

ОБЪЕДИНЕННЫЙ  
ИНСТИТУТ  
ЯДЕРНЫХ  
ИССЛЕДОВАНИЙ  
ДУБНА



4/II-74

S-70

E4 - 7646

446/2-74

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OF THE EXCITED STATES  
OF ATOMIC NUCLEI

**1973**

ЛАБОРАТОРИЯ  
ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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Report submitted to the XXIV All-Union  
Conference on Nuclear Spectroscopy and  
Nuclear Structure, Kharkov, January 29 -  
February 1, 1974.

СОЪЕДИНЕННЫЙ ИНСТИТУТ  
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БИБЛИОТЕКА

## 1. Introduction

The low-lying states of medium-weight and heavy nuclei are rather well described by approximate solving of the problem with the following Hamiltonian defining the interactions of the nucleons in the nucleus:

$$H = H_{av} + H_{pair} + T_{rot} + H_{Cor} + H_A + H_{GA} + H' . \quad (1)$$

Here  $H_{av}$  is the average field of the neutron and the proton systems,  $H_{pair}$  the interactions leading to superconducting pairing correlations,  $T_{rot}$  the kinetic energy of rotation,  $H_{Cor}$  the Coriolis interaction describing the relationship between the internal motion and the rotation,  $H_A$  the multipole-multipole interaction,  $H_{GA}$  the spin-multipole-spin-multipole interaction,  $H'$  some other types of interactions including, e.g., the Gamov-Teller-type interaction. We note that this division is somewhat conditional, as far as, for example, the average field is separated after averaging  $H$  over the ground state.

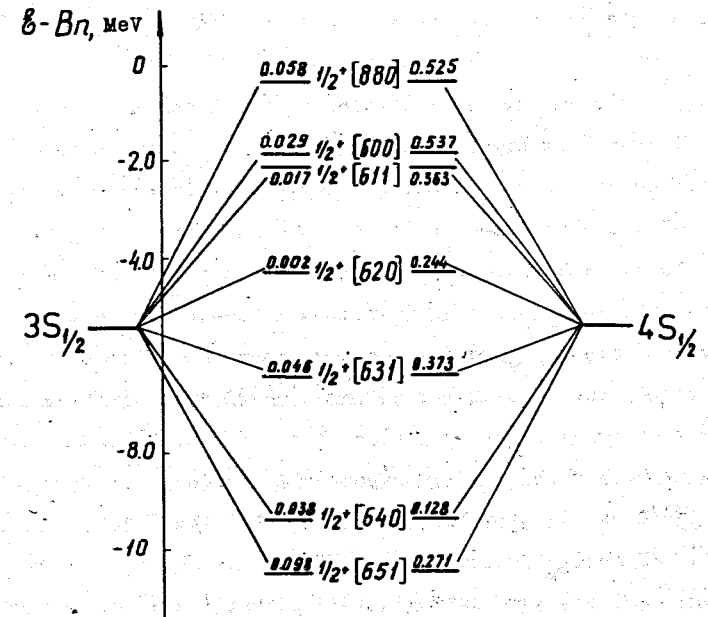
In the present report we analyse which terms of the Hamiltonian (1) are important at intermediate and high excitations, what are the causes of complications of the state structure with increasing excitation energy and what are particular features of the wave functions of highly excited states compared with the wave functions of lowly excited states.

## 2. The Average Field of Spherical and Deformed Nuclei

The decisive steps in the description of the low-lying states of nuclei are the following: firstly, we separate the average nuclear field and, secondly, we postulate that for the ground states of doubly even nuclei in a given region with respect to  $A$  the average field is chosen so that the density matrix is diagonal, and the correlation function assumes a canonical form ( see refs. <sup>1,2</sup> ). This implies that in treating the ground states we should take into account in addition to the average field only one kind of residual interactions, the interactions leading to superconducting pairing correlations.

The conception of the average field is found to be useful in the analysis of the state structure at excitation energies exceeding the neutron binding energy  $B_n$ . Physical grounds for the use of the average field in the description of highly excited states are the large effect of the shells on the density and the widths of neutron resonances and their display in nuclear reactions in which levels of an energy of (10-20) MeV are excited.

