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THE AVERAGE BEHAVIOUR OF THE MOMENT OF INERTIA

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I. Introduction

The influence of the shell structure on the binding energy of the atomic nucleus has been extensively studied in many publications (see the references in the review [1]). Strutinsky [2] proposed a method which allows one to take into account these effects in an accurate manner. In this approach the total energy of the nucleus is divided into two parts. The first one, $\tilde{U}$, depends smoothly on the number of particles and, therefore, does not show any shell effects. The second one, δU , is the so-called shell correction. It depends only on the level density near the Fermi surface and can be calculated from the shell model single particle energies. Within the shell model, it is impossible to calculate the smooth part of the energy with a reasonable accuracy. One rather substitutes the energy U_{LDM} of the liquid drop model for $\tilde{U}$:

$$U = U_{LDM} + \delta U.$$

This method turned out to be sufficiently accurate and very convenient for the investigation of U as a function of the particle number and the shape of the nucleus, so it permits a study of the static properties of the nucleus.

The investigation of the dynamical behaviour of the nuclear shape is a very important but much more complicated problem, which has recently received an impact from nuclear fission studies. Such calculations are usually carried out in the adiabatic approximation. The energy of the collective motion is considered to be the sum of the kinetic energy T and potential energy U , $W = T + U$, where T is a quadratic form whose coefficients are called mass parameters. One possibility of calculating these quantities is the generalized cranking model /1,3/. In this approach one enforces a slow change of the shell model potential and calculates W as the sum of the energies of non-interacting particles (or quasiparticles in the BCS approximation) in this time-dependent potential. In the adiabatic approximation one can introduce single-particle energies in this potential, which depend on time as a parameter. In a number of papers it is shown that the shell structure has a significant influence on the mass parameters /1,3,4/ . Due to the above-mentioned representation of W as the sum of the single-particle energies, it is possible to generalize Strutinsky's idea about the division of the energy U into two parts to the total energy W and therefore to T . The partition of T leads to the corresponding division of the mass parameter into a smooth part and another term which we prefer to call shell contribution, because it can amount to the same order of magnitude as the smooth part (see Section 4).

One can hope to improve the accuracy of calculating mass parameters using different approximations for the smooth part and the shell contributions, because the motion of the nucleons may influence the two parts in a different way.

In the calculations of Baranger and Kumar /4/ an analogous idea is used. In their work the mass coefficients also consist of two parts. One part, which is similar to our shell contribution, is obtained by means of the cranking model with account of only the states of about one major shell around the Fermi level. The other, smooth part, is a phenomenological expression. However, such a division becomes rather arbitrary at larger deformation, when the levels of different spherical shells are no longer separated. Therefore a division by means of Strutiksky's averaging procedure seems to be preferable.

In order to make a first step in this direction, we investigate the suggested division in the simple case of the moment of inertia $\mathcal{J}$ of deformed nuclei calculated by means of the cranking model /5/. We show that the division into the smooth part $\mathcal{J}$ and the shell contribution $\delta\mathcal{J}$ is unambiguous. We investigate the dependence of $\mathcal{J}$ and $\delta\mathcal{J}$ on the number of particles and the nuclear shape. We compare our expressions with the classical expression for the moment of inertia and the moment of inertia of a heated nucleus at different temperatures. The magnitude of $\delta\mathcal{J}$ at the equilibrium deformation is examined in detail. We suggest some conditions for the case where the influence of the shell structure on the moment of inertia is rather strong.

2. Definition of the Averaged Moment of Inertia and the Shell Contribution to the Moment of Inertia

In this work we consider only the moments of inertia of even-even axially symmetric nuclei rotating around an axis perpendicular to the symmetry axis. We do not consider the residual interaction between the nucleons.

Let us consider a system of non-interacting nucleons in an axially symmetric external potential, the z-axis being the axis of symmetry. Within the cranking model the moment of inertia $\mathcal{J}_{SM}$ with respect to the x-axis can be extracted from the energy E_ω in the body fixed frame of reference rotating with the angular velocity ω around the x-axis. The moment of inertia is defined by the quadratic term of a perturbation expansion into powers of ω

$$E_\omega = E_{\omega=0} - \frac{1}{2}\omega^2 \mathcal{J}_{SM} \quad . \tag{1}$$

