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E4 - 7625

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STANDARD-BASIS OPERATOR METHOD IN THE GREEN-FUNCTION DIAGRAM TECHNIQUE OF MANY BODY SYSTEMS. II. APPLICATIONS TO HEISENBERG FERROMAGNET WITH SINGLE-ION ANISOTROPY

1973

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STANDARD-BASIS OPERATOR METHOD IN THE GREEN-FUNCTION DIAGRAM TECHNIQUE OF MANY BODY SYSTEMS.

II. APPLICATIONS TO HEISENBERG FERROMAGNET WITH SINGLE-ION ANISOTROPY

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1. Introduction

In the preceding paper [1] / reffered to as I hereafter / we have presented the essential steps of the constructing of diagram technique for an arbitrary spin Hamiltonian by the example of some subclass of spin one Hamiltonians, being of the considerable physical interest / see corresponding references in I / . The rules for drawing of the graphs, obtained in I, allow us to build up the diagrams for arbitrary interaction and to arbitrary interaction order.

On the basis of the previous approach, in this paper we proceed the study of the Heisenberg Ferromagnet with uniaxial crystal field.

In sec. 2 we calculate the correlation functions-transverse and longitudinal /diagonal /-to order $(1/z)^1$. The effective interaction obtained in sec. 2 is used to derive the $(1/z)^2$ result for the correlation functions in secS. 3,4 and appendices A,B.

2. Effective interaction and $(1/z)^1$ result for the correlation functions

In this section we consider the correlation functions

$$\langle\langle S_k^- ; S_k^+ \rangle\rangle \equiv \bar{G}^{-+}(k, i\lambda_m) \; , \qquad /1a/$$

$$\langle\langle \hat{Q}_k^{13} ; \hat{Q}_k^{13} \rangle\rangle = K(k, i\lambda_m) \qquad /1b/$$

within the Heisenberg Ferromagnet with single-ion anisotropy model, i.e., we analyse the limiting case $E \to 0$, $K_{\varkappa\varkappa'} \to 0$ of the Hamiltonian / I.1,4-6,12-15 / :

$$H_o = -\sum_{\varkappa} \left[(h + J_o \langle Q^{13} \rangle_o) Q_{\varkappa}^{13} + D Q_{\varkappa}^{\overline{13}} \right] , \qquad /2/$$

$$V = -\tfrac{1}{2} \sum_{\varkappa \neq \varkappa'} J_{\varkappa\varkappa'} \left(\hat{Q}_{\varkappa}^{13} \hat{Q}^{13} + S_{\varkappa}^{12} \overset{+}{S}{}_{\varkappa'}^{12} + S_{\varkappa}^{12} \overset{+}{S}{}_{\varkappa}^{13} + S_{\varkappa}^{23} \overset{+}{S}{}_{\varkappa'}^{12} + S_{\varkappa}^{23} \overset{+}{S}{}_{\varkappa'}^{13} \right) =$$

$$= \bigcirc\!\sim\!\sim\!\sim\!\bigcirc + \textcircled{\scriptsize 12}\!\sim\!\sim\!\sim\!\textcircled{\scriptsize 12} + \textcircled{\scriptsize 12}\!\sim\!\sim\!\sim\!\textcircled{\scriptsize 13} + \textcircled{\scriptsize 23}\!\sim\!\sim\!\sim\!\textcircled{\scriptsize 12} + \textcircled{\scriptsize 23}\!\sim\!\sim\!\sim\!\textcircled{\scriptsize 23} \; , \qquad /3/$$

where the circle instead of $\boxed{13}$ is used in this paper to denote the operator $\hat{Q}^{13}$ .According to /3 / we define the 2x2 matrix Green function

$$G^{-+}(k, i\lambda_m) = \begin{pmatrix} G^{-+}_{12,12}(k, i\lambda_m) & G^{-+}_{12,13}(k, i\lambda_m) \\ G^{-+}_{13,12}(k, i\lambda_m) & G^{-+}_{13,13}(k, i\lambda_m) \end{pmatrix} \qquad /4/$$

where the components of the matrix are causal Green functions of the transverse operators and are defined by

$$G_{ij,fp}^{-+}(k,i\lambda_m) \equiv \langle\langle S_k^{ij} ; S_k^{+fp} \rangle\rangle =$$

$$= \frac{1}{2}\sum_{x}\int_{-\beta}^{\beta} d\tau \exp\left[-ik(R_x-R_{x'})+i\lambda_m\tau\right] * \langle T S_x^{ij}(\tau)\, S_{x'}^{+fp}(0)\rangle \quad . \qquad /5/$$

Corresponding matrix for the transverse interaction is

$$\underline{V}^{+-}(k) = V(k)\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}; \quad V(k) = J(k)/2 , \qquad /6/$$

where $J(k)$ is the Fourier transform of $J_{xx'}$.

Denoting by $\underline{\Sigma}^{-+}$ the irreducible polarization part of $\underline{G}^{-+}$ /2-7/ we can write the graphical equation

$$\underline{G}^{-+}(k,i\lambda_m) = \underline{\Sigma}^{-+}(k,i\lambda_m) + \underline{\Sigma}^{-+}(k,i\lambda_m)\, \underline{V}^{+-}(k)\, \underline{G}^{-+}(k,i\lambda_m) \qquad /7/$$

According to the relations $S^- = S^{12} + S^{23}$,/5-7/ we get the solution for the Green function / 1a / in the form

$$\bar{G}^{-+} = \left(\bar{\Sigma}^{-+^{-1}} - V(k)\right)^{-1} , \qquad /8/$$

where

$$\bar{\Sigma}^{-+} = \Sigma_{12,12}^{-+} + \Sigma_{12,23}^{-+} + \Sigma_{23,12}^{-+} + \Sigma_{23,23}^{-+} \qquad /9/$$

and the components of $\underline{\Sigma}^{-+}$ are given in Appendix A up to order $(1/z)^1$, i.e.,they involve one internal momentum summation only.We obtain the similar result for the diagonal Green function /1b/

$$K(k,i\lambda_m) = \left(\Sigma_{0,0}^{-1} - J(k)\right)^{-1} , \qquad /10/$$

where $\Sigma_{0,0}$ is the irreducible polarization part of K and is given in App.B up to order $(1/z)^1$.

