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APPROXIMATE SOLUTION OF THE EQUATIONS OF THE MODEL FOR THE DESCRIPTION OF HIGHLY EXCITED STATES OF DOUBLY EVEN DEFORMED NUCLEI

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1. Introduction

The problem of how the transition from the simple structure and small density of low excitations to the complex structure and large density of high excitations occurs seems to remain still unsolved theoretically. This is explained by the fact that the structure of intermediate and highly excited states is unknown and there is no unified nuclear theory. By tradition, the lower part of the spectrum is described, e.g., in the framework of the superfluid nuclear model [1], while highly excited states are treated by the statistical model [2]. The semi-microscopic approach of the superfluid nuclear model has proved to be good in the region of low excitations. It is important to generalize it so that it would be suitable for a qualitative study of highly excited states. To this end, of much importance was the idea to express the complexity of highly excited states in terms of the operator wave function [3] which was defined in such a manner that there was a hierarchy of its components with different number of quasiparticles. A systematic study of the new approach is given in ref. [4]. It has made a good start for creating models for the description of structure complications with increasing excitations [5-7]. The models mentioned above are based on the account of the quasiparticle-phonon interaction.

In refs. [5-7] systems of the basic equations of the model are obtained for the solution of which it is necessary to diagonalize matrices of the rank 10^4 and higher. This fact forces us to apply to approximate methods of solving. The approximate solution found in ref. [5], when the coherent terms alone were taken into account, was found to be rough. In addition, in this approximation there appeared superfluous solutions which are hardly separable from the true solutions. A decisive step toward solving the basic equations of the model was made in ref. [8], where the so-called one-pole approximation was used to solve the problem in an analytic form.

The aim of the present paper is to generalize the model suggested in ref. [5] to the description of highly excited states of doubly even deformed nuclei and obtain an approximate solution on the basis of the method developed in ref. [8].

2. Formulation of the Model for Doubly Even Deformed Nuclei

The model for the case of a doubly even deformed nucleus is formulated in the framework of the semimicroscopic description in nuclear theory in just the same manner as in refs. [5-7]. The model Hamiltonian is taken in the form of the potential describing the average field of the neutron and proton systems, the interactions leading to superconducting pairing correlations and multipole-multipole interactions. Taking into account the secular equations for determining the phonon energies ω_g (where g denotes $\lambda\mu j$, j being the number of the secular equation root) we can write the Hamiltonian in the form

$$H = H_v + H_{vq} = \sum_g \omega_g Q_g^+ Q_g -$$

$$-\frac{1}{2} \sum_{g,\nu,\nu'} \{ [\Gamma^g(\nu,\nu') B(\nu,\nu') + \bar{\Gamma}^g(\nu,\nu') \bar{B}(\nu,\nu')] (Q_g^+ +$$

$$+ Q_g) + h.c. \} . \qquad (1)$$

The phonon operator Q_g is

$$Q_g = \frac{1}{2} \sum_{\nu,\nu'} [\Psi^g_{\nu\nu'} A(\nu,\nu') - \Phi^g_{\nu\nu'} A^+(\nu,\nu') + \bar{\Psi}^g_{\nu\nu'} \bar{A}(\nu,\nu') -$$

$$- \bar{\Phi}_{\nu\nu'} \bar{A}^{+}(\nu,\nu')] ,$$

$$\Gamma^{g}(\nu,\nu') = \frac{v_{\nu\nu'}}{2\sqrt{Y_g}} \cdot f^{\bar{g}}(\nu,\nu') , \quad \bar{\Gamma}^{g}(\nu,\nu') = \frac{v_{\nu\nu'}}{2\sqrt{Y_g}} \cdot \bar{f}^{\bar{g}}(\nu,\nu') , \quad \bar{g} = \lambda\mu ,$$

$(\nu\sigma)$ is the set of the quantum numbers characterizing the single-particle average-field level, $\sigma = \pm 1$, the remaining notation is the same as in refs.[1,5].