The total energy E_ω is simply the sum of the single particle energies ϵ_i^ω of the occupied levels. The energies ϵ_i^ω are the eigenvalues of the single-particle Hamiltonian H_ω of the nucleons in the rotating frame of reference

$$H_\omega = H_{SM} - \omega j^x \quad , \tag{2}$$

where j^x denotes the x-component of the single particle operator of the angular momentum, and H_{SM} is the shell model Hamiltonian of the system in the absence of rotation. As we are interested only in the second term of expansion (1), we need ϵ_i^{ω} only in the second order of perturbation theory starting with the eigenvalues ϵ_i° of H_{SM}

$$\epsilon_i^{\omega} = \epsilon_i^{\circ} + \omega^2 \frac{d\epsilon_i^{\circ}}{d\omega^2} + \dots , \quad \frac{d\epsilon_i^{\circ}}{d\omega^2} = \sum_{j, j \neq i} \frac{|j_{ij}^{x}|^2}{\epsilon_i^{\circ} - \epsilon_j^{\circ}} . \qquad (3)$$

The total energy U_{ω} can be written down in terms of the occupation numbers n_i^{SM}, which are equal to 0 above the Fermi level and to 1 elsewhere. By making use of eq. (1) we obtain

$$\mathcal{J}_{SM} = 2 \sum_{i \neq j} n_i^{SM} \frac{|j_{ij}^{x}|^2}{\epsilon_j^{\circ} - \epsilon_i^{\circ}} = \sum_{i \neq j} (n_i^{SM} - n_j^{SM}) \frac{|j_{ij}^{x}|^2}{\epsilon_j^{\circ} - \epsilon_i^{\circ}} . \qquad (4)$$

This expression is valid only for small enough values of ω, when there is no crossing of levels with the Fermi level and n^{SM} does not depend on ω. We introduce the averaged moment of inertia $\tilde{\mathcal{J}}$ in the following manner. The eigenvalues ϵ_i^{ω} of the Hamiltonian H_{ω} are characterized by their density $g_{SM}^{\omega}(\epsilon)$. By means of Strutinsky's averaging procedure we divide this density into a smooth part $\tilde{g}^{\omega}(\epsilon)$ and a shell correction $\delta g^{\omega}(\epsilon)$. Thus we get the smooth energy $\tilde{U}_{\omega}$ and the shell correction δU_{ω} /1/. By expanding $\tilde{U}_{\omega}$ into powers of ω we define the averaged moment of inertia as a coefficient of the quadratic term. In order to calculate $\tilde{\mathcal{J}}$, we use the occupation number representation of the Strutinsky procedure /1/, which is especially suitable for our purposes. Following ref. /9/ we have

$$\sum_i \epsilon_i^{\omega} \tilde{n}_i^{\omega} = \tilde{U}_{\omega} - \gamma \frac{d}{d\gamma} \tilde{U}_{\omega} . \qquad (5)$$

The averaged occupation numbers $\tilde{n}_i^{\omega}$ can be expressed in terms of the function $n(x)$ defined in ref. /1/

$$\tilde{n}_i^{\omega} = n\left(\frac{\tilde{\lambda}^{\omega} - \epsilon_i^{\omega}}{\gamma} \right) , \qquad (6)$$

where γ is the averaging width. The chemical potential is fixed by the particle number N

$$\sum_i \tilde{n}_i = N \,. \tag{7}$$

We substitute (3) into (6) and expand it into powers of ω

$$\sum_i \epsilon_i^{\omega} \tilde{n}_i^{\omega} = \sum_i \Big[\epsilon_i^{o} \tilde{n}_i^{o} + \omega^2 \frac{d\epsilon_i^{o}}{d\omega^2} \tilde{n}_i^{o} + \frac{\omega^2}{\gamma} \Big(\frac{d\tilde{\lambda}^{o}}{d\omega^2} - \frac{d\epsilon_i^{o}}{d\omega^2} \Big) \epsilon_i^{o} (\tilde{n}_i^{o})' \Big] . \tag{8}$$

In this expression $\tilde{n}_i^{o}$ stands for $n(x)$ at $x = \frac{\tilde{\lambda}^{o} - \epsilon_i^{o}}{\gamma}$, $(\tilde{n}_i^{o})'$ is $dn(x)/dx$ with the same argument, and $\tilde{\lambda}^{o}$ and $\frac{d\tilde{\lambda}^{o}}{d\omega^2}$ are the solutions of eq. (7) and its derivative at $\omega = 0$. Differentiating (7) with respect to γ and ω^2 we obtain the derivatives of $\tilde{\lambda}^{o}$ and can express the third term in the following manner

$$2\sum_i \epsilon_i^{\omega} \tilde{n}_i^{\omega} = 2 \sum_i \epsilon_i^{o} \tilde{n}_i^{o} + 2\omega^2 \Big(1 - \gamma \frac{d}{d\gamma} \Big) \sum_i \frac{d\epsilon_i^{o}}{d\omega^2} \tilde{n}_i^{o} \,. \tag{9}$$

