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PHENOMENOLOGY OF THE BACK-BENDING AT HIGH SPINS IN NUCLEAR ROTATIONAL BANDS

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1. INTRODUCTION

After the discovery of the drastic increase at high spins of the moment of inertia J in the ground state rotational bands of even-even deformed nuclei by Johnson et al. /1/, which leads to the well-known back-banding behaviour of the moment of inertia J versus angular velocity squared ω^2 curve, this phenomenon has been widely studied by several groups /2-10/ and observed not only for deformed nuclei and not only for ground state bands. Good reviews are to be found in refs. /11-13/.

Two main mechanisms for explanation have been suggested - one of them is the old Coriolis anti-pairing effect /14/ leading to a phase transition from a superfluid to a normal state /15,16/. There are also calculations of this effect using the projection method /17,18/. Another effect is the Coriolis decoupling effect /19/ leading to rotational alignment of the angular momenta of a pair of particles to the total momentum. There are applications of the self-consistent cranking model for treating the phenomenon as well /20/. The decoupling of one particle is experimentally known in odd nuclei /21,22/. Both mechanisms lead to a qualitative agreement with experiment but both effects have not been experimentally distinguished as yet /11/. Also phenomenological fits to the back-bending curve are known /23-26/, in some cases supported by a band hybridization calculation /27/.

The idea of this proposal has come from the observation that the two-parameter formula /28/:

$$E = \frac{J}{c}\left[\sqrt{1 + c\frac{I(I+1)}{J^2}} - 1\right], \qquad (1)$$

which describes rotational energies up to high spins [28] (but below the transition point), working not worse than the well-known two-parameter Harris formula [29], can be deduced [30,31] from the reversed expansion [31]:

$$I(I+1) = f(E) = bE + cE^2 + \ldots, \; b = 2 \qquad (2)$$

if we limit it to the first two terms, and apply the well-known relation [11]:

$$2J = \frac{dI(I+1)}{dE} = f'(E) . \qquad (3)$$

This means that the reversed expansion (2) converges more rapidly and gives a better description of rotational energies up to high spins than the direct expansion E versus $I(I+1)$.

In the region around the transition point one cannot expect a good convergence even from (2). But from the fact that the level energy E increases with the spin I (though the transition energy ϵ may decrease) it follows that the moment of inertia J remains a single-valued function of E (though a multi-valued function of ω^2). Therefore we can hope that we could find a simple good parametrization of $f(E)$ in (2). In such a case E would appear to be a better argument for J than or ω^2 obtained from [11]

$$\omega = \frac{dE}{d\sqrt{I(I+1)}} = \frac{\sqrt{I(I+1)}}{J} . \qquad (4)$$

2. ROTATIONAL ENERGIES PARAMETRIZATION

Inserting (2) with 2 parameters into (3) we obtain that below the transition point $2J = f'(E)$ is a linear function of E. In the region aroung the transition point it increases more or less rapidly towards nearly its rigid-body value [11]. Therefore $f''(E)$ will possess a resonance-

like form with a maximum in the transition point. What should be its exact form is difficult to guess since the transition is a complicated and not well understood. phenomenon. Even in the solid state, where phase transitions are studied by many people (but where they can be due to different physical causes) there is little information about this point. There are some theoretical models leading to formulas of the type $\sim ch^{-2}x$ or $\sim[ch\,x+1]^{-1}$ ($x\sim E-E_0$, E_0 - the transition point energy).

We decided to limit ourselves here to the simplest resonance-type form, the Lorentz-form, which added to the term following from (2) gives:

$$f''(E)=2c+\frac{a}{1+(\frac{E-E_0}{\beta})^2};=2c+\gamma\,\delta(E-E_0) \qquad \text{for } \beta=0 \quad (5)$$

(We denote $\gamma=\pi a\beta$), wherefrom:

$$2J=f'(E)=b+2cE+\frac{\gamma}{\pi}\left[\operatorname{arctg}\frac{E-E_0}{\beta}+\operatorname{arctg}\frac{E_0}{\beta}\right];$$

$$=b+2cE+\gamma\theta(E-E_0) \quad \text{for} \quad \beta=0 \qquad (6)$$

with the notations: $\theta(x)=\begin{cases}1, & x>0\\ 0, & x<0\end{cases}$, $b=2J_0$.

