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THE SINGULARITY SUBTRACTION METHOD FOR EXTRACTING SPECTROSCOPIC INFORMATION FROM NUCLEAR REACTION DATA

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A simple way of exploiting the analytic nature of scattering data is to try to get information on the nearest singularity of the amplitude in the $z=\cos\theta$ plane at fixed energy /1/. The possibilities for this were greatly extended by the introduction of the optimal conformal mapping technique /2,3/. It was proposed that such methods could be useful in nuclear physics, too /4,5/.

The basic ideas have been repeatedly presented in the above-mentioned references, therefore we give only a very concise summary of certain points which we think to be of importance. The reaction amplitude is an analytic function of the $z=\cos\theta$ variable, the singularities of which correspond to certain Feynman graphs, i.e. to certain reaction mechanisms /7/. The strength of these singularities provides information on the structure of nuclei involved in the reaction. It is possible to define an analytic function, which is equal to the square of the absolute value of the amplitude (i.e., to the differential cross section) on that interval of the real axis which lies in the analyticity domain of the amplitude. This function is analytic in the same domain.

The nearly trivial way of determining the strength of the nearest singularity is to continue the experimental data up to this point after removing it by a suitably chosen factor. The powerful technique of optimal conformal mapping made it possible to apply successfully this continuation method in nuclear physics too /4,5/, despite the obvious difficulties in getting error limits /6/ in practice.

Another possibility is to subtract the nearest singularity after the mapping with a guess strength. It is possible to determine when the guess strength is equal to the correct one [2]. We modified the subtraction method of Cutkosky and Deo [2], namely, we avoided the introduction of any model assumption and thus made the method exact.

The reaction we chose for checking the applicability of the method was the $d(d,p)t$ reaction. As a successful peripheral model fit showed, the forward and backward peaks are dominated by the neutron transfer mechanism [13], and good experimental data are available at E_d=25.3MeV [8]; our choice was motivated by these facts. Besides, in this case an exact treatment is possible in a simple way. We notice that neglecting Coulomb effects in ref. [5] makes the result for the $^3He \to d+p$ vertex constant questionable, despite the fact that its numerical value is near to other results *. The resulting information on the structure of the triton is important, this problem is widely discussed in the literature [16].

The singularity structure of the amplitude is as follows. There is a neutron exchange pole near to the physical region (at E_d= 25.3 MeV at z_p=1.334), the next singularity (at z_t= 3.37) corresponds to the proton knock-out mechanism. It generates a branch point singularity of logarithmical character. The location of these singularities can be calculated according to ref. [9]. Due to the identity of the deuterons these singularities lie symmetrically about $z=0$ and only even powers of the variables enter into any expression describing the data.

The differential cross section has a second order pole, which comes from the square of the amplitude, and a first order pole from the interference with the background terms. The location of the branch point was taken as the parameter determining the $x=x(z)$ conformal mapping [2,3].

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* The Coulomb singularity on the edge of the physical region makes any expansion series, which is convergent inside the physical region, divergent outside it. Therefore any extrapolation procedure without taking into account the Coulomb singularity is doubtful.

After the mapping the location of the singularities is changed, the pole lies at x_p=1.36, while the branch point at x_t = 6.63.

Now the experimental data could be described as:

$$\frac{d\sigma}{d\Omega}=\frac{P}{(z_p-z)^2}+\frac{P}{(z_p+z)^2}+\frac{Q}{z_p-z}+\frac{Q}{z_p+z}+\sum_{n=1}^{N} b_n x^{2n-2}(z), \quad (1)$$

where due to the statistical errors in the data only a finite number of the b_n coefficients is significant in the sense that they are larger than their errors. The optimal conformal mapping used by us minimizes the number of terms needed in the sum. As we are not interested in the first order pole, we removed it multiplying with a factor of

$$m(x)=(x_p^2-x^2)/(2x_p x'(z_p)), \quad (2)$$

where $x'(z)$ denotes the derivative of $x(z)$. This factor does not alter the residue at the pole and it is of practical importance that it be linear in x^2. Then one has

$$m(x)\frac{d\sigma}{d\Omega}=m(x)\left(\frac{P}{(z_p-z)^2}+\frac{P}{(z_p+z)^2}\right)+\sum_{n=1}^{N+1} c_n x^{2n-2}. \quad (3)$$

