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An intensive accumulation of experimental data stimulates more and more thorough theoretical investigations of low-lying nuclear excitations. Especially extensive studies are made in the "classical" region of spherical nuclei near $Z = 50$. It has been known long ago that the properties of the lowest excitations of the nuclei of this region are not at all embeded in the framework of the simple picture of harmonic quadrupole oscillations. Apparently, so much time one has tried to account for the properties of these states including in the Hamiltonian different kinds of anharmonic corrections. In this case both phenomenological and semimicroscopic models /1/ * are used. Undoubtedly, there also exist some other approaches to the description of the structure of the excited states of Sn isotopes and the neighbouring nuclei /2/.

At the same time, simple considerations show that the properties of some low-lying states of the Te, Sn and Cd isotopes must exhibit noticeable admixtures of non-collective components. This is also confirmed by experimental data /3/.

There also exists a phenomenological model in which account is taken of the effects of coupling of two-particle and vibrational excitations in the structure of the Te and Cd excited states . This is the so-called Alaga model /4,5/.

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* In the review /1/ one can find references to the main original papers devoted to this problem.

In the framework of the semi-microscopic approach these effects have been studied by Soloviev by the example of deformed nuclei /6/. Recently some papers have appeared which employ the same formalism for spherical nuclei /7,8/.

In the present paper we study the effect of the interaction of two-quasiparticle and vibrational excitations on the properties of the $I^{\pi} = 2^{+}, 3^{-}$ states in tellurium, tin and cadmium. The investigation is performed in the framework of the superfluid nuclear model /9/ on the basis of the equations obtained and investigated in detail in ref. /7/.

The energies of the states, their quadrupole moments and the electric transition probabilities have been calculated. The operator of the electric $E\lambda$ -transition is taken from ref. /9/ (chapter 6, § 3). The Hamiltonian of the spherical nucleus in the superfluid nuclear model is known to include the average field for the proton and neutron systems, the pairing superfluid interaction and the quadrupole-quadrupole and octupole-octupole residual interactions.

The parameters of the average nuclear field are given in Table 1, they are chosen on the basis of the results of papers /10/. The single-particle spectrum and single-particle matrix elements of the multipole operators are calculated by the program of ref. /11/. The pairing constants are taken from ref. /12/, the remaining constants are chosen so that the theoretical and experimental energies of the 2_1^+ and 3_1^- levels coincide. The effective charge necessary for the calculations of the electromagnetic characteristics is chosen to be 0.6.

The wave function of the states with $I^{\pi} = 2^{+}, 3^{-}$ is found to be:

$$\Psi_{\nu}(IM) = \Big\{ \sum_{i} C_i^{\nu} Q^{+}_{IMi} + \sum_{\substack{\lambda_1 i_1 \\ \lambda_2 i_2}} P^{\lambda_2 i_2}_{\lambda_1 i_1}(I\nu) < \lambda_1 \mu_1 \lambda_2 \mu_2 | IM > \times$$

$$\times Q^{+}_{\lambda_1 \mu_1 i_1} Q^{+}_{\lambda_2 \mu_2 i_2} \Big\} | 0 >_{ph} \qquad (1)$$

Table 1
Saxon-Woods potential parameters

Region of nuclei for which the single-particle level scheme was used		A, N, Z		r_0 fm	V_0 Mev	κ fm ^{2}	a fm^{-1}
Isotopes Cd		A=109	N=63	1.29	44.0	0.413	1.613
		A=109	Z=47	1.24	55.0	0.335	1.587
Isotopes Sn		A=121	N=69	1.28	45.5	0.413	1.613
		A=121	Z=69	1.24	55.4	0.341	1.587
Isotopes Te	A ≤ 124	A=121	N=69	1.29	45.5	0.413	1.613
	A > 124	A=127	N=73	1.28	43.8	0.413	1.613
		A=127	Z=53	1.24	59.8	0.350	1.587

The variational procedure is used to obtain the following secular equation for the $\eta_{I\nu}$ state energy

$$\det \{ (\omega_{Ii} - \eta_{I\nu}) \delta_{ii'} - K(ii') \} = 0. \tag{2}$$

The expression for $K(ii')$ is given in ref. [7]. In fms. (1) and (2) the following notation is used: $Q^{+}_{\lambda\mu i}, Q_{\lambda\mu i}$ - the creation and annihilation operators for a phonon with momentum and projection λ, μ and the number i ; ω_{Ii} - the energy of the phonon excitation $Q^{+}_{IMi} |0\rangle_{ph}$; ν - the number of the root of eq. (2); $|0\rangle_{ph}$ - the phonon vacuum, $Q_{IMi} |0\rangle_{ph} = 0$; $\langle \lambda_1 \mu_1 \lambda_2 \mu_2 | IM \rangle$ - the Clebsch-Gordan coefficient.