By comparing (5) and (9) we obtain for the averaged moment of inertia

$$\tilde{\mathcal{J}} = -2 \sum_i \frac{d\epsilon_i^{o}}{d\omega^2} \tilde{n}_i^{o} = \sum_{ij} (\tilde{n}_i^{o} - \tilde{n}_j^{o}) \frac{|j_{ij}^{x}|^2}{\epsilon_j^{o} - \epsilon_i^{o}} . \tag{10}$$

One gets an identical expression for $\tilde{\mathcal{J}}$ as the linear term in the expansion of the averaged expectation value of the angular momentum operator. The shell contribution to the moment of inertia $\delta\mathcal{J}$ is defined as the difference

$$\delta\mathcal{J} = \mathcal{J}_{SM} - \tilde{\mathcal{J}} \tag{11}$$

This expression can be written down in the same form as (4) or (11) using $\delta n_i = n_i^{SM} - n^{o}$.

It is interesting to compare $\tilde{\mathcal{J}}$ with the classical moment of inertia of a system of non-interacting particles, which is given by the rigid body value

$$\mathcal{J}_{RIG} = M \int dr\, r^2 \rho \,/ \int dr\, \rho \,, \tag{12}$$

where M is the total mass, ρ denotes the density of particles, and r is the distance from the axis of rotation. We use the shell model value for the density

$$\rho_{SM}(\vec{r}) = \sum_i n_i^{SM} \,|\, \psi_i(\vec{r}) \,|^2, \tag{13}$$

where $\psi_i(\vec{r})$ are the eigenfunctions of the Hamiltonian H_{SM}. The respective moment of inertia is denoted by $\mathcal{J}^{SM}_{RIG}$. In addition, we define the moment of inertia $\tilde{\mathcal{J}}_{RIG}$ by means of eq. (12) and the averaged density $\tilde{\rho}(\vec{r})$, which is obtained from eq.(13) with the averaged occupation numbers $\tilde{n}_i^o$ instead of n_i^{SM}.

Without any special discussion we quote the known expression for the moment of inertia $\mathcal{J}_T$ of a heated nucleus at the temperature T/1/. It is given by expression (10) with the occupation numbers n_i^T of a Fermi gas in a potential well instead of n_i^o:

$$n_i^T = n^F\left(\frac{\lambda^T - \epsilon_i^o}{T}\right), \quad n^F(x) = (1 + \exp(x))^{-1}. \tag{14}$$

Here λ^T is obtained from eq. (7) with the analogous substitution of the occupation numbers. The expansion (3) (and consequently (4)) is applicable only in the case where $\hbar\omega$ is small compared with the distance between the single particle levels. The expressions for $\tilde{\mathcal{J}}$ and $\mathcal{J}_T$ are valid under the less stringent conditions

$$\hbar\omega < \gamma \approx \hbar\omega_0 \,, \tag{15}$$

and $\hbar\omega < T$, respectively, where $\hbar\omega_0$ means the intershell distance. The condition (15) can be derived in analogy to the case of statistics, which is discussed, e.g., in ref. /7/. A similar discussion for $\tilde{U}$ can be found in ref. /1/

3. Numerical Investigation of the Plateau Condition

The main part of our calculations is done with the Woods-Saxon potential. The parametrization of the nuclear surface and the method of solving the eigenvalue problem are described in refs. /8,9/ . The potential parameters are taken from ref./10/. The calculations are carried out for the nuclei of the rare earth region. The energy levels are calculated for the basic nuclei ^{162}Er and ^{190}Hg. The single particle spectra of neighbouring nuclei are obtained by interpolation assuming the same dependence on the mass number as that for the harmonic oscillator. In order to test our numerical method, we carried out some calculations using the anisotropic harmonic oscillator as the shell model potential. The degeneracies of the oscillator levels did not permit a direct application of our computer code. To avoid these difficulties, we introduced a slight spin-orbital perturbation and a weak anharmonicity of the type R^4 , R being the spherical radius coordinate.