In the present model we take the following wave function

$$\Psi_i(K_0^{\pi_0}) = C^i_{g_0} \Big(Q^+_{g_0} + \frac{1}{\sqrt{2!}} \sum_{g_1, g_2} F^{g_1 g_2}_{g_0 i} Q^+_{g_1} Q^+_{g_2} +$$

$$+ \frac{1}{\sqrt{3!}} \sum_{g_1, g_2, g_3} P^{g_1 g_2 g_3}_{g_0 i} Q^+_{g_1} Q^+_{g_2} Q^+_{g_3} + \qquad (2)$$

$$+ \frac{1}{\sqrt{4!}} \sum_{g_1 g_2 g_3 g_4} R^{g_1 g_2 g_3 g_4}_{g_0 i} Q^+_{g_1} Q^+_{g_2} Q^+_{g_3} Q^+_{g_4} \Big) \Psi_0 ,$$

where Ψ_0 is the phonon vacuum, i is the number of an excited state, the functions F, P and R are completely symmetric with respect to the indices $g_1 g_2 g_3 g_4$. The fact that the wave function (2) has many components makes it suitable for obtaining a model of the complex structure of highly excited states. For example

$$(C^i_{g_0} F^{g_1 g_2}_{g_0 i})^2$$

defines the contribution of the two-phonon component $g_1 g_2$ to the wave function (2) normalization. The latter has the following form

$$(\Psi_i^* \Psi_i) = 1 = (C^i_{g_0})^2 [1 + \sum_{G_2} (F^{G_2}_{g_0 i})^2 + \sum_{G_3} (P^{G_3}_{g_0 i})^2 + \sum_{G_4} (R^{G_4}_{g_0 i})^2] . \qquad (3)$$

Here G_n means $g_1, g_2, \ldots, g_n$ and the summation $\sum_{G_n}$ is a brief notation of $\sum_{g_1, g_2, \ldots, g_n}$.

The average value of the Hamiltonian (1) over the state (2) has the form

$$(\Psi_i^* H \Psi_i) = (C^i_{g_0})^2 [\omega_{g_0} + \sum_{G_2} \Omega_{G_2} (F^{G_2}_{g_0 i})^2 + \sum_{G_3} \Omega_{G_3} (P^{G_3}_{g_0 i})^2 + \sum_{G_4} \Omega_{G_4} (R^{G_4}_{g_0 i})^2 - \qquad (4)$$

$$- 2 \sum_{G_2} U^{G_2}_{g_0} F^{G_2}_{g_0 i} - 2 \sum_{G_3 G_2'} U^{G_3}_{G_2'} P^{C_3}_{g_0 i} F^{G_2'}_{g_0 i} -$$

$$- 2 \sum_{G_4} U^{g_0}_{G_4} R^{G_4}_{g_0 i} - 2 \sum_{G_4, G_3'} U^{G_4}_{G_3'} R^{G_4}_{g_0 i} P^{G_3'}_{g_0 i}] .$$

Here $\Omega_{G_n} = \omega_{g_1} + \omega_{g_2} + \ldots + \omega_{g_n}$, U represent the matrix elements of the second term H between the states with different number of phonons, e.g.,

$$U^{G_3}_{G_2'} = \frac{1}{\sqrt{2!\ \ 3!}} (\Psi_0^* Q_{g_3} Q_{g_2} Q_{g_1} H_{vq} Q^+_{g_1'} Q^+_{g_2'} \Psi_0) ,$$

$U^{G_2}_{g_0}$ corresponds to the quantity $U^{g_1}_{g_2}(g_0)$ introduced in ref. /9 /:

$$U^{G_2}_{g_0} = \frac{1}{\sqrt{2!}} \sum_{\nu,\nu',\nu_2} \{ [\Gamma^{g_1}(\nu,\nu') (\Psi^{g_2}_{\nu\nu_2} \Psi^{g_0}_{\nu'\nu_2} +$$

$$\Phi^{g_2}_{\nu'\nu_2} \Phi^{g_0}_{\nu\nu_2} + \bar{\Psi}^{g_2}_{\nu\nu_2} \bar{\Psi}^{g_0}_{\nu'\nu_2} + \bar{\Phi}^{g_2}_{\nu'\nu_2} \bar{\Phi}^{g_0}_{\nu\nu_2}) +$$

$$+ \bar{\Gamma}^{g_1}(\nu,\nu') (\Psi^{g_2}_{\nu\nu_2} \bar{\Psi}^{g_0}_{\nu_2\nu'} + \Phi^{g_2}_{\nu'\nu_2} \bar{\Phi}^{g_0}_{\nu\nu_2} + \bar{\Psi}^{g_2}_{\nu\nu_2} \Psi^{g_0}_{\nu'\nu_2} +$$