In the wave function (1) $I, \lambda_1, \lambda_2 = 2^{+}, 3^{-}$. This results in the loss of certain components which, however, are found to be important only at energies higher 3.5 MeV for the states with positive parity and 4.0 MeV for the states with negative parity. Therefore we have restricted the summation over i in eq. (1) by the condition $i \le 4$. All these restrictions, including also the neglect of three-phonon components, imply that it is in this energy region alone that we can claim to a correct description of the experimental situation where the main role is attributed to the two-quasipartucle and two-phonon ($Q^{+}_{21} Q^{+}_{21}$ or $Q^{+}_{21} Q^{+}_{31}$) components.

Note that when calculating the two-quasiparticle energies we have taken into account the blocking effect [9] which, in some cases, makes the agreement between theory and experiment better for the second and higher levels with $I^{\pi} = 2^{+}, 3^{-}$.

We now pass to the discussion of the results of our calculations. They are given in Table 2 (states with $I^{\pi} = 2^{+}$) and in Table 3 (states with $I^{\pi} = 3^{-}$). Here are also given experimental data the main fraction of which is taken from refs. [13,14].

In the Te isotopes the 2^{+}_{2} -level energies are in satisfactory agreement with experiment. The exception is the nucleus ^{122}Te alone in which the difference $E(2^{+}_{2}) - \eta_{22}$ reaches 400 keV. According to the coincidence of the theoretical and experimental probabilities for E2 transitions and quadrupole moments, it may be

Table 2
Some experimental and theoretical characteristics of the $I^{\pi} = 2^+$ states in the Te, Sn and Cd isotopes

Nucleus	i	$E(2^+_i)$ KeV exp.	theor.	$B(E2, 0^+_{g.s.} \to 2^+_i)$ e^2b^2 exp.	theor.	$Q_2(2^+_i)$ eb exp.	theor.	$B(E2, 2^+_i \to 2^+_1)$ e^2b^2 exp.	theor.
^{122}Te	1	564	573	0.66	0.46	-0.30 ± 0.22	-0.55		
	2	1360	1661	0.020	0.057	-	0.37	0.35	0.087
	3	1752	1993	-	0.007	-	-0.31	-	0.016
^{124}Te	1	602	604	0.527	0.48	-0.08 ± 0.11	-0.40		
	2	1325	1499	0.0165	0.025	-	0.26	0.34	0.13
	3	2038	1625	-	3×10^{-5}	-	-0.44	-	0.005
^{126}Te	1	666	649	0.47	0.46	-0.16 ± 0.16	-0.11		
	2	1420	1444	0.004	3×10^{-4}	-	0.15	0.17	0.15
	3	2190	1665	-	9×10^{-5}	-	-0.41	-	0.008
^{128}Te	1	743	743	0.38	0.34	-0.14 ± 0.13	0.19		
	2	15[illegible]	1551	0.011	0.013	-	-0.25	-	0.07
	3	-	1757	-	0.003	-	-0.35	-	0.01
^{130}Te	1	839	888	0.30	0.22	-0.19 ± 0.15	0.19		
	2	1597	1823	0.004	0.002	-	-0.18	-	0.004
	3	1664	1877	-	0.029	-	-0.22	-	0.007
^{112}Sn	1	1260	1264	0.257	0.20	-0.15 ± 0.16	0.19		
	2	2160	2488	-	0.014	-	-0.14	-	0.046
	3	2787	2847	-	6×10^{-4}	-	0.09	-	0.007

Table 2 (continued)

^{114}Sn	1	1300	1281	0.23	0.24	-	-0.13		
	2	1926	2428	-	0.0019	-	0.11	-	0.07
	3	2239	3121	-	0.0014	-	0.16	-	0.002
^{116}Sn	1	1293	1269	0.216	0.23	0.08±0.13	-0.25		
	2	2111	2447	0.0012	0.0026	-	0.19	-	0.05
	3	2225	3182	-	4×10^{-4}	-	-0.21	-	5×10^{-6}
^{118}Sn	1	1230	1227	0.218	0.20	-0.23±0.16	-0.23		
	2	2040	2304	0.022	0.0012	-	0.13	-	0.03
	3	2409	2731	-	9×10^{-4}	-	-0.036	-	0.006
^{120}Sn	1	1172	1188	0.206	0.19	0.09±0.10	-0.18		
	2	2361	2394	0.0019	0.0011	-	0.076	-	0.013
	3	2735	2563	-	2×10^{-4}	-	0.012	-	0.038
^{122}Sn	1	1140	1152	0.202	0.180	-0.28±0.17	-0.08		
	2	2412	2405	0.0011	8×10^{-4}	-	0.05	-	0.023
	3	-	2604	-	1.4×10^{-4}	-	0.06	-	0.003
^{124}Sn	1	1139	1134	0.169	0.15	-0.09±0.15	0.021		
	2	2130	2119	4×10^{-4}	0.0024	-	-0.07	-	0.023
	3	2431	2343	-	0.0016	-	0.07	-	0.003
^{114}Cd	1	558	518	0.515	0.43	-0.40±0.15	0.56		
	2	1209	1590	0.0095	0.07	-	-0.45	0.40	0.07
	3	1364	2383	0.007	1×10^{-4}	-	0.33	0.065	0.009
^{116}Cd	1	514	541	0.63	0.48	-0.90±0.25	0.34		
	2	1214	1324	0.019	0.02	-	-0.29	0.36	0.15
	3	-	2508		4×10^{-3}	-	0.34	-	9×10^{-3}