Further on:

$$f(E)=bE+cE^2+\frac{\gamma}{\pi}(E-E_0)\left(\operatorname{arctg}\frac{E-E_0}{\beta}+\operatorname{arctg}\frac{E_0}{\beta}\right)+$$

$$+\frac{\gamma\beta}{2\pi}\ln\frac{1+(\frac{E_0}{\beta})^2}{1+(\frac{E-E_0}{\beta})^2};\ bE+cE^2+\gamma(E-E_0)\theta(E-E_0)$$

$$\text{for} \quad \beta=0. \qquad (7)$$

This parametrization is not intended to describe the "down-bending" /11/ of J (its decrease) which occurs in some of the nuclei after the back-bending. One more

parameter changing the slope $2c$ of the straight line after the transition point could take it into account, but it hardly makes sense in a phenomenological description since this phenomenon is usually observed only at the highest-spin point of the curve.

Thus we have a two-parameter (b, c) smooth component (straight line) - the first two terms of (6), and a three-parameter (E_0, β, γ) step-like component - the last term of (6). Our parameters have the following physical meaning: b is twice the moment of inertia $2J_0$ at zero energy $E=0$, c gives half the slope of the smooth component and measures the usual smooth non-adiabatic deviations from the rigid-rotor. E_0 is the transition point energy, β is the half-width and γ - the amplitude of the step-like component. This is illustrated in fig. 1.

The experimental value for the moment of inertia of the transition $I \to I-2$ is obtained from /11/:

$$2J = \frac{4I - 2}{E_I - E_{I-2}} \qquad (8)$$

at an energy defined from:

$$\frac{f(E_I) - f(E_{I-2})}{E_I - E_{I-2}} = f'(E), \qquad (9)$$

which is better than the simple definition:

$$E = \frac{E_I + E_{I-2}}{2} \qquad (10)$$

for our representation (fig. 1), or at an angular velocity squared ω^2 defined from:

$$\omega = \frac{\sqrt{f(E)}}{2I - 1} \quad (E_I - E_{I-2}) \qquad (11)$$

with E obtained from (9), which is better than the usual definition /11/:

$$\omega = \frac{\sqrt{I^2 - I + 1}}{2I - 1} (E_I - E_{I-2}) \qquad (12)$$

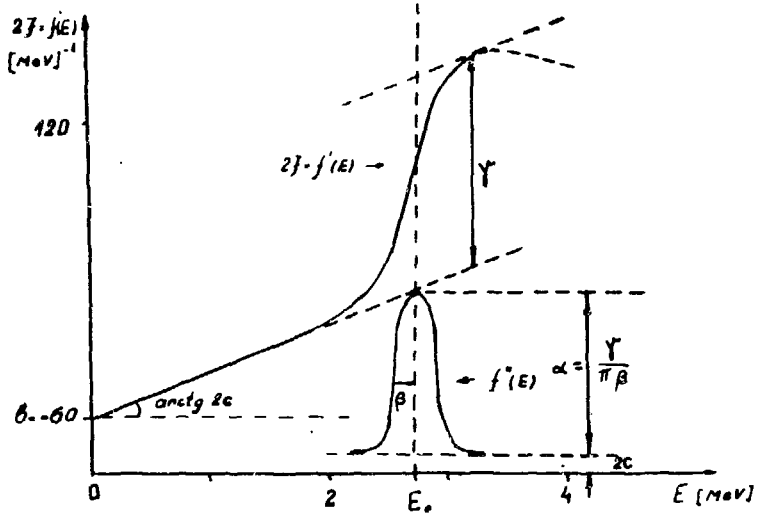

Fig. 1. 2J versus E representation from (6), and meaning of the parameters in (6).

for the usual back-bending representation in the following respect. (12) gives an interpolated ω for the transition $I \to I-2$. It is obtained from (4) for $I(I+1)$ changed to $1/2\,[\,I(I+1)+(I-2)(I-1)] = I^2 - I + 1$. This value of the spin has come from the condition that $2J$ obtained from (8) for the transition $I \to I-2$ should coincide with $2J$, obtained from (3) at the so chosen value of the spin, but with an additional assumption: the direct expansion of E in $I(I+1)$ should be limited to the second quadratic term in the interval $I \to I-2$. Similarly, (10) is an interpolated value of E obtained from the same condition: $2J$ from (8) for the transition $E_I \to E_{I-2}$ to coincide with $2J$ from (3) at the energy E, with the additional assumption: the reversed expansion (2) is limited to the second quadratic term in the interval $I \to I-2$. Both assumptions might work badly around the transition point. Therefore we propose (9) instead of (10) and (11) instead of (12), (9) and (11) being obtained from the same condition, but with the more realistic dependence $I(I+1) = f(E)$ taken from the formula (7). However the difference is usually very small and becomes significant only in some cases for the points near to places of strong bending of the curve.

