else the same as in the ^{42}Ca case.

The results are surprisingly similar to those from Ca. The ratio $\frac{\sigma(0_2^+)}{\sigma(0_1^+)}$ comes out to be ~ 1, whereas the experimental ratio is only ~ 0.03 12), so in this case the spectroscopic amplitude in question should be ~ 0.2.

4. Conclusions

It was shown that the Sturmian basis provides a good method for finding the energies and eigenstates of the system of two nucleons outside an inert core with any desired accuracy.

The question of accuracy can, however, not be phrased in an unambiguous way, since different experiments test different components of the wave function. So, e.g., wave functions which give good agreement with the energy spectrum may still be insufficient for one-particle or two-particle transfer or both.

This ambiguity can, in the Sturmian approach, to some extent be matched with the ambiguity in choice of basis energy. If the energy of single particle states is chosen to correspond to the calculated single-particle separation energies, the convergence for all $r > R$ will be about as good as for $r \gtrsim R$, and this may be important in single particle transfer calculations.

In the two-particle case there are, as mentioned above, some indications that, for projectile energies such as in current use, the region of importance is that where both particles are in the neighbourhood of the nuclear radius. In this region the basis which was used here seems to be sufficient.

A number of questions concerning the reaction mechanisms of two-particle transfer are still unsolved, and it would be premature to claim that the present calculations could give complete predictions of cross sections. On the other hand, realistic form factors are necessary for any understanding of the cross sections.

In heavy ion transfer, the mechanism is although more complicated presumably better understood than for light projectiles. In this case, ratio of experimental cross sections with those obtained with conventional form factors are very large, - 3 - 50 particularly for proton 13). It seems likely that just in this case, the inclusion of quasi-stationary states by the present method might lead to improvements.

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Appendix

In the case of negligible interaction between the two particles, an analytical expression for the asymptotic form factor is obtained. For simplicity only $\ell_1 = \ell_2 = 0$ is considered.

Apart from normalization constants, this asymptotic form is

$$\Phi \approx \sum_i c_i \Phi_i = \sum c_i \frac{K_{\frac{1}{2}}(\varkappa_i r_1) K_{\frac{1}{2}}(\varkappa_{2i} r_2)}{(r_1 r_2 \varkappa_{1i} \varkappa_{2i})^{\frac{1}{2}}}, \qquad \frac{\hbar^2}{2m}(\varkappa_{1i}^2 + \varkappa_{2i}^2) = E. \tag{A.1}$$

In coordinates of center of mass and relative motion Φ_i may be written

$$\Phi_i = \frac{-16\pi^3}{\varkappa_1 \varkappa_2 R \rho} \sum_{m_1 m_2 m_1' m_2'} Y^*_{m_1 m_1'}(\hat{R}) Y_{m_1 m_1'}(\hat{\rho}) Y_{m_2 m_2'}(\hat{R}) Y^*_{m_2 m_2'}(-\hat{\rho})$$
$$I_{m_1+\frac{1}{2}}\left(\frac{\varkappa_1 \rho}{2}\right) I_{m_2+\frac{1}{2}}\left(\frac{\varkappa_2 \rho}{2}\right) K_{m_1+\frac{1}{2}}(\varkappa_1 R) K_{m_2+\frac{1}{2}}(\varkappa_2 R), \tag{A.2}$$

where ρ and R and the length of the relative distance and the center of mass vector, respectively, I and K are modified Bessel and Hankel functions. It is here included in the definition of the asymptotic region that $R > \frac{\rho}{2}$.