Table 3

Some experimental and theoretical characteristics of the $I^\pi = 3^-$ states in the Te, Sn and Cd isotopes

Nucleus	1	$E(3_i^-)$ Kev		$B(E3, 0^+_{g.s.} \to 3_i^-)e^2b^3$		$Q_2(3_i^-)eb$
		exp.	theor.	exp.	theor.	theor.
^{122}Te	1	2170	2182	–	0.11	0.47
	2	–	2632	–	0.013	–0.26
^{124}Te	1	2294	2271	–	0.11	0.20
	2	–	2536	–	0.002	–0.80
^{126}Te	1	2320	2288	–	0.10	–0.34
	2	–	2591	–	0.01	0.45
^{128}Te	1	2500	2503	–	0.10	–0.24
	2	–	2807	–	0.01	0.39
^{130}Te	1	2320	2405	–	0.12	–0.10
	2	–	2866	–	0.005	0.23
^{112}Sn	1	2355	2371	0.049	0.040	0.32
	2	–	3222	–	2x10^{-5}	0.28
^{114}Sn	1	2275	2287	0.062	0.05	0.27
	2	–	3114	–	6x10^{-4}	0.31
^{116}Sn	1	2269	2297	0.067	0.067	0.12
	2	3320?	3200	–	0.001	0.28
	3	–	3806	–	2x10^{-5}	–0.08
^{118}Sn	1	2321	2337	0.072	0.063	–0.09
	2	3362?	3207	–	6x10^{-4}	0.22
	3	–	3812	–	0.003	0.10
^{120}Sn	1	2408	2423	0.069	0.056	–0.24
	2	3467?	3382	–	1x10^{-5}	0.20
	3	–	4107	–	0.[illegible]	0.20
^{122}Sn	1	2499	2525	0.068	0.048	–0.30
	2	3367	3567	–	4x10^{-5}	0.17
	3	–	4478	–	0.012	0.21
^{124}Sn	1	2612	2636	0.069	0.042	–0.31
	2	3009	3750	–	6x10^{-5}	0.11
	3	3516	4790	–	0.017	0.19
^{114}Cd	1	1950	1938	0.09	0.13	0.12
	2	–	2742	–	5x10^{-3}	0.11
^{116}Cd	1	1945	1980	0.075	0.12	–0.40
	2	–	2932	–	2x10^{-3}	0.39

said that we describe the state structure satisfactorily. It contains, on the average, 60-70% of the component $Q^+_{21}\ Q^+_{21}$ and 6-7% of the component Q^+_{21}. It is only in ^{130}Te nucleus that the 2^+_2 states contain mostly noncollective components. The 2^+_3-state structure is a superposition of noncollective one-phonon excitations. The energy of these states is found to be noticeably lower than the experimental one which is associated with the single-particle level scheme.

It is worth noting that in the structure of 2^+_2 states of nuclei with N = 80, 84 which were studied in ref. /15/, the admixture of the $Q^+_{21}\ Q^+_{21}$ component was far smaller which resulted in some cases to an unreally small value of $B(E2, 2^+_2 \to 2^+_1)$. In the Te isotopes the situation is noticeably better. The obtained characteristics of the 2^+ levels of the Te isotopes are, on the average, in satisfactory agreement with the results of calculations in the framework of other models /5/. The only noticeable difference is observed in the probability $B(E2, 2^+_2 \to 0^+_{g.s.})$.

The information about the 3^- levels in the Te isotopes is related only to the energies of the first 3^- states so that we are unable to verify our results. The 3^-_2-level structure is found to be complex, however, the component $Q^+_{21}\ Q^+_{31}$ is the largest one and reaches 40-50%. We want to note here again that in the N 80, 84 isotones the 3^-_2 states are found to be practically two-quasiparticle ones.