3. FITTING PROCEDURE

To fit the parameters to the experimental data we have applied the least-square method, minimizing the expression:

$$S = \sum_I p_I [\, f(E_I) - I(I+1) \,]^2 \,, \qquad (13)$$

where E_I is the experimental energy of the level with spin I. The weights p_I could in principle be chosen different because of the varying experimental accuracy of the different level energies E_I: to a higher I would correspond a lower accuracy of E_I and therefore - a lower value of p_I. But such levels are most important for the description of the transition. Moreover the theoretical description gives differences with experiment

still several times higher than the experimental errors. Therefore we choose simply $p_I = 1$.

To find the experimental accuracy of our fit we can estimate the error of S by:

$$\Delta S = \{ \sum_{I=2}^{I_{max}} [\sum_{I'=1}^{I_{max}} 2[f(E_{I'}) - I'(I'+1)] f'(E_{I'})^2 \overline{\Delta\epsilon_I^2} \}^{1/2} , \quad (14)$$

where $\epsilon_I = E_I - E_{I-2}$ is the transition energy and $\overline{\Delta\epsilon_I^2}$ - its dispersion which can be accepted to be approximately the same for different I: $\overline{\Delta\epsilon_I^2} = \overline{\Delta\epsilon^2}$ and a rough estimate could be $\Delta\epsilon = 0.3$ keV. Thus we can regard two sets of parameters as distinct outside experimental errors if they would give a difference of S higher than ΔS from (14).

To estimate the quality of our fit, we can apply a procedure similar to the χ^2 -test. We will compare $\sqrt{2/I_{max}}S$, which measures the mean square error of $f(E)$ due to our fit, i.e., its mean square deviation from $I(I+1)$, to $\sqrt{(2/I_{max})S_{id}}$, where S_{id} would be the value of S if our fit would be ideal and the deviation $f(E)-I(I+1)=f'(E_I)\Delta E_I$ would be due only to experimental random deviations of ϵ_I, which would mean:

$$S_{id} = \sum_{I=2}^{I_{max}} [\sum_{I'=1}^{I_{max}} f'(E_{I'})^2 \overline{\Delta\epsilon_I^2} . \quad (15)$$

Thus their ratio $\chi = \sqrt{S/S_{id}}$ is the ratio of the errors of $f(E)$ in the cases of the real and of an ideal fit. The best quality would be represented by $\chi = 1$ and worse quality would mean $\chi > 1$.

We shall also separate the most important half-width parameter β to study it in greater detail, and in particular to see if we have a zero or finite transition width. Therefore we shall first minimize over the rest of the parameters, obtaining $S(\beta)$, and find at the end its minimum over β.

4. NUMERICAL RESULTS

We have chosen 4 nuclei in the rare-earth region, which until recently (there have been new contributions in München) were the nuclei with the best expressed S -shape of the curve $2J$ versus ω^2 /11,12/:
^{166}Yb /3/, ^{162}Er /2,11/, ^{160}Er /2,3/, ^{158}Er /3/.

The results of studying S as a function of β are shown in fig. 2. The highest level in the cases of ^{166}Yb, ^{162}Er, ^{158}Er has not been included in (13) since it exhibits a "down-bending" which our formula, as we have mentioned already, does not reproduce.

The experimental accuracy and quality of the fit can be seen from Table 1. If one compares ΔS from Table 1 to the changes of S with β from fig. 2, one finds that in the cases of ^{160}Er, ^{158}Er the experimental accuracy does not allow us to distinguish the half-width β from 0, and in the cases of ^{160}Yb ^{162}Er β is finite.

By comparing the errors of the fit $\sqrt{(2/I_{max})S}$ with the purely experimental errors of an ideal fit $\sqrt{(2/I_{max})S_{id}}$ one sees that the errors of the fit are near to the experimental ones, but still exceed them several (χ) times. If one looks upon the different terms of S from (13) one will find that the errors of the fit come mainly from the highest levels above the transition point, the rest of the levels below and just around the transition being fitted with almost the experimental accuracy.