$$+ \bar{\Phi}^{g_2}_{\nu_2\nu'} \Phi^{g_0}_{\nu\nu_2}) + [g_1 \rightleftarrows g_2]] + \Gamma^{g_0}(\nu,\nu') \times$$

$$\times (\Psi^{g_1}_{\nu\nu_2} \Phi^{g_2}_{\nu'\nu_2} + \Phi^{g_1}_{\nu'\nu_2} \Psi^{g_2}_{\nu\nu_2} + \bar{\Psi}^{g_1}_{\nu\nu_2} \bar{\Phi}^{g_2}_{\nu'\nu_2} + \bar{\Phi}^{g_1}_{\nu'\nu_2} \bar{\Psi}^{g_2}_{\nu\nu_2}) +$$

$$+ \bar{\Gamma}^{g_0}(\nu,\nu') (\Psi^{g_1}_{\nu'\nu_2} \bar{\Phi}^{g_2}_{\nu\nu_2} + \bar{\Psi}^{g_1}_{\nu_2\nu'} \Phi^{g_2}_{\nu\nu_2} + \bar{\Psi}^{g_2}_{\nu_2\nu'} \Phi^{g_1}_{\nu\nu_2} +$$

$$+ \Psi^{g_2}_{\nu'\nu_2} \bar{\Phi}^{g_1}_{\nu\nu_2}) \} . \qquad (5)$$

$$U^{G_3}_{G_2'} = \frac{1}{\sqrt{3!}} (U^{g_1 g_2}_{g_1'} \delta_{g_2' g_3} + U^{g_1 g_3}_{g_1'} \delta_{g_2' g_2} +$$

$$+ U^{g_2 g_3}_{g_1'} \delta_{g_2' g_1} + U^{g_1 g_2}_{g_2'} \delta_{g_1' g_3} + U^{g_1 g_3}_{g_2'} \delta_{g_1' g_2} +$$

$$+ U^{g_2 g_3}_{g_2'} \delta_{g_1' g_1}) \equiv \frac{1}{\sqrt{3!}} S^{G_3}_{G_2'} (U^{g_1 g_2}_{g_1'} \delta_{g_2' g_3}) . \quad (6)$$

$S^{G_3}_{G_2'}$ implies a symmetrization with respect to the indices $g_1' \, g_2'$ and to indices $g_1 g_2 g_3$.

$$U^{g_0}_{G_4} = \frac{1}{\sqrt{4!}} S_{G_4} (V_{g_2 g_3 g_4} \delta_{g_1 g_0}) , \quad (7)$$

$$V_{g_2 g_3 g_4} = \sum_{\nu, \nu', \nu_2} \Gamma^{g_2} (\nu, \nu') (\Psi^{g_3}_{\nu\nu_2} \Phi^{g_4}_{\nu'\nu_2} +$$

$$+ \Psi^{g_4}_{\nu\nu_2} \Phi^{g_3}_{\nu'\nu_2} + \bar{\Psi}^{g_3}_{\nu\nu_2} \bar{\Phi}^{g_4}_{\nu'\nu_2} + \bar{\Psi}^{g_4}_{\nu\nu_2} \bar{\Phi}^{g_3}_{\nu'\nu_2}) +$$

$$+ \bar{\Gamma}^{g_2} (\nu, \nu') (\bar{\Psi}^{g_3}_{\nu_2 \nu'} \Phi^{g_4}_{\nu\nu_2} + \Psi^{g_3}_{\nu_2 \nu'} \bar{\Phi}^{g_4}_{\nu\nu_2} + \Psi^{g_4}_{\nu'\nu_2} \bar{\Phi}^{g_3}_{\nu\nu_2} +$$

$$+ \bar{\Psi}^{g_4}_{\nu_2 \nu'} \Phi^{g_3}_{\nu_2 \nu}) . \quad (8)$$

Finally

$$U^{G_4}_{G_3'} = \frac{1}{\sqrt{3! \; 4!}} S^{G_4}_{G_3'} (V^{g_1 g_2}_{g_1'} \delta_{g_2' g_4} \delta_{g_3' g_3}) , \quad (9)$$