When the number of nucleons in unfilled shells increases there occurs a transition from spherical to deformed nuclei. Because of deformation the subshell strength is distributed over some single-particle levels of the average field potential of a deformed nucleus. Fig.1 gives the energies of the single-particle states with  $K^\pi = 1/2^+$  and the components of the  $3S_{1/2}$  and  $4S_{1/2}$  subshells in the expansion of the single-particle wave functions in the spherical basis. It is seen that the strength



$$A = 239 \quad \beta_{20} = 0.23 \quad \beta_{40} = 0.08$$

Fig. 1. Distribution of the strength of the  $3S_{1/2}$  and  $4S_{1/2}$  subshells over the single-particle states with  $K^\pi = 1/2^+$  in the neutron system with  $A=239$  at  $\beta_{20} = 0.23$ ,  $\beta_{40} = 0.08$ . The numbers labelling the levels indicate the values of the components  $3S_{1/2}$  and  $4S_{1/2}$  in the expansion of the wave function of the deformed Saxon-Woods potential in the spherical basis.

of the  $3S_{1/2}$  and  $4S_{1/2}$  subshells is distributed among many single-particle levels. The subshell splitting due to deformation is one of the first manifestations of the fragmentation process.

Of much importance is the problem of the nuclear shape in excited states. On the basis of a large amount of experimental data (behaviour of the strength functions for the s- and p-wave neutron resonances, probabilities of the alpha and gamma transitions from highly excited states, splitting of the EI giant resonance and so on) it may be concluded that the shape of spherical and deformed nuclei for the majority of states does not change essentially with increasing excitation energy. The problem of the shape of the excited states of the transition-region nuclei is more complicated. Among excited states there are states the equilibrium deformation of which differs strongly from that for the ground states. These are spontaneously fissioning isomers <sup>3</sup> and some quasiparticle states <sup>4,5</sup>.

Thus we can draw the conclusion that the description of the states of an excitation energy somewhat exceeding  $B_n$  should be based on the same average field potentials which are used in the description of the low-lying states. The problem of applicability of the conception of the average field is tightly related to the problem to what excitation energies the shells are displayed. It is very important to answer the latter question in order to understand the nature of highly excited states.

### 3. Superconducting Pairing Correlations

The interactions leading to superconducting pairing correlations strongly affect the structure of low-lying states. In the excited state description use is made of the following quasiparticle operators

$$d_{q\sigma}^+ = \begin{array}{c} | \\ U_q \\ | \\ 0 \end{array} a_{q-\sigma}^+ + G \begin{array}{c} 0 \\ V_q \\ | \\ 1 \end{array} a_{q\sigma} \quad \begin{array}{l} E(q) \gg E_F \\ E(q) \sim E_F \\ E(q) \ll E_F \end{array} \quad (2)$$

For a single-particle level  $q$  lying much above the Fermi level the quasiparticle production operator transforms to the particle production operator. For the level  $q$  with  $E(q) \ll E_F$  the quasiparticle production operator transforms to the hole production operator.

The pairing correlations become less important with increasing excitation energy. The quasiparticle energy  $\mathcal{E}(q) = \sqrt{C^2 + (E(q) - \lambda)^2}$  for  $|E(q) - \lambda| \gg C$  transforms to the single-particle energy  $|E(q) - \lambda|$ . At high excitations pairing should be taken into consideration, because it affects noticeably the state structure and presents a convenient mathematical tool. Troubles connected with the application of the theory of pairing correlations to highly excited states are due to the necessity of taking into account the blocking effect.

#### 4. Multipole-Multipole and Spin-Multipole-Spin-Multipole Interactions

There are two types of low-lying nonrotational states: quasiparticle for which the density matrix is diagonal and collective connected with the nondiagonal part of the density matrix. To describe the collective states we introduce the residual interaction

$$V((\vec{r}_1, -\vec{r}_2)) + V_G((\vec{r}_1, -\vec{r}_2)(\vec{\sigma}^{(1)} \vec{\sigma}^{(2)}), \quad (3)$$

which is presented in the form of the multipole-multipole and spin-multipole-spin-multipole expansions. The angular part of this interaction is determined while in the radial dependence there is an ambiguity. It is essential for mathematical treatment that the radial part is taken in a factorized form.