The overlap integral mentioned above now gives

$$I_i = \iiint \alpha\hat{\beta}\rho^2 \alpha_\rho \exp\left(-\tfrac{3}{2}\varkappa\rho^2\right)\left(\frac{216\varkappa^6}{\pi^3}\right)^{\frac{1}{2}} \Phi_i =$$
$$\frac{8\varkappa^2(-6\pi)^{\frac{1}{2}}}{\varkappa_1 \varkappa_2 R} \exp\left(\frac{E \cdot m}{\hbar^2 \cdot 12\varkappa}\right) \sum K_{m+\frac{1}{2}}(\varkappa_1 R) K_{m+\frac{1}{2}}(\varkappa_2 R)$$
$$I_{m+\frac{1}{2}}\left(\frac{-\varkappa_1 \varkappa_2}{12\varkappa}\right)(2m+1). \tag{A.3}$$

If we sharpen the definition of the asymptotic region by the requirement

$$R > \frac{\varkappa_1+\varkappa_2}{6\varkappa}$$

we get

$$I_i \approx \frac{(8\varkappa^3)^{\frac{1}{2}}\pi}{\varkappa_1\varkappa_2 R^2} \exp\left[\frac{(\varkappa_1-\varkappa_2)^2}{24\varkappa} - (\varkappa_1+\varkappa_2)R\right]$$

$$\approx \frac{(2\varkappa)^{\frac{1}{2}}\cdot\pi}{\varkappa_1\varkappa_2 R^2} \exp\left[-(\varkappa_1+\varkappa_2)R\right] . \qquad (A.4)$$

From this expression, it should be noted that if we compare two components $c_1\phi_1$, $c_2\phi_2$ with $c_1 \sim c_2$ but different partitions of the energy, the one where κ_1 and κ_2 are most unequal will be dominating for large R values.

Another extreme case, which leads to an analytic expression for the asymptotic form factor, is obtained when the interaction between the transferred particles is strong enough to produce a bound state of the relative motion, and at the same time the function with which overlap is taken is identical to this state. Then the form factor is a single Hankel function of κR divided by $\sqrt{R}$. ($\frac{h^2}{4m}K^2 = E_{bind}$). Such a factor has actually been used, but the interaction between two neutrons is so weak that the actual form factor of this reaction mechanism, although containing some correlations in ρ, must be nearer to the first case.

The number of nodes is seen, in both cases, to be limited, so for very large R values the form factor is monotonous and definite. In the weak interaction case, there seems, however, to be no limit of how far out nodes may be found.

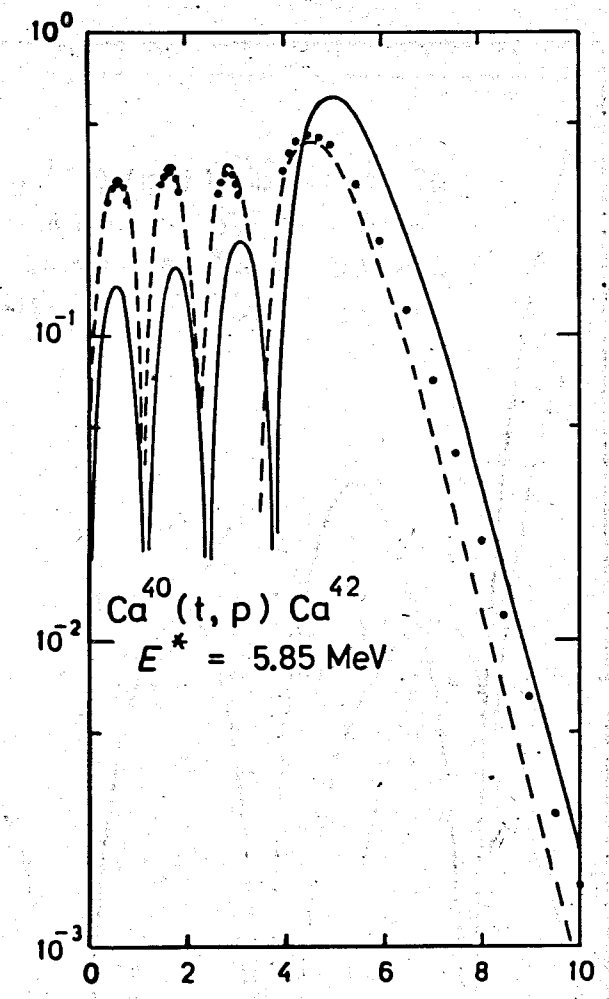

Fig. 1 Form factors for $Ca^{40}(t,p)Ca^{42}$ dots: shell model, dashed line: WDP, full curve: our results, $\ell = 0, 1, 2, 3, 4, 5$.