By means of Strutinsky's averaging procedure we introduced the averaged moment of inertia $\tilde{\mathcal{J}}$ (10). In order that this conception be defined unambiguously, $\tilde{\mathcal{J}}$ must be independent of the averaging width γ with a reasonable accuracy when γ takes on the order of the intershell distance. This is the plateau condition formulated by Strutinsky for the averaged energy /1, 2/. We calculated $\tilde{\mathcal{J}}$ as a function of γ using fourth and sixth order correction polynomials for the proton and the neutron systems, respectively. The results are shown in Fig. 1. As regards the definition of the corrections polynomials, we refer to paper /1/ . It can be seen that in the interval $1.0\hbar\omega_0 < \gamma < 1.5\hbar\omega_0$ the averaged moment of inertia has a plateau indeed. When the number of particles is less than 100, the deviations from the constant value are of the order of 1%. When the particle number is about 114, the plateau becomes somewhat worse, the deviation amounting to 5%. This is connected with the approach of the Fermi level to the unbound levels of the interpolated spectrum. The calculation in the transuranium region with the basic nucleus ^{240}Pu shows that for these nuclei the uncertainties of $\tilde{\mathcal{J}}$ at the plateau

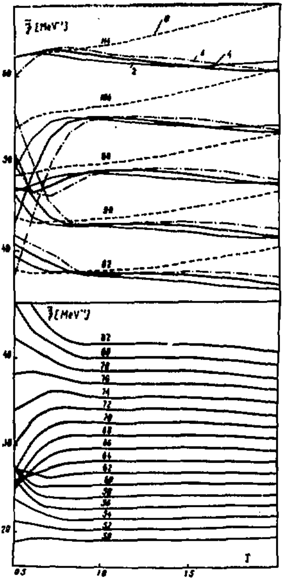

Fig. 1. The averaged moment of inertia as a function of the averaging width γ (in $\hbar\omega_0$ units). The lower part shows the moment of inertia of protons the number of which is indicated above each curve. A 4th order correction polynomial is used. The upper part displays the moment of inertia of neutrons the number of which is also indicated. The different curves refer to different correction polynomials. The order of the polynomial is indicated by the type of the curve and is shown at N= 114. The deformation of the average potential is ϵ =0.28.

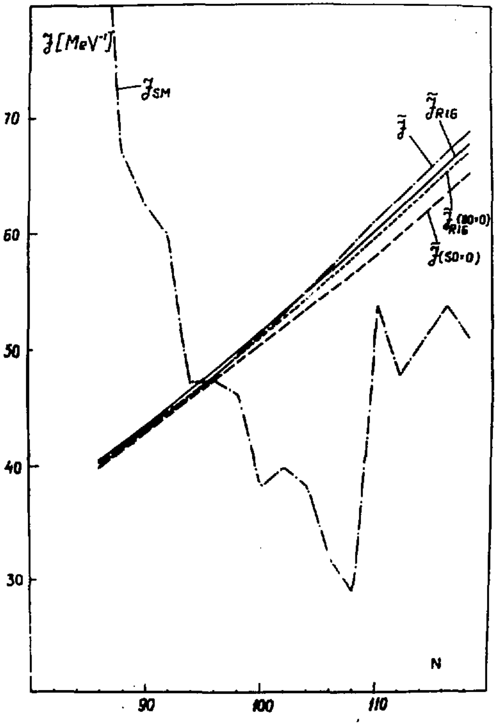

Fig. 2. The moment of inertia of the neutron system as a function of the neutron number N. The different kinds of the moment of inertia are defined in the text, SO=0 means that the spin orbit coupling of the average potential is set approximately equal to zero. The deformation of the average potential is $\epsilon = 0.28$. The averaging width $\gamma = 1.0\hbar\omega_0$.

are smaller than 1%. By comparing with the large fluctuations of $\mathcal{J}_{SM}$ (see, e.g., Fig. 2) one can conclude that the averaged moment of inertia is determined with a good accuracy.

4. Discussion of the Behaviour of the Averaged Moment of Inertia and the Shell Contribution

In Figure 2 we compare the averaged moment of inertia $\bar{\mathcal{J}}$ with the rigid body value $\mathcal{J}_{RIG}$. The calculations show that the difference between $\mathcal{J}^{SM}_{RIG}$ and $\mathcal{J}_{RIG}$ (the definitions are given in Sec. 2) is so small that they would be difficult to distinguish on the scale of Fig. 2. The small difference between $\mathcal{J}^{SM}_{RIG}$ and $\mathcal{J}_{RIG}$ is due to the weak influence of the shell structure on the particle density. As can be seen from Fig. 2, the difference between $\bar{\mathcal{J}}$ and $\mathcal{J}_{RIG}$ is smaller than 1% for $N<100$ and it amounts to 3% for $N>100$. Therefore the two moments of inertia coincide within the limits of accuracy of $\bar{\mathcal{J}}$. This conclusion does not depend on the spin orbit coupling, as shown by the example with a negligible small constant of the spin orbit coupling ($SO = 0$). The difference between $\bar{\mathcal{J}}$ and $\mathcal{J}_{RIG}$ a little bit increases and both moments increase with the particle number somehow more slowly. The calculations show that with a further increase in the number of particles, $\bar{\mathcal{J}}$ oscillates around $\mathcal{J}_{RIG}$.