The fit can be seen in detail for all the 4 nuclei in both representations: the usual one $2J$ versus ω^2, and our - E versus ω^2, in figs. 3-6. The representation $2J$ versus E shows clearly both component of the curve: the smooth one - a straight line below the transition, and a step-like one, describing the increase of $2J$ in the transition region.

Table 1
Estimates of Experimental Accuracy and Quality of the Best Fit

Nucleus	$\overline{S}$ (13)	ΔS (14)	$\sqrt{(2/I_{max})S}$ (13)	$\sqrt{(2/I_{max})S_{id}}$ (15)	$\chi = \sqrt{S/S_{id}}$
^{166}Yb	0.35	0.029	0.198	0.071	2.78
^{162}Er	0.46	0.025	0.225	0.068	3.31
^{160}Er	0.68	0.027	0.276	0.067	4.14
^{158}Er	1.90	0.036	0.484	0.065	7.73

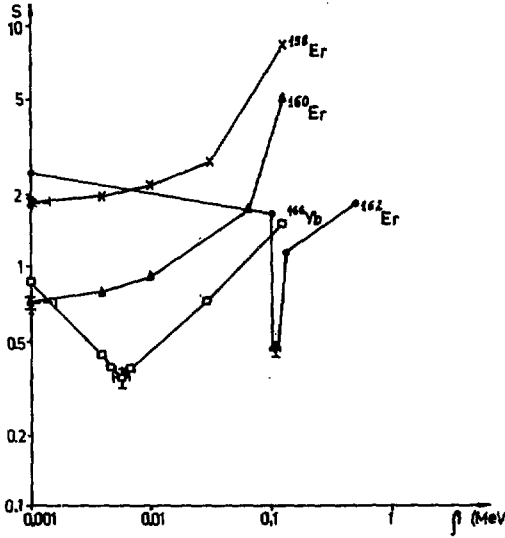

Fig. 2. $S = S(\beta)$ from (13) with $p_I = 1$, minimized over all other parameters at fixed values of β. ΔS from Table 1 and the corresponding errors of β are also indicated.

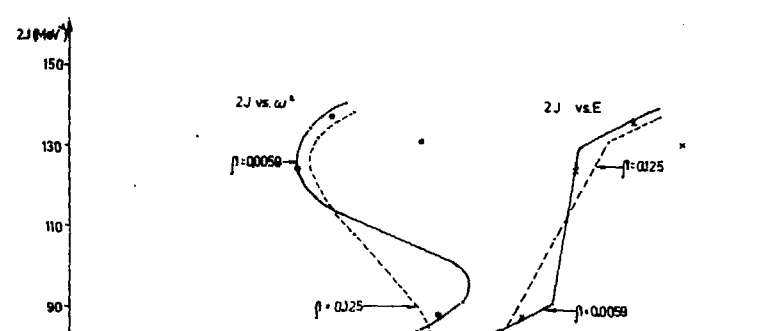

Fig. 3. 2J versus ω^2 and E for ^{166}Yb ——— - best fit; – – – • – – – worse β-fit ; o o o - experiment 2J vs. ω^2 ; x x x experiment 2J vs.E.

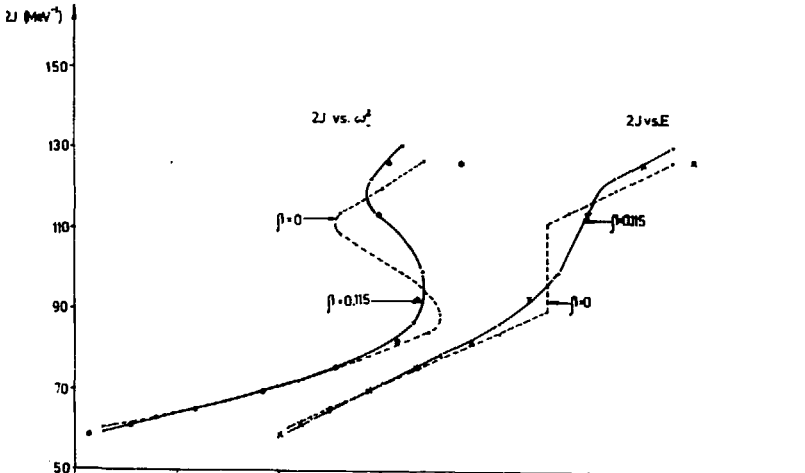

Fig. 4. 2J versus ω^2 and E for ^{162}Er ——•—— best fit; – – – •– – – worse β-fit; o o o - experiment $2J_{vs.\omega^2}$; X X X - experiment $2J_{vs.E}$.