Effective transverse $\widetilde{\underline{V}}^{+-}$ and diagonal $\widetilde{J}$ interactions are defined analogously as in /2-7/ :

$$\widetilde{\underline{V}}^{+-}(k, i\lambda_m) = \underline{V}^{+-}(k) + \widetilde{\underline{V}}^{+-}(k, i\lambda_m)\,\overset{(o)}{\Sigma}{}^{-+}(k, i\lambda_m)\,\underline{V}^{+-}(k), \qquad /11/$$

$$\widetilde{J}(k) = J(k) + \widetilde{J}(k)\,\overset{(o)}{\Sigma}_{o,o}\,J(k) . \qquad /12/$$

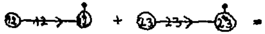
From /11-12/ it follows, that

$$\widetilde{\underline{V}}^{+-}(k, i\lambda_m) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} V(k) \left(1 - \overset{(o)}{\overline{\overline{\Sigma}}}{}^{-+}(k, i\lambda_m)\,V(k)\right)^{-1}, \qquad /11a/$$

$$\widetilde{J}(k) = J(k)\left(1 - \overset{(o)}{\Sigma}_{o,o}\,J(k)\right)^{-1}, \qquad /12a/$$

where the zero order result / Appendix A.1, A.7 and B.1 /

$$\overset{(o)}{\overline{\overline{\Sigma}}}{}^{-+}(k, i\lambda_m) =$$



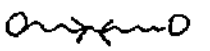
$$= \langle Q^{12}_x \rangle_o \left(i\lambda_m + H'^{12}_{ox}\right)^{-1} + \langle Q^{23}_x \rangle_o \left(i\lambda_m + H'^{23}_{ox}\right)^{-1} , \qquad /13/$$

$$\overset{(o)}{\Sigma}_{o,o}(k, i\lambda_m) = \text{o——o} = \beta\,\delta_{m,o}\,D^{13}_{ox}\,\langle Q^{13}_x \rangle_o \qquad /14/$$

for $\overline{\overline{\Sigma}}{}^{-+}$ and $\Sigma_{o,o}$, respectively, is to be obtained by means of / I.8, 11, 17, 19-22 / in the limit $E \to 0$, $K_o \to 0$ / in the manner described in /7/ / 1973a/.

The components of effective transverse /7/ interaction /11a/ are represented in the Appendices A and B with the help of one directed wavy line in the following way

We mark the diagonal effective interaction /12a/ as follows

$\tilde{J}(\mathbf{k}) \equiv$  .

If we carry out the analytic continuation $i\lambda_m \to \lambda$ of the Bose frequency $i\lambda_m$ into the complex λ plane, the above result /8/ for the Green function $\bar{G}^{-+}$ can be employed to find the poles, which correspond to the excitation energies of the coupled „12" and „23" modes. These energies $\lambda(\mathbf{k})$ will be complex in general since the coupled modes are quasiparticle states with reciprocal lifetimes proportional to the imaginary part of $\lambda(\mathbf{k})$. The complex solutions for $\lambda(\mathbf{k})$ are found from

$$\det\left[1 - \Sigma^{-+}(\mathbf{k}, \lambda(\mathbf{k}))\, \underline{V}^{+-}(\mathbf{k})\right] =$$

$$= 1 - \bar{\Sigma}^{-+}(\mathbf{k}, \lambda(\mathbf{k}))\, \tilde{J}(\mathbf{k}) = 0 ,$$

where the energy and damping of the excitations correspond to the real and imaginary parts, respectively, of $\lambda(\mathbf{k})$.

When $\bar{\Sigma}^{-+}$ in /9/ is approximated by $\bar{\Sigma}^{(0)-+}$ given in /13/ we obtain the results of [8-14] for $\bar{G}^{-+}$ to order $(1/Z)^1$:

$$1 - \bar{\Sigma}^{(0)-+}(\mathbf{k}, i\lambda_m)\tilde{J}(\mathbf{k}) = \frac{(i\lambda_m + \varepsilon_1(\mathbf{k}))(i\lambda_m + \varepsilon_2(\mathbf{k}))}{(i\lambda_m + H_{0x}'^{11})(i\lambda_m + H_{0x}'^{13})} ; \qquad /15/$$

the simple poles $\varepsilon_{(\frac{1}{2})}$

$$\varepsilon_{(\frac{1}{2})}(\mathbf{k}) = h + (J_0 - \tfrac{1}{2}J(\mathbf{k}))\langle Q^{13}_x\rangle_0 + (\pm)\,\delta(\mathbf{k}) ,$$

$$\delta(\mathbf{k}) = \left[D^2 + (\tfrac{1}{2}J(\mathbf{k})\langle Q^{13}_x\rangle_0)^2 - D\,J(\mathbf{k})(3\langle Q^{13}_x\rangle_0 - 2)\right]^{\frac{1}{2}} \qquad /16/$$

correspond to delta function resonance peaks at the unrenormalized spin wave energies of the system.