One can consider the averaged moment of inertia from another, physically more transparent point of view. Strutinsky [2] suggested that the usual averaging over the spectrum of the basis nucleus is approximately equivalent to averaging over the particle number. In this sense we suggest to understand $\bar{\mathcal{J}}$ as the mean value of the moment of inertia of a number of adjacent nuclei of the same shape. The averaging interval is about the number of particles of one shell.

Now we discuss the deviation $\delta\mathcal{J}$ of the moment of inertia from its mean value. Figures 2 and 3 (the case with $T=0$) show the neutron and the proton parts of the moment of inertia $\mathcal{J}_{SM}$ as a function of the number of neutrons N

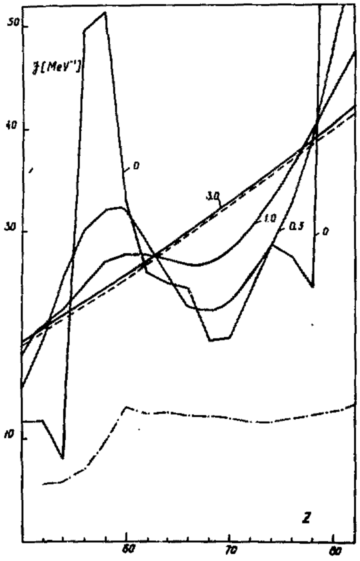

Fig. 3. The moment of inertia of the proton system as a function of the proton number Z. The solid lines show the moment of inertia $\mathcal{J}_T$ calculated at different temperatures. The value of the temperature measured in MeV is denoted by the numbers. The broken line displays the averaged moment of inertia $\bar{\mathcal{J}}$. The dash-dotted line is the moment of inertia in the BCS approximation. The expression for and the value of the strength of the pairing constant are taken from ref. /1/. The deformation of the average potential is $\epsilon = 0.28$. The averaging width $\gamma = 1.0\hbar\omega_0$.

and protons Z, respectively. The intervals considered approximately correspond to one shell. It can be seen that the shell contribution $\delta\mathcal{J} = \mathcal{J}_{SM} - \mathcal{J}$ has a sharp maximum at the beginning of the shell, which is due to the states $\Omega = 1/2$ and $\Omega = 3/2$ descending from the spherical subshell $h_{11/2}$ in the case of protons and $i_{13/2}$ in the case of neutrons. (We denote by Ω the projection of the angular momentum on the symmetry axis). At the end of the shell $\mathcal{J}_{SM}$ becomes smaller than $\mathcal{J}$. The examples shown correspond to a prolate deformation ($\epsilon = 0.28$). For oblate deformations the maximum is shifted to the end of the shell and $\delta\mathcal{J}$ becomes negative at the beginning. Consequently the shell contribution is an oscillating function of the particle number with a period corresponding to the total number of particles in one shell *.

The amplitude of the oscillations depends on the deviation from the spherical shape. With increasing deformation $\delta\mathcal{J}$ decreases, because, as discussed in ref. /11/, each state in the sum (4) couples with an increasing number of other states distributed over a larger and larger energy interval. On the other hand, with decreasing deformation $\delta\mathcal{J}$ increases, because the selection rules for the angular momentum become approximately valid and the distribution of the angular momentum is reduced to only few states near the Fermi surface, i.e., the rotation loses its collective character. In the limit of vanishing deformation, $\delta\mathcal{J}$ becomes infinite reflecting the failure of the perturbation expansion (3) for the determination of $\mathcal{J}_{SM}$. However $\mathcal{J}$ remains finite and equal to the rigid body value. Similarly, in a deformed nucleus there exists a moment of inertia $\mathcal{J}_{11}$ for rotation around the symmetry axis, as the corresponding moment also exists in a heated nucleus, which approaches the rigid body value $\mathcal{J}_{RIG}$ at

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* The moment of inertia may exceed the rigid body value because the nucleus consists of particles with their own angular momenta, i.e., of quantal gyroscopes.

high temperatures. We discussed the reasons for that analogy at the end of Sec. 2. The decrease of $\delta\mathcal{J}$ with increasing deformation is also shown in some figures of ref. /1/. The ratio $\mathcal{J}_{SM}/\mathcal{J}_{RIG}$ plotted there approximately equals $1+\delta\mathcal{J}/\mathcal{J}$. The fission of a nucleus into two nearly spherical fragments is a good example for studying the relation between the shape and the magnitude of $\delta\mathcal{J}$. The results are shown in Fig. 4. It can be seen that $\delta\mathcal{J}$ is very small at large deformations, but it steeply grows when the fragments approach a spherical shape.