where

$$V^{g_1 g_2}_{g_1'} = \sum_{\nu, \nu', \nu_2} \{ [\Gamma^{g_1} (\nu, \nu') (\Psi^{g_1'}_{\nu\nu_2} \Psi^{g_2}_{\nu'\nu_2} + \Phi^{g_1'}_{\nu'\nu_2} \Phi^{g_2}_{\nu\nu_2} +$$

$$+ \bar{\Psi}^{g_1'}_{\nu\nu_2} \bar{\Psi}^{g_2}_{\nu'\nu_2} + \bar{\Phi}^{g_1'}_{\nu'\nu_2} \bar{\Phi}^{g_2}_{\nu\nu_2}) + \bar{\Gamma}^{g_1}(\nu,\nu') \times$$

$$\times (\Psi^{g_1'}_{\nu\nu_2} \bar{\Psi}^{g_2}_{\nu_2\nu'} + \bar{\Psi}^{g_1'}_{\nu\nu_2} \Psi^{g_2}_{\nu'\nu_2} + \Phi^{g_1'}_{\nu'\nu_2} \bar{\Phi}^{g_2}_{\nu\nu_2} + \bar{\Phi}^{g_1'}_{\nu_2\nu'} \Phi^{g_2}_{\nu\nu_2})] +$$

$$+ \frac{1}{2} [\Gamma^{g_1'}(\nu,\nu') (\Psi^{g_1}_{\nu\nu_2} \Phi^{g_2}_{\nu'\nu_2} + \Phi^{g_1}_{\nu'\nu_2} \Psi^{g_2}_{\nu\nu_2} + \bar{\Psi}^{g_1}_{\nu\nu_2} \bar{\Phi}^{g_2}_{\nu'\nu_2} +$$

$$+ \bar{\Phi}^{g_1}_{\nu'\nu_2} \bar{\Psi}^{g_2}_{\nu\nu_2}) + \bar{\Gamma}^{g_1'}(\nu,\nu') (\Phi^{g_1}_{\nu\nu_2} \bar{\Psi}^{g_2}_{\nu_2\nu'} + \bar{\Phi}^{g_1}_{\nu\nu_2} \Psi^{g_2}_{\nu'\nu_2} +$$

$$+ \Psi^{g_1}_{\nu'\nu_2} \bar{\Phi}^{g_2}_{\nu\nu_2} + \bar{\Psi}^{g_1}_{\nu_2\nu'} \Phi^{g_2}_{\nu\nu_2})] \} . \qquad (10)$$

3. Exact Solutions for the Model

The energies η_i of nonrotational states and the functions $C^i_{g_0}$, $F^{G_2}_{g_0 i}$, $P^{G_3}_{g_0 i}$ and $R^{G_4}_{g_0 i}$ are determined from the variational principle:

$$\delta \{ (\Psi_i^* H \Psi_i) - \eta_i [(\Psi_i^* \Psi_i) - 1] \} = 0 . \qquad (11)$$

After performing some transformations we get the following system of basic equations

$$F^{G_2}_{g_0 i} = (\Omega_{G_2} - \eta_i)^{-1} (U^{G_2}_{g_0} + \sum_{G_3'} U^{G_3'}_{G_2} P^{G_3'}_{g_0 i}) , \qquad (12)$$

$$R^{G_4}_{g_0 i} = (\Omega_{G_4} - \eta_i)^{-1} \left(U^{g_0}_{G_4} + \sum_{G'_3} U^{G_4}_{G'_3} P^{G'_3}_{g_0 i} \right) , \quad (13)$$

$$(\Omega_{G_3} - \eta_i) P^{G_3}_{g_0 i} - \sum_{G'_3} \left[\sum_{G''_2} \frac{U^{G_3}_{G''_2} U^{G'_3}_{G''_2}}{\Omega_{G''_2} - \eta_i} + \sum_{G''_4} \frac{U^{G''_4}_{G_3} U^{G''_4}_{G'_3}}{\Omega_{G''_4} - \eta_i} \right] P^{G'_3}_{g_0 i} = L^{G_3}_{g_0} (\eta_i) , \quad (14)$$

$$\omega_{g_0} - \eta_i - K_{g_0}(\eta_i) - \sum_{G_3} L^{G_3}_{g_0}(\eta_i) P^{G_3}_{g_0 i} = 0 . \quad (15)$$