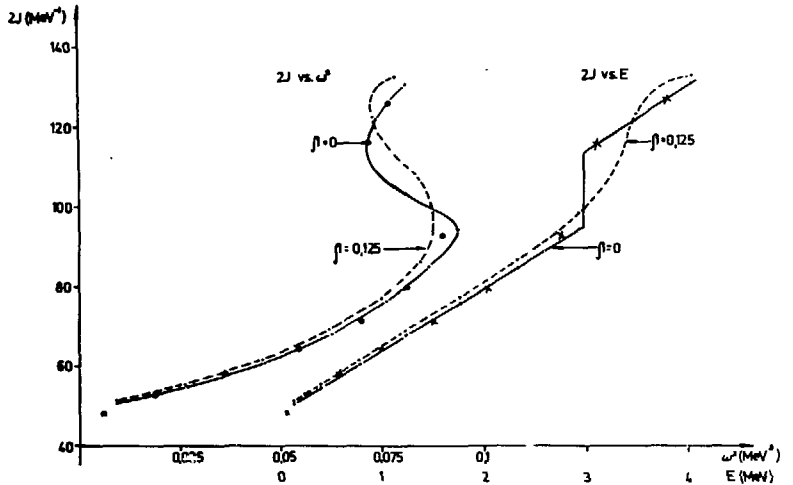

Fig. 5. $2\mathcal{J}$ versus ω^2 and E for ^{160}Er. ——●—— best fit; - - - ● - - - worse β – fit; o o o - experiment $2\mathcal{J}$ vs. ω^2; X X X - experiment $2\mathcal{J}$ vs. E.

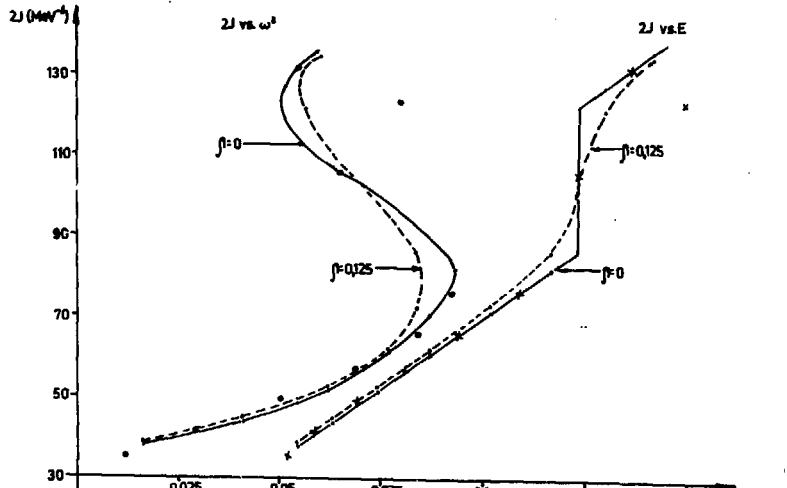

Fig. 6. 2J versus ω^2 and E for ^{158}Er. ——●—— best fit; - - - - ●- - - worse β-fit; o o o - experiment 2J vs. ω^2; X X X - experiment 2J vs. E.

5. DISCUSSION

The parameters for our 4 nuclei are compared in Table 2. One can make the following points:

1. The moment of inertia J_0 takes usual values (around 30 MeV^{-1}), and as usually decreases with A, i.e., when going into the transitional region.

2. The coefficient of the smooth non-adiabatic deviations c also takes usual values (around 5 MeV^{-2} /28/) and shows the expected increase when going into the transitional region.

3. The transition point E_0 remains almost at the same place for all 4 nuclei. Its position in energy E_0 is compared to its position in spin I_0, (obtained from E_0 by (2), (7) and in angular velocity (obtained from E_0, I_0 by (4), (6)). The relative variation $\Delta E_0/E_0 = \pm 0.056/2.90 = \pm 0.019$ can be compared to the relative variations $\Delta\omega_0 / \omega_0 = \pm 0.012/0.27 = \pm 0.045$ or $\Delta I_0/I_0 = \pm 0.77/13.88 = \pm 0.055$ which shows that the transition point energy E_0 is more stable than its angular velocity ω_0 or spin I_0.