In all the nuclei there are observed low-lying collective quadrupole ( $\lambda=2$ ) and octupole ( $\lambda=3$ ) states. The wave functions of the collective one-phonon states are a superposition of a large number of two-quasiparticle components. The presence of the nondiagonal part of the density matrix leads to a distribution of the two-quasiparticle state strength over the collective levels in spherical and deformed nuclei. This is the second type of fragmentation.

Among highly excited states there are collective states commonly called giant resonances. The giant resonances must have different multipolarities and different isoscalar and isovector parts. There is sufficient information about the E1 giant resonances, some data are also obtained on the E2 and M1 giant resonances. Of much interest is the problem of monopole states with high multipolarities.

The presence of the quadrupole and octupole collective low-lying states and high-lying states of the type of giant resonances is the universal property of all nuclei.

Consideration of the multipole-multipole and spin-multipole-spin-multipole interactions is important when one treats the excited states in a large energy interval.

#### 5. The Density of Excited States

A correct description of the density of the highly excited state serves as a criterion of a wide use of a given model. The general laws of the density behaviour are given by the Fermi-gas model. An essentially improved description of the state density is obtained in the independent quasiparticle model with the use of the methods of statistical averaging<sup>6</sup>. Good description of the density up to the neutron-binding energy  $B_n$  is also obtained in the framework of the semi-microscopic approach to the superfluid nuclear model<sup>7</sup>, accounting for quasiparticle and phonon excitations.

The wave functions of the independent quasiparticle model and the wave functions in the form of the product of the quasiparticle and phonon operators do not describe the structure of states of intermediate and high energies. The correct description of the state density provides only evidence that this model may serve as a basis for describing the structure of highly excited states as far as the configuration space of the model is large enough to cover the whole complexity of highly excited states.

## 6. Complication of the Level Structure with Increasing Excitations

If the nuclear states are considered as states having a definite number of quasiparticles and phonons then it is impossible to explain a large amount of experimental facts on the excited states of complex nuclei. For example, if a nucleus in the ground or one-quasiparticle state captures a slow neutron this cannot result in the production of a many-quasiparticle state, and in the framework of such an approach it is impossible to explain the density of neutron resonances and their widths.

In the understanding of the structure of highly excited states and the description of them in the language of quasiparticles and phonons in which the low-lying states are also treated, the main role is attributed to the process called fragmentation or fractionalization. By fragmentation we mean the distribution of the strength of a one-, two- or many-particle states over many nuclear levels. In other words, fragmentation is responsible, for example, for the distribution of the single-particle state being the solution of the Schrödinger equation with the Saxon-Woods potential over a number of nuclear levels.

There are two main causes leading to complication of the state structure with increasing excitations: the first one is the interaction of the single-particle and collective motions described as the interaction of quasiparticles with phonons;

the second cause is the connection of the intrinsic and rotational motions described via the Coriolis interaction.

The interaction of quasiparticles with phonons is of much importance in the process of fragmentation. It leads to mixing of the components differing by one phonon or by two quasiparticles. As a result of consideration of the quasiparticle-phonon interaction the wave function has the form of a sum of one-, two-, three- and higher phonon components in the case of doubly even nuclei. In odd nuclei the wave function consists of the following components: one-quasiparticle, quasiparticle plus phonon, quasiparticle plus two phonons, etc. These facts underlie the model suggested in refs. <sup>8,9</sup> for describing highly excited states. Much progress is made in developing an approximate method for reducing the determinant of the order  $10^4$  to a secular equation.

As investigations in the framework of this model show, the first quadrupole and octupole phonons strongly affect the process of fragmentation. The stronger the collectivization of a one-phonon state the greater its effect on the process of fragmentation. Fragmentation is noticeably affected by higher phonons. Of interest is the problem of the influence of the collective states of the type of giant resonances on the fragmentation process. It seems necessary to study the effect of the collective phonons describing giant resonances on fragmentation and the complication of the structure of highly excited states.

It should be noted that the complication of the structure with increasing excitation energy is universal, it occurs in all nuclei. Therefore much attention should be paid to the investigation of this process.