where

$$L^{G_3}_{g_0}(\eta_i) = \sum_{G'_2} \frac{U^{G_3}_{G'_2} U^{G'_2}_{g_0}}{\Omega_{G'_2} - \eta_i} + \sum_{G'_4} \frac{U^{G'_4}_{G_3} U^{g_0}_{G'_4}}{\Omega_{G'_4} - \eta_i} . \quad (16)$$

$$K_{g_0}(\eta_i) = \sum_{G_2} \frac{(U^{G_2}_{g_0})^2}{\Omega_{G_2} - \eta_i} + \sum_{G_4} \frac{(U^{g_0}_{G_4})^2}{\Omega_{G_4} - \eta_i} . \quad (17)$$

The function $C^i_{g_0}$ is determined by the formula (3). The secular equation for finding η_i is written in the form (15). It is obvious that in order to solve it we have at first to find $P^{G_3}_{g_0 i}$ This coefficients obey the linear inhomogeneous system (14). Owing to a huge rank of the matrix of this system (larger than 10^4) the solution according to the Krammer theorem is cumbersome. We have therefore to apply to an approximate method of solving (14).

4. Approximate Solutions

We rewrite the system (14) as follows

$$\left[\Omega_{G_3} - \eta_i - \sum_{G_2''} \frac{(U^{G_3}_{G_2''})^2}{\Omega_{G_2''} - \eta_i} - \sum_{G_4''} \frac{(U^{G_4''}_{G_3})^2}{\Omega_{G_4''} - \eta_i} \right] P^{G_3}_{g_0 i} -$$

$$- \sum_{G_3' \neq G_3} \left[\sum_{G_2''} \frac{U^{G_3}_{G_2''} \, U^{G_3'}_{G_2''}}{\Omega_{G_2''} - \eta_i} + \sum_{G_4''} \frac{U^{G_4''}_{G_3} \, U^{G_4''}_{G_3'}}{\Omega_{G_4''} - \eta_i} \right] P^{G_3'}_{g_0 i}$$

$$= L^{G_3}_{g_0} (\eta_i) \; ,$$

where $G_3' \neq G_3$ implies that in the sum over G_3' only one term with $G_3' = G_3$ is absent.

The expressions of the type $\sum_{G_n'} \frac{(U^{G_3}_{G_n'})^2}{\Omega_{G_n'} - \eta_i}$ $(n = 2,4)$ contain the squared values of $U^{G_3}_{G_n'}$ and, therefore, such sums are called coherent, while the sums of the type $\sum_{G_n''} \frac{U^{G_3}_{G_n''} \, U^{G_3'}_{G_n''}}{\Omega_{G_n''} - \eta_i}$, $G_3 \neq G_3'$ are called noncoherent. When we neglect the noncoherent sums we get the function $P^{G_3}_{g_0 i}$ which is, in this case denoted as $\tilde{P}^{G_3}_{g_0 i}$

$$\tilde{P}^{G_3}_{g_0 i} = \frac{L^{G_3}_{g_0}(\eta_i)}{b^{G_3}(\eta_i)} , \tag{18}$$

$$b^{G_3}(\eta_i) = \Omega_{G_3} - \eta_i - \sum_{G_2'} \frac{(U^{G_3}_{G_2'})^2}{\Omega_{G_2'} - \eta_i} - \sum_{G_4'} \frac{(U^{G_4'}_{G_3})^2}{\Omega_{G_4'} - \eta_i} . \tag{19}$$

Inserting $\tilde{P}^{G_3}_{g_0 i}$ in (15) we obtain the secular equation in the explicit form. Such type solutions for an odd deformed nucleus were first obtained in ref. /5/ and were called coherent. They are undoubtedly invalid when the noncoherent sums $\sum_{G_n''} \frac{U^{G_3}_{G_n''} U^{G_3'}_{G_n''}}{\Omega_{G_n''} - \eta_i}$ contain at least one-pole term, and superfluous solutions appear in this case.

In ref. /8/ the idea about the necessity of taking into account the pole noncoherent terms was suggested. This means that one does not reject the noncoherent sum as a whole but does reject its non-pole part alone. If we restrict ourselves to the one-pole approximation then the contribution of the term $\frac{U^{G_3}_{G_n^0} U^{G_3'}_{G_n^0}}{\Omega_{G_n^0} - \eta_i}$ from the pole $\Omega_{G_n^0}$ in $\sum_{G_n''} \frac{U^{G_3}_{G_n''} U^{G_3'}_{G_n''}}{\Omega_{G_n''} - \eta_i}$ is considered essential and is taken into account. Following ref. /8/, we shall refer to $\Omega_{C_n^0}$ as an n-phonon fundamental pole. For each fundamental pole we find its approximate expression for the system of equations (14) and solve it.