4. The amplitude γ of the step-like component of the transition varies by a factor of about 2 decreasing in the cases in which the back-bending seems weaker (^{162}Er, ^{160}Er). It gives about half of the increase of the moment of inertia from the zero-point $E=0$ to its value above the transition. The other half is given by the smooth component. Both components make the total increase of J nearly equal to its zero-point value /11/.

5. The half-width β of the transition is rather small. In some cases it cannot be distinguished from 0 (^{160}Er, ^{158}Er), but increases in other cases (^{166}Yb, ^{162}Er), in one of them the back-bending seeming weaker (^{162}Er).

This fit can be applied to study the systematics of the parameters in all the back-bending nuclei and to try to correlate them to their microscopic (e.g., single-particle) properties.

Thanks are due to Dr. I.N.Mikhailov for helpful discussions.

Table 2
Best Fit Parameters

Nucleus	$b=2J_0$ Mev^{-1}	C Mev^{-2}	E_0 Mev	ω_0 Mev	I_0	γ Mev^{-1}	β Mev
^{166}Yb	58.88	5.98	2.88	0.265	14.33	37.39	0.0059
^{162}Er	58.50	6.24	2.84	0.279	14.34	24.93	0.1150
^{160}Er	48.76	7.77	2.96	0.279	14.07	19.10	0.0000
^{158}Er	34.72	8.80	2.92	0.255	12.80	36.09	0.0000

$b=2J_0$ - initial (at $E=0$) doubled moment of inertia;
c - non-adiabatic deviations parameter;
E_0, ω_0, I_0 - transition point resp. in energy, angular velocity and spin;
γ - transition amplitude;
β - transition half-width.

REFERENCES

1. A.Johnson, H.Ryde, J.Sztarkier. Phys.Lett., 34B, 605 (1971).
2. A.Johnson, H.Ryde, S.A.Hjorth. Nucl.Phys., A179, 753 (1972); H.Ryde, S.A.Hjorth, D.Barbeoud, A.Johnson, G.B.Hagemann, B.Herskind. Nucl. Phys., A207, 513 (1973).
3. R.M.Lieder, H.Beuscher, W.F.Davidson, P.Jahn, H.-J.Probst, C.Mayer-Boricke. Phys. Lett., 39B, 196 (1972); Z. f. Physik, 257, 147 (1972); H.Beuscher, W.F.Davidson, R.M.Lieder, G.Mayer-Boricke. Phys. Lett., 40B, 449 (1972); W.F.Davidson, R.M.Lieder, H.Beuscher, C.Mayer-Boricke. Physica Scripta, 6, 251 (1972); J.C.Lisle, F.Kearns, G.D.Dracoulis, J.C.Willmott, W.F.Davidson, R.M.Lieder, H.Beuscher, A.Neskakis, C.Mayer-Boricke. Proc. Int. Conf. Nucl. Phys. (München 1973), v. 1, p. 187; R.M.Lieder, H.Beuscher, W.F.Davidson, A.Neskakis, C.Mayer-Boricke. Proc. Int. Conf. Nucl . Phys., (München 1973), v. 1, p. 188.
4. P.Thieberger, A.Sunyar, P.Rogers, N.Lark, O.Kistner, E. der Mateosian, S.Cochavi, E.Auerbach. Phys. Rev. Lett., 28, 972 (1972).
5. P.Taras, W.Dehnhardt, S.J.Mills. M.Veggian, J C.Merdinger, U.Neumann, B.Povh. Phys.Lett., 41B, 295 (1972); W.Dehnhardt, J.C.Merdinger, S.J.Mills, M.Muller-Veggian, U.Neumann, D.Pelte, G.Poggi, B.Povh, J.Sann, P.Taras. Proc. Int. Conf. Nucl. Phys., (München 1973), v. 1, p. 180.
6. J.N.Mo, R.Chapman, C.G.Dracoulis, W.Gelletly, A.J.Hartley. Particles and Nuclei, 4, 126 (1972); A.J.Hartley, R.Chapman, G.D.Gracoulis, S.Flanagan, W.Gelletly, J.N.Mo. Journ. of Phys., A6, L60, (5) (1973).
7. D.Ward, R.L.Graham, J.S.Geiger, H.R.Andrews. Phys. Lett., 44B, 39 (1973).
8. R.A.Warner, F.M.Bernthal, J.S.Boyno, T.L.Khoo, G.Sletten. Phys.Rev.,Lett., 31, 835 (1973); T.L.Khoo, F.M.Bernthal, J.S.Boyno, R.A.Warner. Phys.Rev. Lett., 31, 1146 (1973).
9. K.Krien, R.A.Naumann, J.O.Rasmussen, I.Rezanka. Nucl. Phys., A209 (1973).
10. G.Scharf-Goldhaber et al. Proc. Int. Conf. Nucl. Phys. (München 1973), v. 1, p. 177; J. de Phys., 33, 102, 104 (1972).
11. A.Johnson, Z.Szymanski. Phys. Rep., 7C, 181 (1973).
12. O.Saethre, S.A.Hjorth, A.Johnson, S.Jagare, H.Ryde,