In Figure 3 we show the dependence of the moment of inertia on nuclear temperature. One can see that in the case of $T=3$ MeV the influence of the shell structure on the moment of inertia completely vanishes. This reflects the already mentioned result /12/ that in the limit of high temperature $\mathcal{J}_T$ tends to the rigid body value. The difference between $\mathcal{J}$ and $\mathcal{J}_T$ at $T=3$ MeV is connected with the fact that the high temperature limit of the moment of inertia is the rigid body moment corresponding to the particle density in the heated nucleus, whereas $\mathcal{J}$ corresponds to the cold nucleus. Therefore it is not quite accurate to call $\mathcal{J}_T-\mathcal{J}$ the shell contribution in a heated nucleus, but the difference between $\mathcal{J}$ and $\mathcal{J}_{T=3\,MeV}$ is small and lies within the limits of the accuracy of determining $\mathcal{J}$.

It is known that heavy nuclei are no longer in the superfluid state at $T \gtrsim 0.7$ MeV /14/. In the case of rotating nuclei the transition temperature decreases with increasing angular velocity /14/. In the region above the transition our calculations have a direct physical meaning. The magnitude of the shell contribution $\delta\mathcal{J}$ in the region of superfluidity can be estimated very crudely from the amplitude of the oscillations of $\mathcal{J}_T$ at the transition temperature. To illustrate this, we have plotted the moment of inertia in the BCS approximation in Fig. 2. An expression for it can be found in ref. /1/, from which we have also taken the parameters of the BCS model. The "average value" of the moment of inertia in the BCS approximation (this quantity is not determined in this work and should

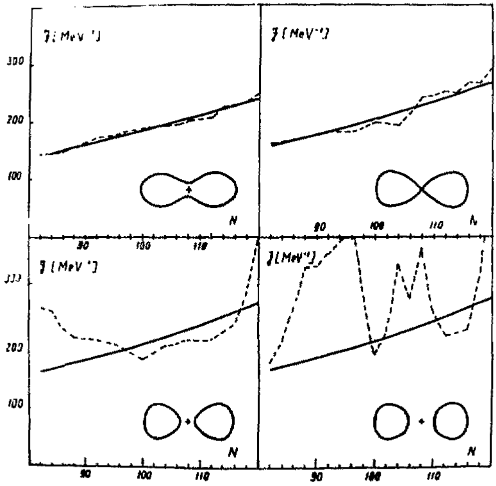

Fig. 4. The moment of inertia of the neutron system as a function of the neutron number at different deformations. The solid line corresponds to $\mathcal{J}$ the broken one to $\mathcal{J}_{SM}$. The deformation parameters are: ϵ = 1.0 and upper left a_4 = -0.05; upper right a_4 = 0.0; lower left a_4 = 0.05; lower right a_4 = 0.10. In the right corner of each figure a cross section through the symmetry axis of the nucleus is shown. As for the definition of ϵ and a_4 see ref. [9]. The averaging width $\gamma = 1.0\,\hbar\omega_0$.

be understood in a heuristic sense) lies considerably below $\mathfrak{J}$.