3. Transverse correlation function to order $(1/z)^2$

The matrix self-energy / irreducible polarization parts / con - tributions / App. A / can be now used to obtain the $(1/z)^2$ contribution to the Green function $\bar{G}^{-+}$. The calculation involves merely replacement of $\bar{\bar{\Sigma}}^{-+}$ in /8/ by $\bar{\bar{\Sigma}}^{(0)}{}^{-+} + \bar{\bar{\Sigma}}^{(1)}{}^{-+\prime}$, where

$$\bar{\bar{\Sigma}}^{(1)}{}^{-+\prime} = \sum_{i=1}^{6} Z_i \quad . \qquad /17/$$

The sum of diagrams in / A.2 / and / A.8 / gives the contributions

$$Z_1 = \left(i\lambda_m + H'^{12}_{0x}\right)^{-1} N^{-1} \sum_q \tfrac{1}{2} J(q) \Big\{ -4\langle Q^{12}_x\rangle_0 D^{23}_{12}(q)\, n(12) + 2\langle Q^{23}_x\rangle_0 \,*$$

$$*\, D^{12}_{23}(q)\, n(23) + \left(E^{12}_{23,1}(q) + \beta F^{12,2}_{23,1}(q)\right) n(\varepsilon_1(q)) +$$

$$+ \left(E^{12}_{23,2}(q) + \beta F^{12,1}_{23,2}(q)\right) n(\varepsilon_2(q)) + \beta D^{12}_x D^{13}_x \langle Q^{13}_x\rangle_0 \,*$$

$$*\, \left[4\left(1 - \beta J(q) D^{13}_x \langle Q^{13}_x\rangle_0\right)\right]^{-1} \Big\} \qquad /18/$$

and

$$Z_2 = \left(i\lambda_m + H'^{23}_{0x}\right)^{-1} N^{-1} \sum_q \tfrac{1}{2} J(q) \Big\{ -4\langle Q^{23}_x\rangle_0 D^{12}_{23}(q)\, n(23) + 2\langle Q^{12}_x\rangle_0 \,*$$

$$*\, D^{23}_{12}(q)\, n(12) + \left(E^{23}_{12,1}(q) + \beta F^{23,2}_{12,1}(q)\right) n(\varepsilon_1(q)) +$$

$$+ \left(E^{23}_{12,2}(q) + \beta F^{23,1}_{12,2}(q)\right) n(\varepsilon_2(q)) + \beta D^{23}_x D^{13}_x \langle Q^{13}_x\rangle_0 \,*$$

$$*\, \left[4\left(1 - \beta J(q) D^{13} \langle Q^{13}_x\rangle_0\right)\right]^{-1} \Big\} , \qquad /19/$$

respectively, where

$$D_{12}^{23}(q) = (H'^{23}_{ox} - H'^{12}_{ox})\left[(\varepsilon_1(q) - H'^{12}_{ox})(\varepsilon_2(q) - H'^{12}_{ox})\right]^{-1} ,$$

$$F_{23,1}^{12,2}(q) = (\varepsilon_2(q) - \varepsilon_1(q))^{-1}\left\{ H'^{23}_{ox} D^{12}_{x} \langle Q^{12}_{x}\rangle_o + H'^{12}_{ox} D^{12}_{x} \langle Q^{23}_{x}\rangle_o + \right.$$

$$\left. - \varepsilon_1(q) D^{12}_{x} \langle Q^{12}_{x} + Q^{23}_{x}\rangle_o \right\} ,$$

$$E^{12}_{23,\binom{1}{2}} = (\varepsilon_1(q) - \varepsilon_2(q))^{-1}(\pm)\left\{ 2\langle Q^{12}_{x}\rangle_o - \langle Q^{23}_{x}\rangle_o + 2\langle Q^{12}_{x}\rangle_o (H'^{12}_{ox} - H'^{23}_{ox}) * \right.$$

$$\left. * (\varepsilon_{\binom{1}{2}}(q) - H'^{12}_{ox})^{-1} + \langle Q^{23}_{x}\rangle_o (H'^{12}_{ox} - H'^{23}_{ox})(\varepsilon_{\binom{1}{2}}(q) - H'^{23}_{ox})^{-1} \right\} .$$

For the sum of diagrams /A.3/, /A.9/,/A.13/ and /A.17/ we obtain the expression

$$Z_3 = -(i\lambda_m + H'^{12}_{ox})^{-2} N^{-1} \sum_q J(q) \left\{ G^{12,2}_{23,1}(q)\, n(\varepsilon_1(q)) + G^{12,1}_{23,2}(q)\, n(\varepsilon_2(q)) + \right.$$

$$\left. + D^{11}_{x} \langle Q^{13}_{x}\rangle_o \cdot (1 - \beta J(q) D^{13}_{x} \langle Q^{13}_{x}\rangle_o)^{-1} \right\} - (i\lambda_m + H'^{23}_{ox})^{-2} N^{-1} \sum_q J(q) *$$

$$* \left\{ G^{23,2}_{12,1}(q)\, n(\varepsilon_1(q)) + G^{23,1}_{12,2}(q)\, n(\varepsilon_2(q)) + D^{23}_{x} \langle Q^{13}_{x}\rangle_o * \right.$$

$$\left. * (1 - \beta J(q) D^{13}_{x} \langle Q^{13}_{x}\rangle_o)^{-1} \right\} +$$

$$+ 2\left[(i\lambda_m + H'^{12}_{ox})(i\lambda_m + H'^{23}_{ox})\right]^{-1} N^{-1} \sum_q J(q) \left\{ G^2_1(q)\, n(\varepsilon_1(q)) + G^1_2(q)\, n(\varepsilon_2(q)) \right\} +$$

$$+ \left[(i\lambda_m + H'^{12}_{ox})^{-1} - (i\lambda_m + H'^{23}_{ox})^{-1}\right]^2 N^{-1} \sum_q \tfrac{1}{2} J(q) \left\{ a_1^2(q) \left(n(\varepsilon_1(q)) - n(13) \right) * \right.$$