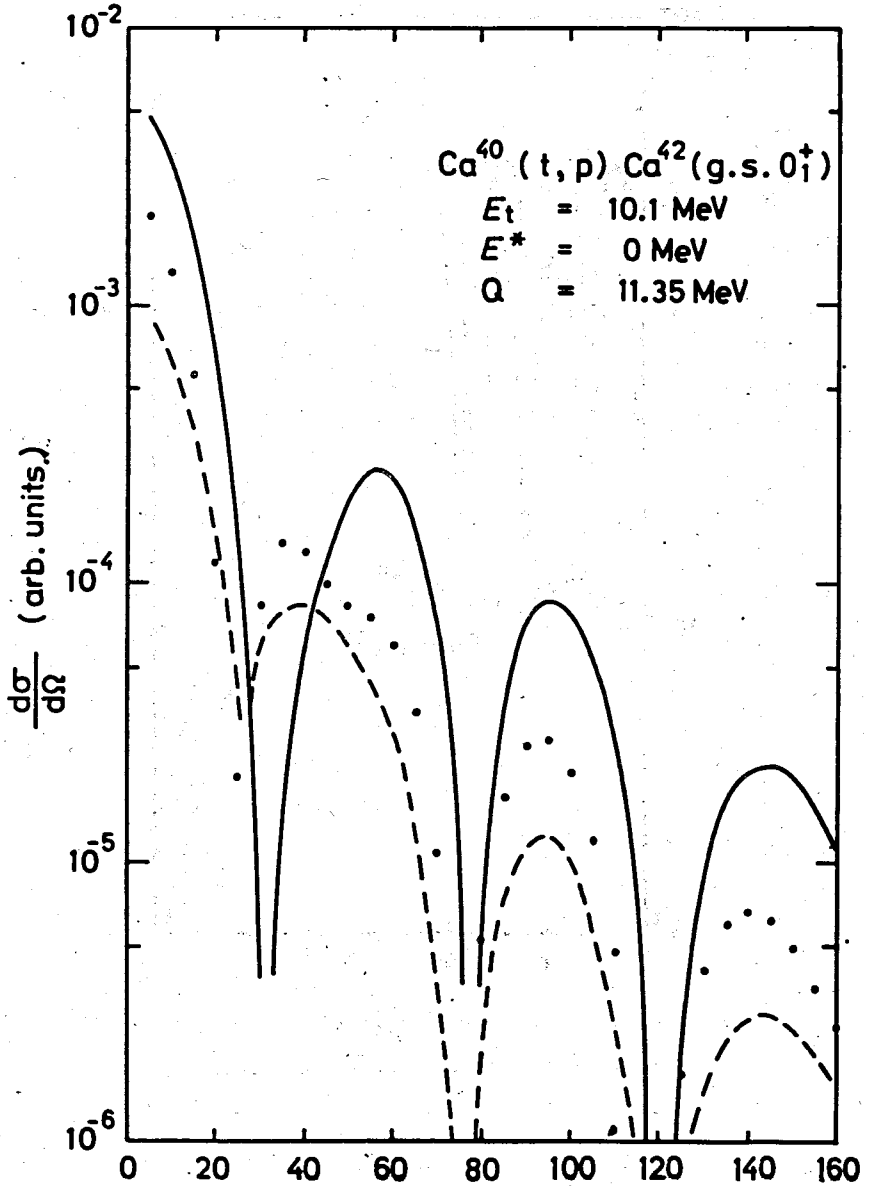

Fig. 2 Cross section for $Ca^{40}(t,p)Ca^{42}$(g.s.). Dots: Shell model, dashed line: WDP, full curve: our method.