We find an approximate solution for eqs. (14) corresponding to the functional pole $\Omega_{G_2^0} = \omega_{g_1^0} + \omega_{g_2^0}$. In eqs. (14) we reject all the noncoherent terms for the exception of those which contain $(\Omega_{G_2^0} - \eta_i)^{-1}$ and after simple transformation we have

$$P^{G_3}_{g_0 i} - \frac{1}{\Omega_{G_2^0} - \eta_i} \cdot \frac{U^{G_3}_{G_2^0}}{b^{G_3}(\eta_i\, ; G_2^0)} \sum_{G_3'} U^{G_3'}_{G_2^0} P^{G_3'}_{g_0 i} = \frac{L^{G_3}_{g_0}(\eta_i)}{b^{G_3}(\eta_i\, ; G_2^0)} \; ; \tag{20}$$

where

$$b^{G_3}(\eta_i\, ; G_2^0) = \Omega_{G_3} - \eta_i - \sum_{G_2' \neq G_2^0} \frac{(U^{G_3}_{G_2'})^2}{\Omega_{G_2'} - \eta_i} - \sum_{G_4'} \frac{(U^{G_4'}_{G_3})^2}{\Omega_{G_4'} - \eta_i} \, . \tag{21}$$

In what follows when the arguments of the functions L, K and b contain in addition to η_i the quantity G_n^0 this means that the functions L, K and b do not contain pole terms $(C_n^0 - \eta_i)^{-1}$.

To solve the system (20) we use the result obtained in ref. /8/ according to which the solution for the system

$$\sum_j (\delta_{ij} - a_{ij})\, x_j = y_i \tag{22}$$

under the condition

$$a_{ij}\, a_{i'j'} = a_{ij'}\, a_{i'j} \tag{22'}$$

reads

$$x_i = y_i + \frac{\sum_j a_{ij}\, y_j}{\Delta}, \tag{23}$$

where the determinant of the system (22) is equal to

$$\Lambda = 1 - \sum_i a_{ii} \; . \tag{24}$$

For the system of equations (22) the condition (22') holds. In fact,

$$\frac{U^{G_3}_{G_2^0}\; U^{G_3'}_{G_2^0}}{(\Omega_{G_2^0} - \eta_i)\, b^{G_3}(\eta_i\, ; G_2^0)} \cdot \frac{U^{\tilde{G}_3}_{G_2^0}\; U^{\tilde{G}_3'}_{G_2^0}}{(\Omega_{G_2^0} - \eta_i)\, b^{\tilde{G}_3}(\eta_i\, ; G_2^0)} =$$

$$= \frac{U^{G_3}_{G_2^0}\; U^{G_3'}_{G_2^0}}{(\Omega_{G_2^0} - \eta_i)\, b^{G_3}(\eta_i\, ; G_2^0)} \cdot \frac{U^{\tilde{G}_3}_{G_2^0}\; U^{G_3'}_{G_2^0}}{(\Omega_{G_2^0} - \eta_i)\, b^{\tilde{G}_3}(\eta_i\, ; G_2^0)},$$

therefore the system (20) can be solved in an analytic form. Taking into consideration (24) we get

$$\Lambda(G_2^0\, ; \eta_i) = 1 - \frac{1}{\Omega_{G_2^0} - \eta_i} \sum_{G_3} \frac{\left(U^{G_3}_{G_2^0}\right)^2}{b^{G_3}(\eta_i\, ; G_2^0)} \; . \tag{25}$$

From eqs. (15) and (23) it follows that the poles of the secular equation (15) are determined from the condition $\Lambda = 0$ from where

$$\Omega_{G_2^0} - \eta^{pol} = \sum_{G_3} \frac{(U^{G_3}_{G_2^0})^2}{b^{G_3}(\eta^{pol}; G_2^0)} . \qquad (25')$$