By a least squares procedure we fitted $m(x)d\sigma/d\Omega$ according to the well known orthonormal polynomials which are orthonormal with the weigths of the least squares procedure:

$$m(x)\frac{d\sigma}{d\Omega}=\sum_n A_n B_n(x^2). \quad (4)$$

The $B_n(x^2)$ polynomials have the advantage that the A_n coefficients are uncorrelated, their rms error is 1. This also means that one can take superfluous terms in the sum, their presence has no effect on the significant terms at all. As the pole lies nearer than the other singularities, it determines the A_n coefficients with sufficiently large n. It can be understood easily as the $B_n(x^2)$ polynomials are regular functions, a finite number of them is unable to reproduce the pole, therefore the asymptotics of the coefficients is determined by the location and the strength of the pole. If the experimental data are accurate enough, one can always reach this asymptotic region. If the pole is subtracted from the data with the correct strength, then these coefficients disappear. Therefore, we extracted the pole with a guess strength and analysed the result, i.e.:

$$m(x)\left(\frac{d\sigma}{d\Omega}-\frac{G}{(z_p-z)^2}-\frac{G}{(z_p+z)^2}\right)=\sum_n A_n B_n(x^2) \qquad (5)$$

and determined the G value at which the (N+ 2)-th and higher order coefficients disappear (see also eq. (3)).

One has to determine the value of N. For this purpose we multiplied the experimantal data by a factor of $m^2(x)$ and analysed them. From (3) it follows:

$$m^2(x)\frac{d\sigma}{d\Omega}=\sum_{n=1}^{N+2} d_n B_n(x^2), \qquad (6)$$

therefore, determining the significant terms one can find the value of N as well. Strictly speaking the $B_n(x^2)$ polynomials in (6) differ from the polynomials in (5) because the weights are different, but this does not alter the value of N.

The method proposed by us is similar to the subtraction method of Cutkosky and Deo /2/. We, however, determine the correct strength of the subtracted pole differently. Cutkosky and Deo made a model assumption about the behaviour of the A_n coefficients in (5) when the pole is correctly subtracted. They defined a convergence test function, which is the measure to which extent the pole not being correctly subtracted destroys the expected behaviour of the coefficients. The strength of the pole is determined by the minimum of this function. In this way the exactness of the method was spoiled.

We also determined the strength of the pole by the continuation model, i.e., by inserting $x=x_p$ into the right-hand side of (6). It is obvious that this procedure has a serious shortcoming. Without additional information on the properties of the other singularities nothing assures that the neglected terms are small not only in the physical region but in the pole, as well. To have bound estimates for them one needs certain information, which comes comple-

tely outside from the differential cross section to be analyzed (see /6/). To be clear, here we speak not of the error-dominated sum $\sum_{n=N+3}^{\infty} d_n B_n(x^2)$, but of a similar sum with the "true" value of the d_n coefficients. As in the subtraction method we all the time work in the physical region, moreover, we consider only certain coefficients in the series (5), therefore our results are free of any systematical error of this kind.

Once the strength of the pole is found, one has to calculate the spectroscopic information from it. For this purpose the formulae of the peripheral model can be used /10,12/ with no cut-off one gets back the pure pole. The identity of the deuterons can be taken into account according to the general formulae of ref. /15/, the result is:

$$\frac{(z_p-z)^2(z_p+z)^2}{4z_p^2}\frac{d\sigma}{d\Omega}\xrightarrow[z\to\pm z_p]{}\frac{5}{24\pi^2}\frac{m_n^2c^4}{E_iE_f}\frac{k_f}{k_i}G_d^2G_t^2, \qquad (7)$$

where k_i, k_f and E_i, E_f are the relative momenta and kinetic energies in the initial and final channels, G_d and G_t are the deuteron and triton vertex constants in square root of fermi, while $d\sigma/d\Omega$ is in mb/sr. The information on the structure of the triton is represented by the G_t^2 vertex constant. If the wave function of the bound system has an asymptotic form of $A\cdot e^{-kr}/r$ in the corresponding channel, then

$$G_t = A\hbar\sqrt{\pi}/\mu c, \qquad (8)$$

where μ is the reduced mass. Later on we use the notation $G = G_d^2 G_t^2$.

In Table 1 we present the expansion coefficients for $m(x)d\sigma/d\Omega$ and $m^2(x)d\sigma/d\Omega$. From (6) the surprising result follows that $N=2$, i.e., two terms are enough to represent the background in (1) to the accuracy given by the data. The differential cross section contains four parameters at all, though judging by the data at lower energies /17/, a Legendre polynomials fit would require seven terms. This demonstrates the power of the optimal conformal mapping technique in eliminating superfluous parameters which are correlated by the common physical information they correspond to. It follows that the $n=4,5,6$ terms can be used for determining G, the results are presented in Table 2. The weighted average is $G = 0.4942 \pm 0.0069\ f^2$. The result of the continuation method is practically the same, $G = 0.4943 \pm 0.0069\ f^2$. The error comes only from the significant terms in (6). The coincidence of the results shows that in this case the truncated terms are small in the pole, though in the pole with n the $B_n(x^2)$ polynomials increase very fast, while their values remain approximately the same in the physical region.