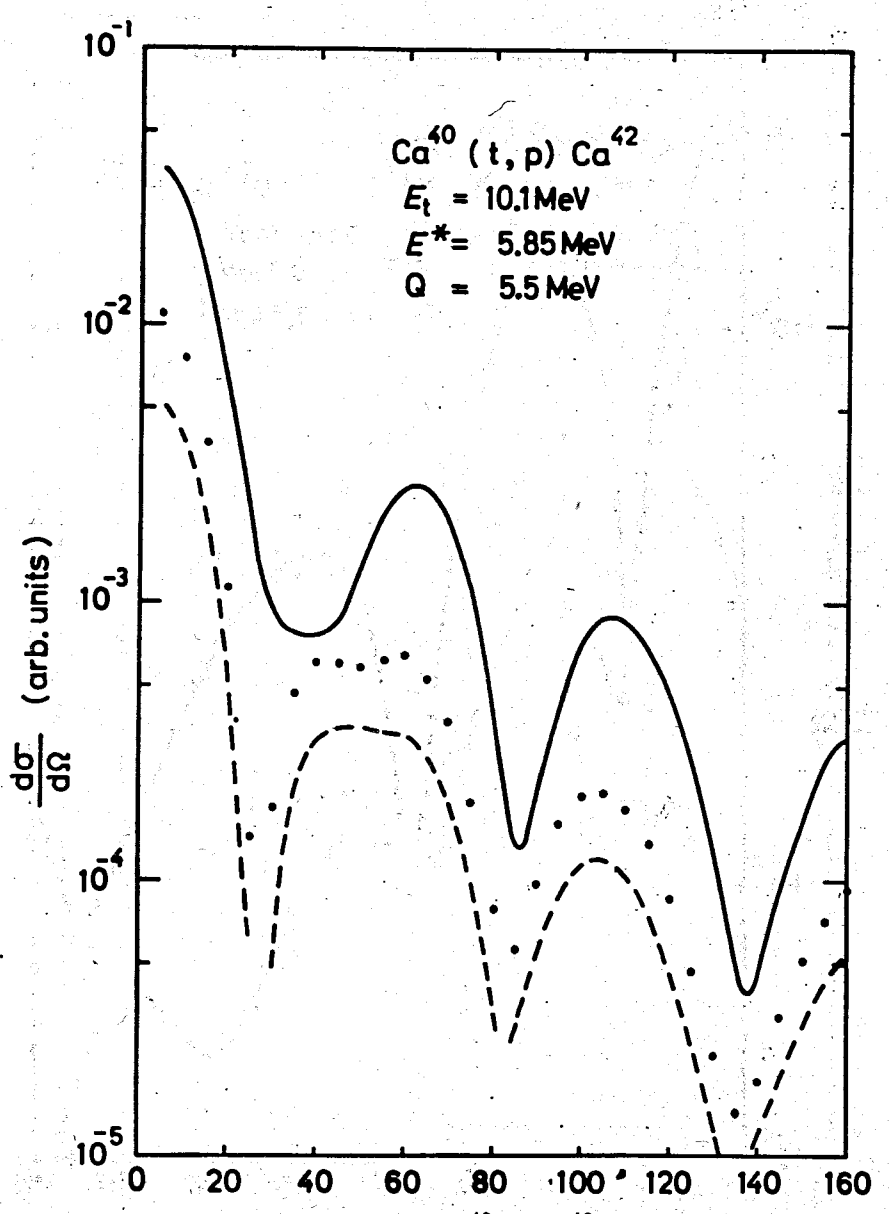

Fig. 3 Cross section for $Ca^{40}(t,p)Ca^{42}$(5.85 MeV), notations as in fig. 2.