5. The Shell Effects at Equilibrium Deformation

Now we shall discuss the relation between the shell contribution to the moment of inertia and the equilibrium deformation. In the case of the anisotropic harmonic oscillator potential one can prove that at those values of deformation, where the sum of the single particle energies of the occupied levels has a minimum or a maximum, the shell model value $\mathfrak{J}_{SM}$ is equal to the rigid body moment $\mathfrak{J}^{SM}_{RIG}$ (a closed shell is an exception)[12]. As the differences between $\mathfrak{J}^{SM}_{RIG}$, $\tilde{\mathfrak{J}}_{RIG}$ and $\tilde{\mathfrak{J}}$ are very small, we expect $\delta\mathfrak{J}$ to be approximately equal to 0 at those equilibrium deformations. Figure 5 shows that this result of Bohr and Mottelson is obtained with a good accuracy. The remaining differences should be connected with the slight perturbation of the oscillator potential and the uncertainties in the determination of $\mathfrak{J}$. The shell model value $\mathfrak{J}_{SM}$ shows the typical jumps connected with the crossing of levels. In the case of the realistic Woods-Saxon potential the difference between the minimum of the sum of the single particle energies and the point where $\delta\mathfrak{J}$ vanishes is somewhat larger than for the oscillator potential. According to ref. [1] one should calculate the equilibrium deformation from the minimum of the shell correction to the energy δU for the neutrons and the protons plus the liquid drop energy. In Figure 5 δU for the neutron system is also shown. One can see that all the minima are shifted to a somewhat larger deformation. If the liquid drop energy is taken into account, this effect is compensated for to some extent. To summarize this, one can say that at the equilibrium deformation the shell contribution to the moment of inertia is finite though not very large. The pairing causes an additional reduction of the shell effects. Hence we can expect that the moments

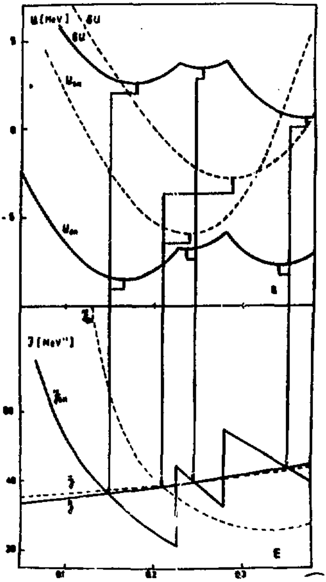

Fig. 5. The averaged moment of inertia $\tilde{\mathfrak{J}}$, the shell model value of the moment of inertia $\mathfrak{J}_{SM}$, the sum of the energies of the occupied levels U_{SM} and the shell correction to the energy δU as a function of the deformation ϵ. All quantities refer to the neutron system with $N=100$. The solid lines correspond to the (slightly disturbed) harmonic oscillator, the broken ones to the Woods-Saxon potential. The arrows indicate the minima of U_{SM} and δU and the crossing points $\mathfrak{J}_{SM} = \tilde{\mathfrak{J}}$. The averaging width $\gamma = 1.0\hbar\omega_0$.

of inertia of the even-even nuclei in the rare earth and actinide region are mainly determined by the nuclear shape and the strength of the pair correlations, because for these nuclei the minima of the shell correction δU of the neutron and proton systems are close to each other and U_{LDM} plays a relatively insignificant role.

There arises the question as when one can expect larger values of $\delta \mathcal{J}$. We would like to point out to some possibilities.

1) A classical example from atomic physics is the magnetic moment of the electron cloud of a rotating molecule. The magnetic moment can be calculated using the cranking model in analogy to the moment of inertia. It has been found experimentally that the moment deviates from the value corresponding to the rigid rotation of the electron cloud /15/.

2) One can expect an increase of $\delta \mathcal{J}$ with increasing difference between the deformations, at which the shell corrections δU to the neutron and the proton systems have their minima. Such a situation may exist in the case of shape isomers /16/.

3) If two nuclei are very close, the Coulomb interaction polarizes the nuclei and their shape does not correspond to the minimum of δU. This situation, which is somewhat analogous to case 1), can be encountered in the fission process or in the scattering of heavy ions.

4) In the case of high spin states the equilibrium deformation deviates from the minimum of δU, and, in addition, the pair correlations are reduced by the rotation. Of course, one cannot introduce the moment of inertia by the perturbation theory, but rather one has to take nonadiabatic effects into account.

5) In the case of large deformations (fissioning isomers), there are two compensating effects. On the one hand, as the numerical investigations of ref. /1/ show, the zero points of $\delta \mathcal{J}$ (in ref. /1/ $\mathcal{J}_{SM} / \mathcal{J}_{RIG} = 1 + \delta \mathcal{J} / \mathcal{J}$ is plotted) and the extrema of δU become uncorrelated at large deformations. On the other hand, the magnitude of $\delta \mathcal{J}$ is strongly reduced at these deformations (see above). For a more accurate estimate one should carry out detailed calculations.