Our result is subject to an additional error of 2.5% due to the uncertainties in the absolute value of the cross section /8/. If one assumes $G_d^2 = 0.40\ f$ /11/, then $G_t^2 = 1.235 \pm 0.035\ f$ is the $t \to d+n$ vertex constant derived by us. It is in very good agreement with other results, a review of which (except the peripheral model results) can be found in ref. /16/. There different notations and units are used, D^2 is, however, proportional to our G_t^2. The D^2 value given in ref. /16/ corresponds to $G_t^2 = 1.28 \pm 0.24\ f$. As peripheral model results are not presented in ref. /16/ we quote two of them: i) from the comparison of (p,d) and (d,t) reactions on ^{18}O we got $G_t^2 = 1.35\ f$ /12/ ; ii) from a peripheral model fit to the neutron transfer peaks in the $d(d,p)t$ reaction at $E_d = 6,8,12,14$ and 25 MeV we got the average $G_t^2 = 1.22\ f$ /13/. Further references can be found in ref. /14/.

We conclude that we have proved in practice the applicability of the singularity subtraction method by the example of the $d(d,p)t$ reaction at $E_d = 25$ MeV. The numerical value of $G_t^2 = 1.235 \pm 0.035\ f$ for the $t \to d+n$ vertex constant is in an excellent agreement with other results, none of which has such a small error. We emphasize that we have arrived at it in a completely model independent way, it is free of any systematical error (if the experimental data are free, of course). The only assumptions made by us were about the analytic structure of the amplitude. We also used an explicit expression for the nearest singularity. These are, however, general and exact properties.

An early variant of this work was completed in the Central Research Institite for Physics in Budapest, and thanks are due to O.Dumbrais, who called our attention to the analytic continuation method, and who later provided us with a subroutine for the calculation of the mapping based on formulae of ref. /2/

References

1. G.F.Chew. Phys.Rev., 112, 1380 (1958).
   J.G.Taylor, M.J.Moravcsik, and J.L.Uretsky. Phys. Rev., 113, 689 (1959).
2. R.E.Cutkosky, B.B.Deo. Phys.Rev.Letters, 20, 1272 (1968); Phys.Rev., 174, 1859 (1968).
3. S.Ciulli. Nuovo Cimento, A61, 787 (1969), A62, 301 (1969).
4. O.V.Dumbrais, F.Nichitiu, Yu.A.Scherbakov. Yadern. Fiz., 16, 327 (1973).
   S.Dubnicka, O.V.Dumbrais, F.Nichitiu. Nucl.Phys., A, 217, 535 (1973).
5. L.S.Kisslinger. Phys.Rev.Letters, 29, 505 (1972); Phys.Letters, 47B, 93 (1973).
6. K.Miller, G.A.Viano. J.Math.Phys., 14, 1037 (1973).
7. I.S.Shapiro. JETF (USSR), 41, 1616 (1961); Selected topics in nuclear theory (IAEA, Vienna, 1963), p. 85; Proc.Int.School Phys.Enrico Fermi, course 38 (Academic Press, New York, 1966), p. 210; H.J.Schnitzer. Rev.Mod.Phys., 37, 666 (1965).
8. W.T.M. van Oers, K.W.Brockman. Nucl.Phys., 48, 622 (1963).
9. L.D.Blokhintsev, E.I.Dolinsky, V.C.Popov. JETF (USSR), 42, 1636 (1962); 43, 2290 (1962).
10. I.Borbely, E.I.Dolinsky, V.V.Turovtsev. Yadern.Fiz., 8, 492 (1968).
11. E.I.Dolinsky, V.V.Turovtsev. Yadern.Fiz., 9, 515 (1969).
12. I.Borbely, E.I.Dolinsky. Yadern.Fiz., 10, 299 (1970).
13. I.Borbely. Phys.Lett., 35B, 388 (1971).
14. E.I.Dolinsky, P.O.Dzhamalov, A.M.Mukhamedzhanov. Nucl.Phys., A202, 97 (1973).

15. I.Borbely, E.I.Dolinsky. Yadern.Fiz., 7, 554 (1967).
16. L.J.B.Goldfarb, J.A.Gonsales, A.C.Phillips. Nucl. Phys., A209, 77 (1973).
17. J.E.Brolley et al. Phys.Rev., 107, 820 (1957).

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Table 1
The expansion coefficients in formulae (4) and (6) with the first and second order pole removed

n	$m(x)\ d\sigma / d\Omega$	$m^2(x)\ d\sigma / d\Omega$
1	175.2	168.9
2	-21.6	-50.7
3	-4.4	11.8
4	67.0	70.8
5	13.1	0.4
6	3.8	1.2
7	0.8	0.2

Table 2
Pole strength (see (5)) when the corresponding coefficients disappear

n	$G\ f^2$	f^2	ΔG %
4	0.4937	0.0071	1.4
5	0.5006	0.0383	7.2
6	0.7341	0.193	34