In the framework of the semi-microscopic approach in the superfluid nuclear model we construct<sup>10,11</sup> the wave function of a highly excited state. The wave function is presented in the form of an expansion in the number of quasiparticles. The complication of the state structure with increasing excitations exhibits in the fact that in the wave function the role of the components grows as the number of quasiparticles increases gradually. When constructing such a wave function we start from the Hamiltonian (1) and use the single-particle states of the average field potential and the mathematical apparatus of the theory of superconducting pairing correlations. We bear in mind the residual interactions in (1) which are employed to describe effects caused by the nondiagonal parts of the density matrix and the correlation function.

The wave function of, e.g., the highly excited state of an odd spherical nucleus is of the form

$$\begin{aligned} \Psi_i(I^\pi M) = & b_I^\lambda \mathcal{L}_{IM}^\lambda \Psi_0 + \\ & + \sum_{\substack{j_1 j_2 j_3 \\ m_1 m_2 m_3}} b_I^\lambda (j_1 m_1 j_2 m_2 j_3 m_3) d_{j_1 m_1}^+ d_{j_2 m_2}^+ d_{j_3 m_3}^+ \Psi_0 + \\ & + \sum_{\substack{j_1 j_2 j_3 j_4 j_5 \\ m_1 m_2 m_3 m_4 m_5}} b_I^\lambda (j_1 m_1 j_2 m_2 j_3 m_3 j_4 m_4 j_5 m_5) d_{j_1 m_1}^+ d_{j_2 m_2}^+ d_{j_3 m_3}^+ d_{j_4 m_4}^+ d_{j_5 m_5}^+ \Psi_0 + \dots \end{aligned} \quad (4)$$

We should add to this expression terms with the operators of pairing vibrational phonons which replace the operators

$(d_{j m}^+ d_{j-m}^+)_{I=0}$ . In addition, we can introduce explicitly in (4) the operators of any phonons. In (4) the summation is performed over the single-particle states of the neutron and proton system,  $\Psi_0$  is the wave function of a quasiparticle or phonon vacuum.

When constructing the wave function (4) we assume that the density matrix is diagonal for the ground state of the nucleus. In this representation the wave function of a highly excited state must contain thousands of different components. The use of this representation for the highly excited state wave function is physically justified. In the majority of cases the formation of a highly excited state occurs due to capture of a slow neutron or a high energy  $\gamma$ -ray by a target-nucleus in the ground zero-quasiparticle or one-quasiparticle states. Therefore the expansion (4) is made as if in the basis functions of the target-nucleus.

The operator form of the wave function is used in refs.<sup>10,11</sup> to express the reduced neutron radiational and alpha widths in neutron resonances in terms of the coefficients  $b_I^\lambda$ . In this way there arises the problem of experimental determination of the coefficients  $b_I^\lambda$ . The coefficients  $b_I^\lambda$  can be found from the spectroscopic factors of the reactions of the types  $(d,p)$  and  $(d,t)$  from the probabilities of  $\beta$  decays, from the probabilities of  $\gamma$  transitions between the excited states, etc.

Consider, for example, one-quasiparticle components. The strength of each single-particle state is distributed over several levels. At low energies this fragmentation is manifested in  $(d,p)$  and  $(d,t)$  reactions when some levels are excited.



With increasing excitation energy the level density grows and it is difficult to find them experimentally. Therefore, in ref. <sup>11</sup> it is proposed to determine experimentally the strength functions in the ( $\alpha p$ ) reaction with momentum transfer  $\ell=1$  and compare them with the strength functions for  $p$ -wave neutrons. This way can be utilized to study the strength function as a function of the excitation energy. The ( $d t$ ) reaction cross sections can yield information about the strength functions associated with the fragmentation of hole states. It is of particular concern to clarify how the fragmentation of the strength of the hole states differs from that of the particle states.

#### 7. The Structure of the Highly Excited State and the Compound State

At excitation energies close to and higher than the neutron binding energy the wave functions(4) contain thousands of different few-quasiparticle and many-quasiparticle components. These wave functions possess the properties of the compound states suggested by N.Bohr. In fact, the formation of a highly excited state can proceed through some components of the wave function and the decay through some others. Therefore in the majority of cases, the basic condition of the compound state, independence of its decay of the method of formation, is valid. Because of a very large number of the components of the wave function some few-quasiparticle components must have, as a rule, small values. This results in an essential hindrance of the probabilities of gamma transitions to low-lying states. Therefore the half-life of a highly excited state must be much longer than that of a one-quasiparticle or two-quasiparticle state.