6. Summary and Conclusions

By means of Strutinsky's averaging procedure we proposed a partition of the moment of inertia into an average part $\tilde{\mathcal{J}}$ and a shell contribution $\delta\mathcal{J}$. We carried out numerical calculations demontsrating that $\tilde{\mathcal{J}}$ (and therefore $\delta\mathcal{J}$) as a function of the averaging width γ has a plateau. This means that the conception of the averaged moment of inertia $\tilde{\mathcal{J}}$ is defined unambiguosly. This is a new result, because the expression for the moment of inertia deviates considerably from that for the binding energy. The moment of inertia has the form of a double sum containing an energy denominator, which restricts the sum only to states near the Fermi surface. Therefore it is possible to calculate $\mathcal{J}_{SM}$ and $\tilde{\mathcal{J}}$ within the shell model with sufficient accuracy in order to be able to speak about their physical meaning. In this sense the situation with the moment of inertia substantially differs from that with the averaged binding energy.

By analogy with the binding energy, $\tilde{\mathcal{J}}$ can be interpreted as the average value of the moment of inertia of a group of nuclei of the same shape. This mean value coincides with the rigid body value with a very good accuracy. This coincidence permits the interpretation of the investigations, in which the rigid body value is used, as those relevant to the average behaviour. For instance, in ref. [17] the nuclear shape of a rotating nucleus is investigated. In this paper the energy is assumed to consist of the potential energy of a liquid drop and the kinetic energy corresponding to the rigid body moment of inertia. The equilibrium shape is found as the minimum of the total energy at a given angular momentum. In this way the yrast line $E(I)$ is found. Now we can interprete this calculation in the following way. The calculated yrast line describes the mean behaviour obtained by averaging over the large (1 shell) number of nuclei of the same shape. The real yrast levels in each nucleus may be widely scattered around this value [18].

In this work we consider only the division of $\mathcal{J}_{SM}$ calculated in the cranking model, into $\tilde{\mathcal{J}}$ and $\delta\mathcal{J}$, but we hope that an analogous partition is also possible in other models. The coincidence of $\mathcal{J}$ with the rigid body value justifies the suggestion that one can calculate $\mathcal{J}$ with the usual approximations employed for a large system, e.g., for nuclear matter. It is known that the moment of inertia of a gas of non-interacting fermions in a large cubic box equals the rigid body value /19/. Long range correlations of the RPA-type do not alter the result /20/. Only short range correlations of the pairing type considerably reduce the moment of inertia (refs. /21-23/), but the influence of the pairing correlations is not investigated in this work systematically.

In a finite system the moment of inertia even without pairing differs from the rigid body value. The shell contribution $\delta\mathcal{J}$ is an oscillating function of the particle number, as well as of the parameters determining the nuclear shape. However this fluctuating behaviour is difficult to observe: it is shown for the anisotropic harmonic oscillator that the difference $\mathcal{J}_{SM} - \mathcal{J}_{RIG}^{SM} \approx \delta\mathcal{J}$ vanishes at the equilibrium deformation /12/. Our calculations show that this is approximately valid also for the realistic Woods-Saxon potential with spin-orbit coupling if the deformation is not too large ($\epsilon < 0.4$). At large deviations from the spherical shape the correlation gets loose. But one can expect a significant influence of the shell structure on the moment of inertia under certain circumstances, some of which are discussed at the end of Section 5.

As was mentioned in the introduction, we consider the investigation of the moments of inertia as a first, simple step in the study of the mass coefficients of other collective modes. Finally, we would like to discuss the general conclusions that can be drawn for the mass parameters from our investigation of the moment of inertia.

The main difference between the moment of inertia and the other mass parameters consists in the fact that the latter ones have the form of a sum whose terms are divided by the difference of the single particle energies in the third power. This leads to a more rapid reduction of the contributions of the levels with their increasing distance from the Fermi surface. Therefore the shell contribution becomes of still greater relative importance. However we hope that the averaging procedure is applicable to other mass parameters with the same success as to the moment of inertia. In the case of the moment of inertia the averaged value of $\mathcal{J}$ is equal to the rigid body one. As regards the other types of mass parameter, it is unclear both what kind of average behaviour should be expected, and what role the interaction between the particles plays.

As is shown, e.g., by the calculations in ref. /1/, the oscillating dependence of the mass parameters on the particle number and nuclear shape is a general feature. However the oscillations of the mass parameter of the fission mode are in another way correlated with the shell correction to the binding energy.

In the case of the moment of inertia the amplitude of these oscillations rapidly decreases with inreasing deformation, but after fission the magnitude of $\delta\mathcal{J}$ is determined by the deformation of the two fragments. This distinction of the spherical shape is probably restricted to the rotational motion. The damping out of the shell effects with increasing temperature is certainly a common feature of all mass parameters.

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