In the Sn isotopes the mixing of the vibrational and two-quasiparticle components is the strongest one in the region of nuclei in question. The Sn isotopes are divided into two groups, according to the agreement of the 2^+_2 - state energy with experiment. The light isotopes $^{112-118}$Sn belong to the first group. Here one observes experimentally pairs of closely spaced 2^+ levels which, in our calculations, are at a large distance from one another. At the same time, the energies of the second and third 2^+ states in the second group of the isotopes agree well with experiment. A disagreement observed in light Sn isotopes appears to be due also to a not quite suitable choice of the single-particle scheme of levels (in the given case - the neutron scheme) in these nuclei. The experimental data on the quadrupole moments of the 2_1^+ levels of the Sn isotopes are rather contradictory. On the whole, they point to the smallness of the quadrupole moments in their absolute magnitude (at least, compared with tellurium and cadmium) and to an irregular change of the sign of $Q_2(2_1^+)$ from isotope to isotope. Our results reflect this fact, although quantitative agreement between theory and experiment is of an irregular character. Satisfactory agreement with experiment /16/ is also exhibited by the quantity $B(E2, 2_1^+ \to 0_{g.s.}^+)$. There are no data on the E2 transitions $2_2^+ \to 2_1^+$ in the Sn isotopes. However, in ^{118}Sn the ratio $\frac{B(E2, 0_{g.s.}^+ \to 2_2^+)}{B(E2, 2_2^+ \to 2_1^+)}$ is known to be 0.05. Our results give for this ratio the value 0.04. All these properties of the 2^+ states are directly associated with the contribution of different components to their structure. For example, a comparatively small $Q_2(2_1^+)$ is connected with that the contribution of $Q_{21}^+ Q_{21}^+$ to the wave function of the 2_1^+ level is not large, 2-4%. On the other hand, the component Q_{21}^+ little contributes to the 2_2^+-state wave function which leads consequently to a small probability of the transition $2_2^+ \to 0_{g.s.}^+$. The admixture of the two-phonon component $Q_{21}^+ Q_{21}^+$ in the 2_2^+ - state structure ranges between 20-50%, and the remaining part is related to the noncollective phonons with $i > 1$. That is here we have states of a very complicated structure.

The 3^- states in the Sn isotopes are studied experimentally better than in tellurium. Here we know many 3_2^- states the energy of which is quite correctly described in our calculations. The exception is the ^{124}Sn nucleus where in addition to the 3_2^- state a 3_3^- state is known whose energy is much lower than that calculated by us. The 3_2^--state structure is mainly of a two-quasipartucle character (contrary to the Te isotopes), the admixture of the $Q_{21}^+ Q_{31}^+$ component varies in the interval 5-10%. Of all the electromagnetic characteristics of the 3^- states the $B(E3, 0_{g.s.}^+ \to 3_1^-)$ alone have been studied experimentally. They are satisfactory described in our calculations, though agreement with experiment becomes worse in heavy isotopes.

The low-lying states with $I^{\pi} = 2^{+}$ in the Cd isotopes have been well studied experimentally. However, in these nuclei anharmonisity of quadrupole vibrations is found to be very strong (the ratio $\omega_{21}/\eta_{21} \approx 1.9$), and our results are, on the whole, unsatisfactory. In these nuclei the energy gap is large, and it seems that in the wave function (1) we should take into account the three-phonon component $Q^{+}_{21} Q^{+}_{21} Q^{+}_{21}$ the effect of which is expected to be strong. The necessity of such a complication is seen from a bad description of the 2^{+}_{3} –state energy even in one of the satisfactorily described isotopes, ^{114}Cd. But in this nucleus the two-phonon component $Q^{+}_{21} Q^{+}_{21}$ in the 2^{+}_{2} – state structure turns out to be too small which leads to a wrong result for the quantity $B(E2, 2^{+}_{2} \to 2^{+}_{1})$. A somewhat better situation is for the ^{116}Cd nucleus.

Thus, our analysis has shown that in the region of nuclei under consideration mixing of vibrational and two-quasiparticle excitations is of much importance in the formation of the state structure. This is especially clearly seen in the Sn isotopes. Inclusion of the two-quasiparticle components to the 2^{+} -state wave function has resulted in noticeable improvement in the description of the 2^{+}_{2}-and 2^{+}_{3}– level energies, as well as of the quantity $B(E2, 0^{+}_{g.s.} \to 2^{+}_{3})$ It is seen from the comparison of the results of our paper and paper /18/ for the Sn and Te isotopes that the values of the energy and the quantity $B(E2, 0^{+}_{g.s.} \to 2^{+}_{2})$ came, in all the cases, nearer to the experimental ones. In many cases this change is appreciable, for example, in the ^{120}Sn isotopes the theoretical value $B(E2, 0^{+}_{g.s.} \to 2^{+}_{2})$ decreased by a factor of 10, and the 2^{+}_{2} – level energy - by 600 keV.

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