Our interpretation of the highly excited state differs from the conception of the compound state suggested by N.Bohr. The latter is based on the smallness of the mean free path of the nucleon in the nucleus and, according to it, a compound nucleus is formed as a result of numerous collisions after penetration of the particle into the nucleus. Our treatment of the highly excited state is based on the model of independent quasiparticles and residual interaction described by the Hamiltonian. (1). We start from the fact that in our representation the wave function is a many-component one. It enables all the effects interpreted by means of the compound state to be understood. It can be used to explain the particular features of excitation as a result of the capture of a nucleon or absorption of a gamma ray. However, in such an approach we do not put the question as to how such a complex state is formed dynamically from the simple state due to a capture of a nucleon or a gamma ray.

The experiments, which study excitation and decay of intermediate and highly excited states, involve, as a rule, only few-quasiparticle components of the wave function. So, for example, few-quasiparticle components amounting to a  $10^{-4}$ - $10^{-8}$  part of the normalization of the wave functions of the neutron resonances are displayed in the study of the neutron resonances. It should be noted that it is just for these few-quasiparticle components alone that the laws of the statistical nuclear model are valid.

In ref. <sup>12</sup> the problem is whether all the components of the wave function of a highly excited state are small or whether among them there are relatively large components. Experiments on gamma transitions between highly excited states capable of revealing large many-quasi-components in their wave functions are suggested.

In some cases experimental data make it possible to extract evidence about some components of the wave functions (4). Of much convenience are the  $(\gamma n)$  reactions since the known structure of the initial state permits reconstruction possible components in the excited state. In some cases it is possible to obtain information on three- and five- quasiparticle components of the wave functions of highly excited states. In this respect the most favourable is the study of the EI transitions in <sup>177</sup>Lu from the neutron resonances with  $I^\pi = 13/2^-$  and  $15/2^-$  to three-quasiparticle states.

The available experimental data enable us to give some conclusions about the main laws of the fragmentation process. This problem is discussed in detail in ref. <sup>13</sup> where it is noticed that the degree of fragmentation is A-dependent. Fragmentation is strongly weakened in doubly closed shell nuclei and in nuclei differing from the latter by one nucleon. Certain approximate laws make fragmentation less strong which is observed in isobar analogue states.

In the collective states of the type of giant resonances the few-quasiparticle components to which there correspond phonons of a definite multipolarity are of greater importance. Because of the quasiparticle-phonon interaction the strength of a

collective branch is distributed over many nuclear levels. The wave functions of the states of the type of giant resonances contain a large number of different many-quasiparticle components.

It should be noted that the division of the wave function into simple and more complicated parts is widely used in the interpretation of the excitation and decay processes of highly excited states. However, the mathematical procedure of successive introduction of simple and complicated components should not be understood literally. We cannot impart to this procedure the physical meaning of the transitions from simple configurations to more complicated ones.

#### 8. Conclusion

It follows from the analysis made here that the consideration of nuclear interaction in the form of the Hamiltonian (1) and the suggested method of solving the nuclear many-body problem may serve as a basis for describing low, intermediate and high nuclear states. It is undoubtful that further study of excited states will lead to the necessity of adding to the Hamiltonian (1) new terms, e.g., corresponding to tensor forces and to improving the mathematical methods.

With increasing excitation energy the state density increases and their structure becomes more complicated. This is the universal property of all nuclei. The complication of the structure is caused by the fact that with increasing excitation energy the number of the degree of freedom responsible for the formation of a given state increases, and the interactions of the

noncoherent type become more and more important. It is shown that the coupling of the single-particle and collective motions presented in the form of the quasiparticle-phonon interaction play the important part in the process of complication of the state structure. Of importance is also the relationship between rotation and intrinsic motion. Searches for forces and mechanism leading to complications of the structure with increasing excitations are for the time being the principal task.

The detailed study of the structure of intermediate and high energy states is still in its initial stage. Because of the complexity of these states it is necessary to make a combined study by various experimental methods. It is undoubtful that to determine many-quasiparticle components the information needed can be extracted from many-nucleon transfer reactions in the interaction of heavy ions with the nuclei.

It may be concluded that at present there is a general basis for describing low, intermediate and high excitations of atomic nuclei.

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Received by Publishing Department  
on December 28, 1973.