$$* \left(i\lambda_m + H'^{13}_{ox} - \mathcal{E}_1(q)\right)^{-1} + \; a^1_2(q)\left(n(\mathcal{E}_2(q)) - n(13)\right) *$$

$$* \left(i\lambda_m + H'^{13}_{ox} - \mathcal{E}_2(q)\right)^{-1} \Big\} \, 4 \langle Q^{13}_x \rangle_o \quad ,$$

where

$$G^{12,1}_{23,2}(q) = \left(\mathcal{E}_1(q) - \mathcal{E}_2(q)\right)^{-1} \Big[2 H'^{23}_{ox} \langle Q^{12}_x \rangle_o + H'^{12}_{ox} \langle Q^{23}_x \rangle_o +$$

$$- \mathcal{E}_2(q) \left(2 \langle Q^{12}_x \rangle_o + \langle Q^{23}_x \rangle_o \right) \Big] \; ,$$

$$G^2_1(q) = \left(\mathcal{E}_2(q) - \mathcal{E}_1(q)\right)^{-1} \Big[H'^{23}_{ox} \langle Q^{12}_x \rangle_o + H'^{12}_{ox} \langle Q^{23}_x \rangle_o + $$

$$- \mathcal{E}_1(q) \langle Q^{12}_x + Q^{23}_x \rangle_o \Big] \; ,$$

$$a^1_2(q) = \left(\mathcal{E}_1(q) - \mathcal{E}_2(q)\right)^{-1} \Big[H'^{12}_{ox} H'^{23}_{ox} - \left(H'^{13}_{ox} - \mathcal{E}_2(q)\right) \mathcal{E}_2(q) \Big] \quad .$$

Collecting the diagrams /A.4/, /A.10/,/A.14/ and /A.18/ we get the result

$$Z_4 = N^{-1} \sum_q \Big\{ \left(i\lambda_m + H'^{12}_{ox}\right)^{-2} b_{12,12}(q) + \left(i\lambda_m + H'^{23}_{ox}\right)^{-2} b_{23,23}(q) +$$

$$+ 2 \left[\left(i\lambda_m + H'^{12}_{ox}\right)\left(i\lambda_m + H'^{23}_{ox}\right) \right]^{-1} b_{12,23}(q) \Big\} J(k-q) \left[2\left(1 - \beta J(q) D^{13}_x \langle Q^{13}_x \rangle_o\right) \right]^{-1} +$$

$$+ N^{-1} \sum_q \Big\{ \left(i\lambda_m + H'^{12}_{ox}\right)^{-2} b_{12,12}(k-q) + \left(i\lambda_m + H'^{23}_{ox}\right)^{-2} b_{23,23}(k-q) +$$

$$+ 2 \left[\left(i\lambda_m + H'^{12}_{ox}\right)\left(i\lambda_m + H'^{23}_{ox}\right) \right]^{-1} b_{12,23}(k-q) \Big\} J(q) \left[2\left(1 - \beta J(q) D^{13}_x \langle Q^{13}_x \rangle_o\right) \right]^{-1} *$$

$$* \Big[a^2_1(q) \left(i\lambda_m + \mathcal{E}_1(q)\right)^{-1} + a^1_2(q) \left(i\lambda_m + \mathcal{E}_2(q)\right)^{-1} \Big] \; ,$$

where

$$b_{12,23}(q) = D^{12}_{x}\langle Q^{23}_{x}\rangle_0 + \beta J(q)\left[D^{12}_{x}\langle Q^{13}_{x}\rangle_0 \cdot D^{23}_{x}\langle Q^{13}_{x}\rangle_0 + \right.$$

$$\left. - D^{12}_{x}\langle Q^{23}_{x}\rangle_0 \cdot D^{13}_{x}\langle Q^{13}_{x}\rangle_0 \right] .$$

The contribution coming from the diagrams /A.5/,/A.11/,/A.15/ and /A.19/ take the form

$$Z_5 = (i\lambda_m + H'^{12}_{0x})^{-2}\langle Q^{12}_{x}\rangle^2 N^{-1}\sum_q J(k-q)J(q)\tfrac{1}{2}\left\{ D^{23}_{12}(q)\, n(12) + \right.$$

$$\left. + D^{23,2}_{12,1}(q)\, n(\varepsilon_1(q)) + D^{23,1}_{12,2}(q)\, n(\varepsilon_2(q)) \right\} + (i\lambda_m + H'^{23}_{0x})^{-2} *$$

$$* \langle Q^{23}_{x}\rangle_0^2 N^{-1}\sum_q J(k-q)J(q)\tfrac{1}{2}\left\{ D^{12}_{23}(q)\, n(23) + D^{12,2}_{23,1}(q)\, n(\varepsilon_1(q)) + \right.$$

$$\left. + D^{12,1}_{23,2}(q)\, n(\varepsilon_2(q)) \right\} + \left[(i\lambda_m + H'^{12}_{0x})(i\lambda_m + H'^{23}_{0x})\right]^{-1}\langle Q^{12}_{x}\rangle_0 *$$

$$* \langle Q^{23}_{x}\rangle_0 N^{-1}\sum_q J(k-q)J(q)\left(\varepsilon_1(q) - \varepsilon_2(q)\right)^{-1}\left(n(\varepsilon_2(q)) - n(\varepsilon_1(q)) \right) +$$

$$+ N^{-1}\sum_q D^{13}_{x}\langle Q^{13}_{x}\rangle_0 J^2(k-q)\left(1 - \beta J(k-q) D^{13}_{x}\langle Q^{13}_{x}\rangle_0\right)^{-1} *$$