The quasiparticle-phonon interaction has led to a shift of η^{pol} with respect to the fundamental pole. In the expression for $b^{G_3}(\eta_i; G_2^0)$ we replace η_i by η^{pol} and use the formula (25'). Then for the determinant we get the following expression

$$\Delta(G_2^0; \eta_i) = \frac{\eta^{pol} - \eta_i}{\Omega_{G_2^0} - \eta_i} . \qquad (26)$$

Utilizing eq. (23) we find the solution for eq. (20) in the form

$$P^{G_3}_{g\,i} = \frac{L^{G_3}_{g_0}(\eta_i)}{b^{G_3}(\eta_i; G_2^0)} + \frac{1}{b^{G_3}(\eta_i; G_2^0)} .$$

$$\cdot \frac{U^{G_3}_{G_2^0}}{\eta^{pol} - \eta_i} \sum_{G_3'} \frac{U^{G_3'}_{G_2^0}}{b^{G_3'}(\eta_i; G_2^0)} L^{G_3'}_{g_0}(\eta_i) . \qquad (27)$$

If either g_1 or g_2 or g_3 are not equal to g_1^0 or g_2^0 then from (6) it follows that the second terms of (27) vanishes and then the function $P^{G_3}_{g_0 i}$ is close to $\tilde{P}^{G_3}_{g_0 i}$ defined by eq. (18).

We find the solution for eq. (15) corresponding to the fundamental pole $\Omega_{G_2^0}$ and lying near the pole η^{pol} defined by eq. (25). Inserting the function $P^{G_3}_{g_0 i}$ in the form (27) in eq. (15) we single out the pole terms

$$L_{g_0}^{G_3}(\eta_i) = \frac{U_{G_2^0}^{G_3} \, U_{g_0}^{G_2^0}}{\Omega_{G_2^0} - \eta_i} + L_{g_0}^{G_3}(\eta_i ; G_2^0) \qquad (28)$$

and after simple transformations we get the following secular equation

$$\omega_{g_0} - \eta_i - K_{g_0}(\eta_i ; G_2^0) - \sum_{G_3} \frac{[\,L_{g_0}^{G_3}(\eta_i ; G_2^0)\,]^2}{b^{G_3}(\eta_i ; G_2^0)} -$$

$$- \frac{1}{\eta^{pol} - \eta_i} [\, U_{g_0}^{G_2^0} + \sum_{G_3} \frac{U_{G_2^0}^{G_3} \, L_{g_0}^{G_3}(\eta_i ; G_2^0)}{b^{G_3}(\eta_i ; G_2^0)} \,]^2 = 0 \;,$$

(29)

where $K(\eta_i ; G_2^0)$ is defined by eq. (28). Perform a further simplification of eq. (29). In all the terms but $\eta^{pol} - \eta_i$ we replace η_i by η^{pol} and obtain

$$\eta^{pol} - \eta_i =$$

$$= \frac{[\, U_{g_0}^{G_2^0} + \sum_{G_3} \dfrac{U_{G_2^0}^{G_3}}{b^{G_3}(\eta^{pol} ; G_2^0)} L_{g_0}^{G_3}(\eta^{pol} ; G_2^0) \,]^2}{\omega_{g_0} - \eta^{pol} - K_{g_0}(\eta^{pol} ; G_2^0) - \sum_{G_3} \dfrac{[\,L_{g_0}^{G_3}(\eta^{pol} ; G_2^0)\,]^2}{b^{G_3}(\eta^{pol} ; G_2^0)}}$$

(30)

The value of η_i found from eq. (30) should be considered as the first approximation. Then by the iteration method it is possible to obtain the solution for eq. (29).

We find the approximate solution for eqs. (14) and (15) corresponding to the fundamental pole $\Omega_{G_4^0} = \omega_{g_1^0} + \omega_{g_2^0} + \omega_{g_3^0} + \omega_{g_4^0}$. We perform the same transformations as in the case of finding the expressions (26) and (27) and equation (29). As a result, we obtain

$$P^{G_3}_{g_0 i} = \frac{L^{G_3}_{g_0}(\eta_i ; G_4^0)}{b^{G_3}(\eta_i ; G_2^0)} + \frac{1}{\eta^{pol} - \eta_i} \cdot \frac{U^{G_4^0}_{G_3}}{b^{G_3}(\eta_i ; G_2^0)} [U^{g_0}_{G_4^0} + \sum_{G_3'} \frac{U^{G_4^0}_{G_3'} L^{G_3'}_{g_0}(\eta_i ; G_4^0)}{b^{G_3'}(\eta_i ; G_4^0)}] , \qquad (31)$$