Z.Szsymanski. Nucl.Phys., A207, 486 (1973).
13. R.A.Sorensen. Rev. Mod. Phys., 45, 353 (1973).
14. B.R.Mottelson, J.C.Valatin. Phys.Rev.Lett., 5, 511 (1960).
15. R.A.Sorensen. Phys.Lett., 38B, 376 (1972).
16. J.Krumlinde, Z.Szymanski. Phys.Lett., 36B, 157 (1971); 40B, 314 (1972); Annals of Physics, 79, 201 (1973).
17. A.Faessler, L.Lin, F.Wittmann. Phys.Lett., 44B, 127 (1973).
18. S.Frauendorf. Preprint JINR, P4-6990, Dubna, 1973; S.Frauendorf, P.Gippner, K.-H.Kaun, W.Neubert, F.Stary, W.Schulze. Proc. Int. Conf. Nucl . Phys. (München 1973), v. 1, p. 191.
19. F.S.Stephens, R.Simon. Nucl.Phys., A183, 257 (1972); F.S.Stephens, P.Kleinheinz, R.A.Sheline, R.S.Simon. To be published (1973).
20. B.Banerjee, H.J.Mang, P.Ring. Nucl. Phys., A215, 366 (1973).
21. F.S.Stephens, R.M.Diamond, J.R.Leigh, T.Kammuri, K.Nakai. Phys.Rev.Lett., 29, 438 (1972); K.Nakai, P.Kleinheinz, J.R.Leigh, K.H.Maier, F.S.Stephens, R.M.Diamond, G.Lovhoiden. Phys.Lett., 44B, 443 (1973); F.S.Stephens, R.M.Diamond, D.Benson Jr., M.R.Maier. Phys.Rev., C7, 2163 (1973); J.R.Leigh, K.Nakai, K.H.Maier, F.Puhlhofer, F.S.Stephens, R.M.Diamond. Nucl.Phys., A213, 1 (1973).
22. D.Habs et al. Proc. Int. Conf. Nucl. Phys. (München 1973), v. 1, p. 183; H.Beuscher et al., ibid., p. 189; D.Proetel et al., ibid., p. 194; S.Andre et al., ibid., p. 196.
23. S.Wahlborn, R.K.Gupta. Phys.Lett., 40B, 27 (1972); R.K.Gupta, S.Wahlborn, L.Hjertman. Physica Scripta, 6, 261 (1972).
24. J.E.Draper. Phys.Lett., 41B, 105 (1972).
25. C.K.Ross, Y.Nogami. Nucl.Phys., A211, 145 (1973).
26. T.K.Das, B.Banerjee. Phys.Rev., C7, 2590 (1973).
27. A.Molinari, T.Regge. Phys.Lett., 41B, 93 (1972).
28. I.N.Mikhailov, E.Nadjakov, R.N.Safarov. Preprint JINR, P-2866, Dubna, 1966; E.Nadjakov, I.N.Mikhailov. Nucl. Phys., A107, 92 (1968).
29. S.M.Harris. Phys.Rev.Lett., 13, 663 (1964); Phys. Rev., 138, B509 (1965).
30. P.Holmberg, P.O.Lipas. Nucl.Phys., A117, 552 (1968).
31. E.Nadjakov, I.N.Mikhailov. Preprint IC/71/111 (Trieste 1971).

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