$$* \left\{ \langle Q^{12}_{x}\rangle_0 (i\lambda_m + H'^{12}_{0x})^{-2} d^{23}_{12}(q,m) + \langle Q^{23}_{x}\rangle_0 (i\lambda_m + H'^{23}_{0x})^{-2} * \right.$$

$$* d^{12}_{23}(q,m) + 2\langle Q^{12}_{x}\rangle_0 \langle Q^{23}_{x}\rangle_0 J(q)\left[2(i\lambda_m + H'^{12}_{0x})(i\lambda_m + H'^{23}_{0x}) * \right.$$

$$\left. \left. * (i\lambda_m + \varepsilon_1(q))(i\lambda_m + \varepsilon_2(q)) \right]^{-1} \right\} ,$$

where

$$D^{-3,2}_{12,1}(q) = \left(H'^{23}_{0x} - \varepsilon_1(q)\right)\left[\left(H'^{12}_{0x} - \varepsilon_1(q)\right)\left(\varepsilon_2(q) - \varepsilon_1(q)\right)\right]^{-1} \quad ,$$

$$d^{23}_{12}(q,m) = \left(i\lambda_m + H'^{12}_{0x}\right)^{-1} + \langle Q^{12}_x\rangle_o J(q)\left(i\lambda_m + H'^{23}_{0x}\right) *$$

$$* \left[2\left(i\lambda_m + H'^{12}_{0x}\right)\left(i\lambda_m + \varepsilon_1(q)\right)\left(i\lambda_m + \varepsilon_2(q)\right)\right]^{-1} .$$

The diagrams indicated as / A.6/, /A.12/, /A.16/ and / A.20 / contribute to Z_6 as follows

$$Z_6 = -N^{-1}\sum_q J(k-q)J(q)\left[\left(1-\beta J(k-q)\,D^{13}_x\langle Q^{13}_x\rangle_o\right)\left(i\lambda_m + \varepsilon_1(q)\right) *\right.$$

$$\left. *\left(i\lambda_m + \varepsilon_2(q)\right)\right]^{-1}\left\{\langle Q^{12}_x\rangle_o\left[D^{12}_x\langle Q^{13}_x\rangle_o\left(i\lambda_m + H'^{23}_{0x}\right)\left(i\lambda_m + H'^{12}_{0x}\right)^{-1} +\right.\right.$$

$$\left. + D^{23}_x\langle Q^{13}_x\rangle_o\right]\left(i\lambda_m + H'^{12}_{0x}\right)^{-1} + \langle Q^{23}_x\rangle_o\left[D^{12}_x\langle Q^{13}_x\rangle_o + D^{23}_x\langle Q^{13}_x\rangle_o *\right.$$

$$\left.\left. * \left(i\lambda_m + H'^{12}_{0x}\right)\left(i\lambda_m + H'^{23}_{0x}\right)^{-1}\right]\left(i\lambda_m + H'^{23}_{0x}\right)^{-1}\right\} .$$

The right-hand side in / 18/ and / 19/ without the factor $\left(i\lambda_m + H'^{12}_{0x}\right)^{-1}$ and $\left(i\lambda_m + H'^{23}_{0x}\right)^{-1}$,respectively,gives the average of Q^{12} and Q^{23} ,correspondingly,in the first order. Hence we obtain the average of $S^2 = Q^{13}$ in the same order due to the relation $Q^{12} + Q^{23} = 2\,Q^{13}$.

4. Diagonal correlation function to order $(1/z)^2$

In the approximation presented in Appendix B , $\sum_{o,o}$ appeared in / 10/ ,is equal to

$$\Sigma_{0,0} = \Sigma^{(0)}_{0,0} + \Sigma^{(1)\,\prime}_{0,0} \quad ,$$

where $\Sigma^{(0)}_{0,0}$ is given in / 14 / and

$$\Sigma^{(1)\,\prime}_{0,0} = \sum_{i=1}^{5} Y_i \quad .$$

The diagrams in / B.2 / give the contribution

$$Y_1 = \beta\,\delta\,\omega_{,0}\,N^{-1}\sum_{q} \beta J(q)\tfrac{1}{2}\left(1-\beta J(q)\,D^{13}_{x}\langle Q^{13}_{x}\rangle_0\right)^{-1} *$$

$$* \left[D^{13}_{x}\langle Q^{13}_{x}\rangle_0 + \beta J(k+q)\left(1-\beta J(k+q)\,D^{13}_{x}\langle Q^{13}_{x}\rangle_0\right)^{-1} D^{13}_{x}\langle Q^{13}_{x}\rangle_0^{2}\right] .$$

The sum of diagrams in / B.3 / is equal to

$$Y_2 = \beta\,\delta\,\omega_{,0}\,N^{-1}\sum_{q} \beta J(q)\tfrac{1}{2}\left(\varepsilon_1(q)-\varepsilon_2(q)\right)^{-1}\Big\{\left[(\varepsilon_1(q)-H'^{23}_{0x})D^{12}_{x}D^{13}_{x}\langle Q^{13}_{x}\rangle_0 +\right.$$

$$\left. + (\varepsilon_1(q)-H'^{12}_{0x})D^{23}_{x}D^{13}_{x}\langle Q^{13}_{x}\rangle_0\right] n(\varepsilon_1(q)) + \left[(H'^{13}_{0x}-\varepsilon_2(q))D^{12}_{x}\,D^{13}_{x}\langle Q^{13}_{x}\rangle_0 +\right.$$

$$\left. + (H'^{12}_{0x}-\varepsilon_2(q))D^{23}_{x}\,D^{13}_{x}\langle Q^{13}_{x}\rangle_0\right] n(\varepsilon_2(q)) \Big\} .$$