$$\omega_{g_0} - \eta_i - K_{g_0}(\eta_i ; G_4^0) - \sum_{G_3} \frac{[L^{G_3}_{g_0}(\eta_i ; G_4^0)]^2}{b^{G_3}(\eta_i ; G_4^0)} -$$

$$- \frac{1}{\eta^{pol} - \eta_i} [U^{g_0}_{G_4^0} + \sum_{G_3} \frac{U^{G_4^0}_{G_3} L^{G_3}_{g_0}(\eta_i ; G_4^0)}{b^{G_3}(\eta_i ; G_4^0)}]^2 = 0 , \qquad (32)$$

$$\Omega_{G_4^0} - \eta^{pol} = \sum_{G_3} \frac{(U^{G_4^0}_{G_3})^2}{b^{G_3}(\eta^{pol} ; G_4^0)} \qquad (33)$$

In solving eq. (32) as the first approximation we can take the value $\eta^{pol} - \eta_i$ similar to eq. (30).

Consider the case when the solution for eqs. (14) and (15) corresponding to the three-phonon pole is derived. Since the system (14) contains no noncoherent three-phonon poles then we keep only the coherent terms and obtain $P^{G_3}_{g_0 i}$ in the form (18). The poles of the secular equation are found from the equation $b^{G_3^0}(\eta^{pol}) = 0$ which we rewrite as

$$\Omega_{G_3^0} - \eta^{pol} = \sum_{G_2'} \frac{\left(\Gamma^{G_3}_{G_2'}\right)^2}{\Omega_{G_2'} - \eta_i} + \sum_{G_4'} \frac{\left(\Gamma^{G_4'}_{G_3}\right)^2}{\Omega_{G_4'} - \eta_i} . \tag{34}$$

The explicit form of the secular equation is as follows

$$\omega_{g_0} - \eta_i - K_{g_0}(\eta_i) - \frac{L^{G_3^0}_{g_0}(\eta_i)}{\eta^{pol} - \eta_i} - \sum_{G_3 \neq G_3^0} \frac{\left[L^{G_3}_{g_0}(\eta_i)\right]^2}{b^{G_3}(\eta_i)} = 0 . \tag{35}$$

For the sake of completeness we give the secular equation corresponding to the one-phonon pole

$$\omega_{g_0} - \eta_i - K_{g_0}(\eta_i) - \sum_{G_3} \frac{L^{G_3}_{g_0}(\eta_i)}{b^{G_3}(\eta_i)} = 0 , \tag{36}$$

the function $P^{G_3}_{g_0 i}$ is of the form (18).

Thus, the method of one-pole approximation allows constructing the operator wave function in the analytic form in all the cases of interest. The obtained secular equations differ mainly by the form of the pole terms.

It is interesting to note that in the framework of the one-pole approximation it is not difficult to show that

$$(\Psi^*_{i_1} \Psi_{i_2}) = 0 , \quad i_1 \neq i_2 , \tag{37}$$

$$(\Psi^*_i H \Psi_i) = \eta_i , \tag{38}$$

i.e. the orthogonality of the states (2) and the meaning of the Lagrange multiplier η_i as the energy of an excited state remain valid in this approximation.

5. Conclusion

The main result of the present paper is the obtaining of rather good approximate solutions for the system of equations (14) and of the secular equation (15) in the explicit form. The approximate secular equations (29), (32), (35), and (36) contain no superfluous solutions and there is an effective method of solving them numerically. The approximate secular equations are also obtained for a simple case when in the wave function (2) the four-phonon terms are absent, i.e. $R^{G_4}_{g_0 i} = 0$.

In ref. /8/ the exact and approximate solutions are compared for the case of a small basis. It is shown that the large wave function components for the exact and approximate solutions are close to each other. The approximate solutions for eqs. (14) and (15) we have got are expected to be close to the exact ones as far as we employ the same approximation as in ref. /8/.

The obtained approximate equations are applicable to the study of the structure of intermediate and highly excited states of doubly even deformed nuclei and, first of all, to the study of the fragmentation of one-, two-, three and four-phonon states over many nuclear levels.

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