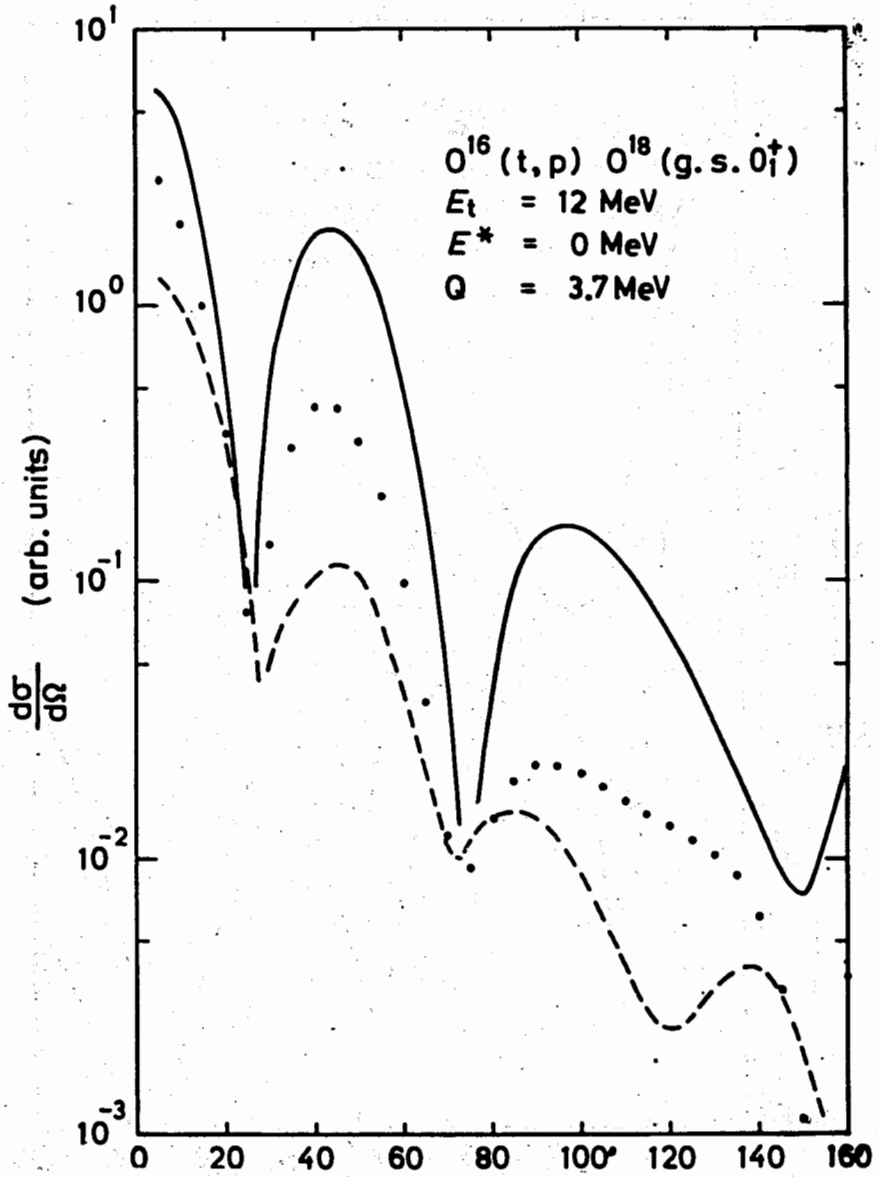

Fig. 4 Cross section for $0^{16}(t,p)0^{18}$(g.s.), notations as in fig. 2.