Summing the diagrams in / B.4 / we get

$$Y_3 = \beta\,\delta\,\omega_{,0}\,N^{-1}\sum_{q}\tfrac{1}{4}\beta J(q)J(k+q)\left[A^2_1\binom{k+q}{q} n(\varepsilon_1(q)) + A^2_1\binom{q}{k+q} n(\varepsilon_1(k+q)) +\right.$$

$$\left. + A^1_2\binom{k+q}{q} n(\varepsilon_2(q)) + A^1_2\binom{q}{k+q} n(\varepsilon_2(k+q))\right] ,$$

where

$$A^2_1\binom{k+q}{q} = \left[(\varepsilon_1(q)-\varepsilon_2(q))(\varepsilon_1(k+q)-\varepsilon_1(q))(\varepsilon_1(q)-\varepsilon_2(k+q))\right]^{-1}\left[(\varepsilon_1(q)-H'^{23}_{0x})^2 *\right.$$

$$* \left(D^{12}_{x}\langle Q^{13}_{x}\rangle_0\right)^2 + 2(\varepsilon_1(q)-H'^{12}_{0x})(\varepsilon_1(q)-H'^{23}_{0x})D^{12}_{x}\langle Q^{13}_{x}\rangle_0\cdot D^{23}_{x}\langle Q^{13}_{x}\rangle_0 +$$

$$+\left(\varepsilon_1(q)-H'^{12}_{0x}\right)^2\left(D^{23}_x\langle Q^{13}_x\rangle_0\right)^2\Big]\quad .$$

Collecting the diagrams / B.5 / we obtain

$$Y_4=-\beta\,\delta_{m,0}N^{-1}\sum_q J(q)\Big\{A^{23}_{12}(q)D^{12}_x\langle Q^{13}_x\rangle_0\cdot n(12)+A^{12}_{23}(q)D^{23}_x\langle Q^{13}_x\rangle_0 *$$

$$*\,n(23)+\left[B^{23,2}_{12,1}(q)D^{12}_x\langle Q^{13}_x\rangle_0+B^{12,2}_{23,1}(q)D^{23}_x\langle Q^{13}_x\rangle_0\right]n(\varepsilon_1(q))\;+$$

$$+\left[C^{23,2}_{12,1}(k+q)D^{12}_x\langle Q^{13}_x\rangle_0+C^{12,2}_{23,1}(k+q)D^{23}_x\langle Q^{13}_x\rangle_0\right]n(\varepsilon_1(k+q))\;+$$

$$+\left[B^{23,1}_{12,2}(q)D^{12}_x\langle Q^{13}_x\rangle_0+B^{12,1}_{23,2}(q)D^{23}_x\langle Q^{13}_x\rangle_0\right]n(\varepsilon_2(q))\quad+$$

$$+\left[C^{23,1}_{12,2}(k+q)D^{12}_x\langle Q^{13}_x\rangle_0+C^{12,1}_{23,2}(k+q)D^{23}_x\langle Q^{13}_x\rangle_0\right]n(\varepsilon_2(k+q))\Big\}\;,$$

where

$$A^{23}_{12}(q)=\left(H'^{23}_{0x}-H'^{12}_{0x}\right)\left[\left(\varepsilon_1(q)-H'^{12}_{0x}\right)\left(\varepsilon_2(q)-H'^{12}_{0x}\right)\right]^{-1} *$$

$$*\left\{1+\tfrac{1}{2}J(k+q)\langle Q^{12}_x\rangle_0\left(H'^{23}_{0x}-H'^{12}_{0x}\right)\left[\left(\varepsilon_1(k+q)-H'^{12}_{0x}\right)\left(\varepsilon_2(k+q)-H'^{12}_{0x}\right)\right]^{-1}\right\},$$

$$B^{23,2}_{12,1}(q)=\left(\varepsilon_2(q)-\varepsilon_1(q)\right)^{-1}\left\{\left(H'^{23}_{0x}-\varepsilon_1(q)\right)\left(H'^{12}_{0x}-\varepsilon_1(q)\right)^{-1}+\tfrac{1}{2}J(k+q)X^{23,2}_{12,1}(q) *\right.$$

$$\left. *\left[\left(\varepsilon_2(k+q)-\varepsilon_1(q)\right)\left(\varepsilon_1(k+q)-\varepsilon_1(q)\right)\right]^{-1}\right\},$$

$$C^{23,2}_{12,1}(k+q)=\tfrac{1}{2}J(k+q)\,X^{23,2}_{12,1}(k+q)\left[\left(\varepsilon_2(k+q)-\varepsilon_1(k+q)\right)\left(\varepsilon_2(q)-\varepsilon_1(k+q)\right) *\right.$$

$$* \left(\mathcal{E}_1(q) - \mathcal{E}_1(k+q)\right)\Big]^{-1} ,$$

$$X^{23,2}_{12,1}(q) = \langle Q^{12}_x\rangle_o \left(H'^{23}_{ox} - \mathcal{E}_1(q)\right)^2 \left(H'^{12}_{ox} - \mathcal{E}_1(q)\right)^{-1} + \langle Q^{23}_x\rangle_o \left(H'^{23}_{ox} - \mathcal{E}_1(q)\right)$$

The diagrams/B.6/ contribute to Y_5 as follows

$$Y_5 = \beta\, \delta_{m,0} N^{-1} \sum_q \tfrac{1}{2} J(q) \Big\{ \langle Q^{12}_x\rangle_o\, n(12)\left(n(12)+1\right) D^{23}_{12}(q) \left[2 + \tfrac{1}{2} J(k+q) *\right.$$

$$\left. * \langle Q^{12}_x\rangle_o D^{23}_{12}(k+q)\right] + \langle Q^{23}_x\rangle_o\, n(23)\left(n(23)+1\right) D^{12}_{23}(q)\left[2 + \tfrac{1}{2} J(k+q) *\right.$$

$$\left. * \langle Q^{23}_x\rangle_o D^{12}_{23}(k+q)\right]\Big\} + N^{-1} \sum_q \tfrac{1}{2} J(q) \Big\{ \langle Q^{12}_x\rangle_o \left[1 + \tfrac{1}{2} J(k+q) \langle Q^{12}_x\rangle_o *\right.$$

$$* D^{23}_{12}(k+q)\Big] \Big[D^{23,2}_{12,1}(q)\left(n(\mathcal{E}_1(q)) - n(12)\right)\left(\left(i\lambda_m + H'^{12}_{ox} - \mathcal{E}_1(q)\right)^{-1} +\right.$$

$$\left. - \left(i\lambda_m + \mathcal{E}_1(q) - H'^{12}_{ox}\right)^{-1}\right) + D^{23,1}_{12,2}(q)\left(n(\mathcal{E}_2(q)) - n(12)\right)\left(\left(i\lambda_m + H'^{12}_{ox} - \mathcal{E}_2(q)\right)^{-1} +\right.$$

$$\left. - \left(i\lambda_m + \mathcal{E}_2(q) - H'^{12}_{ox}\right)^{-1}\right)\Big] + \langle Q^{23}_x\rangle_o \left[1 + \tfrac{1}{2} J(k+q) \langle Q^{23}_x\rangle_o D^{12}_{23}(k+q)\right] *$$

$$* \Big[D^{12,2}_{23,1}(q)\left(n(\mathcal{E}_1(q)) - n(23)\right)\left(\left(i\lambda_m + H'^{23}_{ox} - \mathcal{E}_1(q)\right)^{-1} - \left(i\lambda_m + \mathcal{E}_1(q) - H'^{23}_{ox}\right)^{-1}\right) +$$

$$+ D^{12,1}_{23,2}(q)\left(n(\mathcal{E}_2(q)) - n(23)\right)\left(\left(i\lambda_m + H'^{23}_{ox} - \mathcal{E}_2(q)\right)^{-1} - \left(i\lambda_m + \mathcal{E}_2(q) - H'^{23}_{ox}\right)^{-1}\right)\Big]\Big\} +$$

$$+ N^{-1} \sum_q \tfrac{1}{4} J(q) J(k+q) \Big\{ \Big[\langle Q^{12}_x\rangle_o^2 D^{23,1}_{12,2}(q) D^{23,2}_{12,1}(k+q) - 2 \langle Q^{12}_x\rangle_o \langle Q^{23}_x\rangle_o *$$

$$* \left(4\, \delta(q)\, \delta(k+q)\right)^{-1} + \langle Q^{23}_x\rangle_o^2 D^{12,1}_{23,2}(q) D^{12,2}_{23,1}(k+q)\Big] *$$

$$* \Big(n(\varepsilon_1(k+q)) - n(\varepsilon_2(q)) \Big) \Big((i\lambda_m + \varepsilon_2(q) - \varepsilon_1(k+q))^{-1} +$$

$$- (i\lambda_m + \varepsilon_1(k+q) - \varepsilon_2(q))^{-1} \Big) + \Big[\langle Q_x^{12} \rangle_0^2 \, D_{12,1}^{23,2}(q) \, D_{12,1}^{23,2}(k+q) +$$

$$+ 2 \langle Q_x^{12} \rangle_0 \langle Q_x^{23} \rangle_0 \big(4 \, \delta(q) \, \delta(k+q) \big)^{-1} + \langle Q_x^{23} \rangle_0^2 \, D_{23,1}^{12,2}(q) \, D_{23,1}^{12,2}(k+q) \Big] *$$

$$* \Big(n(\varepsilon_1(k+q)) - n(\varepsilon_1(q)) \Big) \big(i\lambda_m + \varepsilon_1(q) - \varepsilon_1(k+q) \big)^{-1} + \Big[\langle Q_x^{12} \rangle_0^2 \, D_{12,2}^{23,1}(q) \, D_{12,2}^{23,1}(k+q) +$$

$$+ 2 \langle Q_x^{12} \rangle_0 \langle Q_x^{23} \rangle_0 \big(4 \, \delta(q) \, \delta(k+q) \big)^{-1} + \langle Q_x^{23} \rangle_0^2 \, D_{23,2}^{12,1}(q) \, D_{23,2}^{12,1}(k+q) \Big] *$$

$$* \Big(n(\varepsilon_2(k+q)) - n(\varepsilon_2(q)) \Big) \big(i\lambda_m + \varepsilon_2(q) - \varepsilon_2(k+q) \big)^{-1} \Big\} \quad .$$

5. Conclusions

The results obtained in sections 3 and 4 for the correlation functions may be interesting, when anisotropy constant D is large/1/. These solutions might be helpful if one would like to derive them within the framework of the method of /15/ or /16/. Following Plakida /16/, who has got the result corresponding to that in /2/ for the correlation function $\vec{G}^{-+}$ in the approach of the irreducible Green functions, it is quite realistic to generalize his method in order to obtain the corresponding result in our case. Furthemore, it is possible to establish the bridge between these two methods if we employ the alternative way of the reduction of averages due to the relation / 2[illegible] ref. /7/ / 1973a/.

As it may be verified, in the limit of the anisotropy constant $D \to 0$, the correlation functions, obtained in secs.3 and 4, take the form of those in /2/.

Acknowledgements

The author would like to express his sincere thanks to N.M. Plakida for critical reading of the manuscript and valuable comments.

He is also indebted to A.I.Larkin and A.Pawlikowski for usefull discussions.