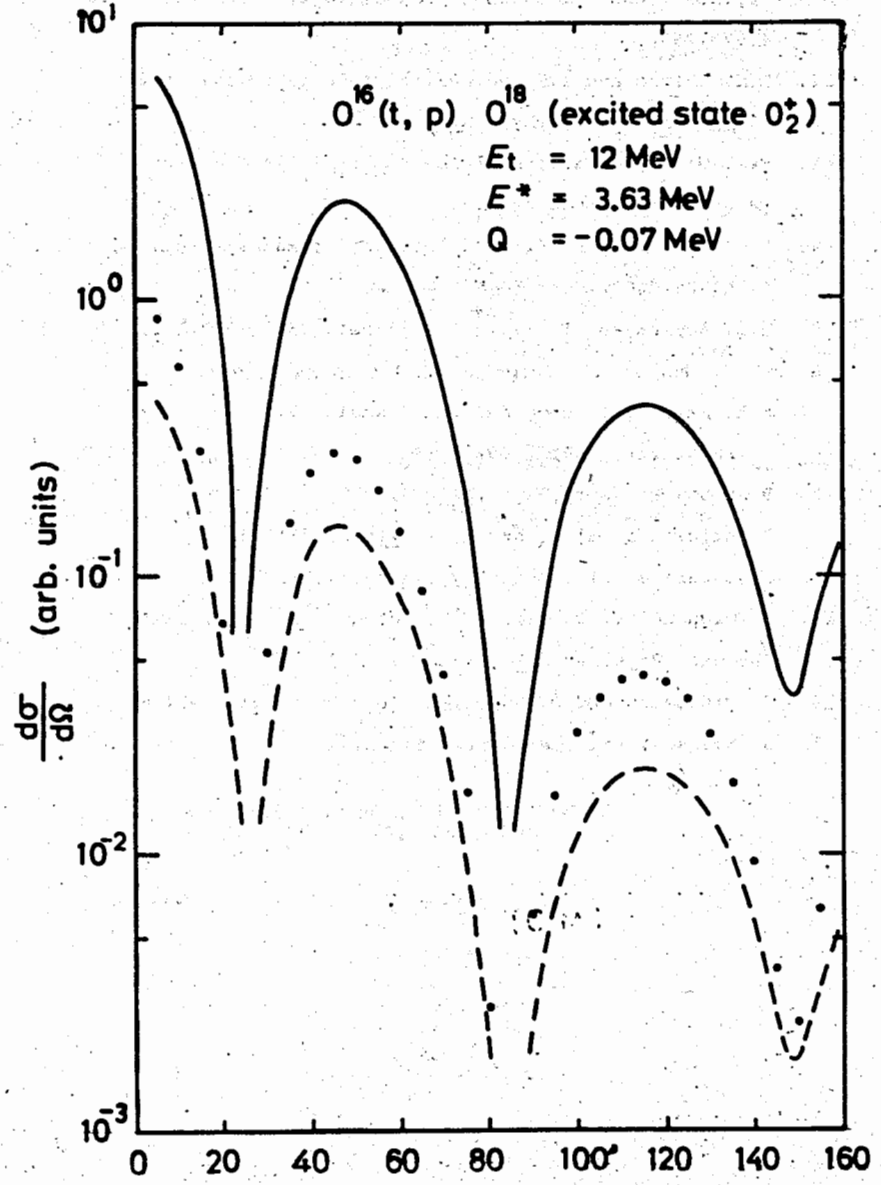

Fig. 5 Cross section for $0^{16}(t,p)0^{18}$(3.63 MeV), notations as in fig. 2.

References

1. R.A. Broglia, O. Hansen and C. Riedel, "Advances in Nuclear Physics", Eds. M. Baranger and E. Vogt, New York, Vol.6 (1973)283.
2. R.H. Ibarra and B.F. Bayman, Phys.Rev., C1 (1970)1786.
2a. R.H. Ibarra, Nucl.Phys., A211 (1973)317.
3. F.A. Gareev et al., Nucl.Phys., A215 (1973)570 and references of this article
4. R. Courant and D. Hilbert "Methods of Mathematical Physics" Intersci.Publ., New-York - London, 1953.
5. B.E. Bayman and A. Kallio, Phys.Rev., 156 (1967)1121.
6. V. Furman et al. Preprint P4-8110,Dubna (1974).
7. F.A. Gareev, J. Bang and R.M. Jamalejev, Phys.Lett., 49B (1974) 239.
8. B.F. Bayman, Nucl.Phys., A168 (1971)1.
9. T. Ericson et al., Phys.Lett., 46B (1973)173.
10. H.W. Barz et al., Nucl.Phys., A125(1969)577.
11. M. Pignanelli, J. Gosset, F. Resmini, B. Mayer and J.L. Escadie, Phys.Rev., C8 (1973)2120.
12. R. Middleton and D.J. Pullen, Nucl.Phys., 51 (1964)63.
13. B. Nilsson, private communication

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