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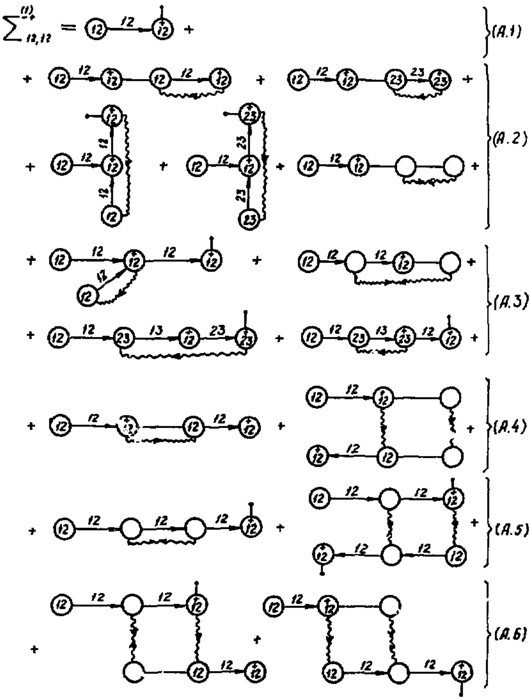
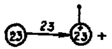
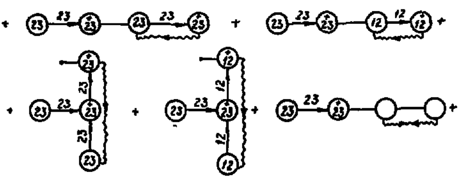
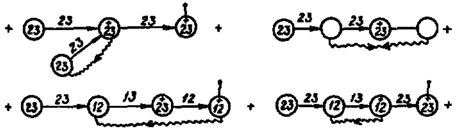
Received by Publishing Department
on December 21, 1973.

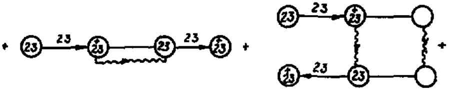
Appendix A. Irreducible polarization part components of Σ^{-+} up to first order

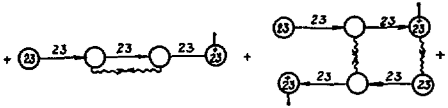


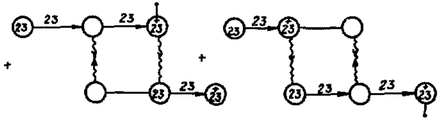
$\sum^{(1)}_{23,23} =$  (A.7)

 (A.8)

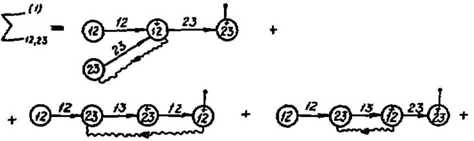
 (A.9)

 (A.10)

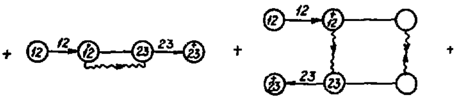
 (A.11)

 (A.12)

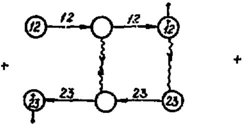
$$\sum\nolimits^{(1)}_{12,23} =$$



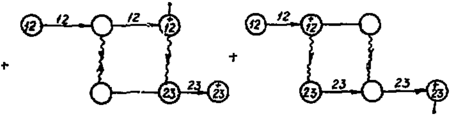
(A.13)



(A.14)

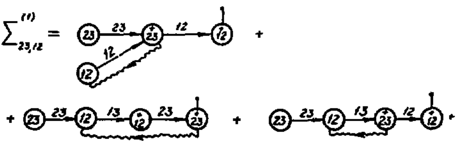


(A.15)

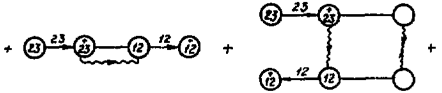


(A.16)

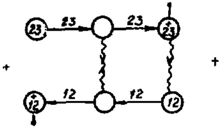
$$\sum\nolimits^{(1)}_{23,12} =$$



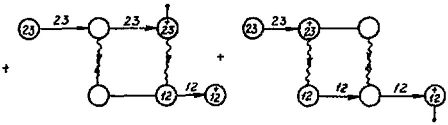
(A.17)



(A.18)

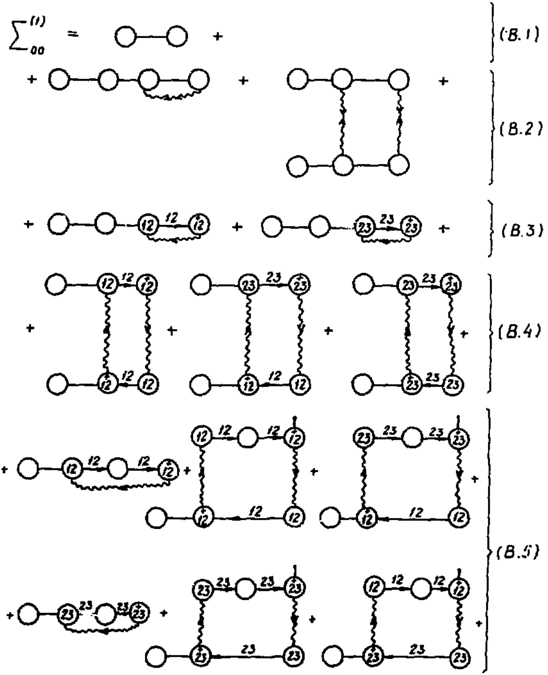


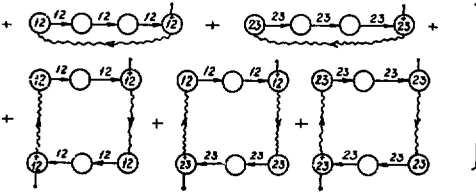
(A.19)



(A.20)

Appendix B. Irreducible polarization part $\Sigma_{0,0}$ up to first order





(B.6)