СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
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ДУБНА



E4 - 7624

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# STANDARD-BASIS OPERATOR METHOD IN THE GREEN-FUNCTION DIAGRAM TECHNIQUE OF MANY BODY SYSTEMS. I

1973

ЛАБОРАТОРИЯ
ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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## 1. Introduction

The importance of crystal field effects in magnetic materials has long been recognized. For certain rare-earth compounds, such as those of NaCl structure with group-V anions, it is possible for the crystal field to be compared to or even dominant over the exchange interaction between the rare-earth ions. Thus, the crystal field is expected to play a very important role in the nature of the macroscopic magnetic properties in such materials [1]. Theory of these substances is given and quoted in [2]. More complicated mechanisms, e.g., orbital effects on exchange[3], biquadratic exchange [4] can be as large as, or even larger than the exchange interaction.

Isotropic biquadratic interactions of the form $K(\vec{S}_x \vec{S}_{x'})^2$, in addition to the usual Heisenberg bilinear ones $J \vec{S}_x \cdot \vec{S}_{x'}$ have been of current theoretical interest [5], since Harris and Owen, Rodbell et al., Bertaut and Rubinshtein [6] poited out that they could have significant effects on the magnetic properties of antiferromagnets, like MnO, $\alpha$ -MnS, EuSe, and ferrimagnets.

The authors of /7/ noted that since two order parameters at least are to be considered for such systems / that is $\langle S^z \rangle = m$ and $\langle S^{z2} \rangle - \frac{1}{3} S(S+1) = Q$ / , ordering in the quadrupolar uniaxial parameter $Q$ might occur as a separate phase transition.

Quadrupolar ordering in magnetic crystals has been investigated theoretically /8–9/ in connection with experimental results on magnetic and crystallographic/multiple, separate / phase transitions in some rare-earth compounds such as /10/ $DyVO_4$. A general purely quadrupolar system may have up to 2l+1 = 5 independent, nonzero order parameters /9/, though this number may often be reduced by a judicious choice of axes, when the system possesses appreciable symmetry. The asymmetry biaxial parameter $P = \langle S^{-2} + S^{+2} \rangle = \langle S^{x2} - S^{y2} \rangle$ does not, in general, vanish and both $P$ and $Q$ may well be necessary /9/ to describe states of quadrupolar order in low-symmetry magnetic crystals such as the rare-earth vanadates, phosphates, and arsenates.

The mathematical methods employing in the theoretical papers cited above and in /11–16/ are those of the double-time temperature dependent Green functions /17/ or Bogoliubov variational principle /18/.

The diagrammatic method of Vaks et al. /19/ was applied to the investigation of Heisenberg /Anti-/ Ferromagnet with single-ion anisotropy in /20–22/ : Solyom considers single-ion anisotropy as a perturbation, Ginzburg has been calculated correlation function $\langle\langle S^-; S^+ \rangle\rangle$ in molecular field approximation taking into account the anisotropy term exactly and matrix form for the operators $S^{\pm}, S^z$.

Keschenko et al. /22/ have also considered this term exactly and analyzed collective excitations and their damping. However they state that the inclusion of the single-ion anisotropy term into zero order Hamiltonian gives rise to the absence of an analog of Wick theorem and in order to calculate blocks, containing the transverse components of spin operators it is required to use the definition of T-ordered product of operators:

$$T\left(S^{\alpha_1}(t_1)\cdots S^{\alpha_n}(t_n)\right)=\sum_{P}\theta(t_1-t_2)\cdots\theta(t_{n-1}-t_n)\,S^{\alpha_1}(t_1)\cdots S^{\alpha_n}(t_n),$$

where summing runs over all possible permutations of the „times" $t_j$ and $\theta(t_i-t_j)$ is Heaviside's function.

Kitaev et al. /23/ have calculated irreducible polarization part $\Sigma$ in zero order and obtained excitation spectrum, when zero order Hamiltonian contains $-\varepsilon\sum_f\left(S_f^{x^2}-S_f^{y^2}\right)$ in addition to that in /22/ : they diagonalize this zero order Hamiltonian and then apply the procedure of /22/ .

In this paper we present the perturbation theory for arbitrary spin Hamiltonians with S= 1 on the basis of general statistical Wick theorem / GSWT / /24/ and its application to spin Hamiltonians for S = 1 /25/ .According to /24-25/ *the statement of the authors of* /22/ on the nonexistence of an analog of Wick theorem is not valid.

*Detailed discussion of GSWT* /24/ for transverse and diagonal operators is given by the example of the Hamiltonians of Anderson and *Hubbard type in* /26-27/ *: this case* may be considered as a general one in such a sense, that the set of the transverse operators consists of the Fermi and spin type operators, that has an influence on the character of realization of GSWT.

This article is organized as follows: In sec.2 and 3 we decompose the Hamiltonian into the unperturbed and interaction parts and represent the last term by the standard-basis operators /15,24-25/ , respectively.In sec.4 the realization of GSWT for transverse and disgonal operators is commented. Finally, in Appendix B the rules for drawing of the graphs, holding for arbitrary interaction, are obtained.

## 2. Decomposition of Hamiltonian

Let us consider a system of ions, on translationally invariant lattice in uniaxial $D$ and biaxial $E$ crystal fields, interacting in pairs according to the following Hamiltonian

$$\bar{H} = \sum_{\varkappa} \left[ -h S^z_{\varkappa} - D S^{z\,2}_{\varkappa} - \tfrac{1}{2} E \left( S^{-\,2}_{\varkappa} + S^{+\,2}_{\varkappa} \right) \right] +$$

$$- \tfrac{1}{2} \sum_{\varkappa \neq \varkappa'} \left[ J_{\varkappa\varkappa'} \vec{S}_{\varkappa} \cdot \vec{S}_{\varkappa'} + K_{\varkappa\varkappa'} \left( \vec{S}_{\varkappa} \cdot \vec{S}_{\varkappa'} \right)^2 \right] , \qquad /1/$$

where $h$ is an external magnetic field, $J_{\varkappa\varkappa'}$ and $K_{\varkappa\varkappa'}$ are usual Heisenberg and biquadratic exchange interactions, respectively. The above Hamiltonian represents and contains the cases of considerable physical interest /2-9,11-16,20-22,23/. Therefore it seems very required to construct a systematic perturbation theory for such Hamiltonians, due to which a part of Hamiltonian /1/ as large as possible could be included into zero order Hamiltonian. This becomes possible because of the formulation of GSWT in /24/. According to /24/ by unperturbed Hamiltonian we can mean an arbitrary part of Hamiltonian, which has the form

$$\bar{H}_0 = \sum_{\varkappa} \bar{H}_{0\varkappa} \, , \qquad /2/$$

where $\bar{H}_{0x}$ is one-ion Hamiltonian and can be a linear self-adjoint combination of nxn matrices, or that of spin operators. Of course $\bar{H}_{0x}$ can contain, frequently, some mean fields corresponding to some order parameters. For example, the mean field $\sum_{x'} J_{xx'} \langle S^z \rangle_0$ in /19/ corresponds to the order parameter $\langle S^z \rangle$.

From the mathematical point of view one may considered the more general single-system operator /15/ than that in /1/. But then, as it was pointed out in /25/, we can always find a unitary transformation $U_x$ and constant $C$ so that $U_x^+ \bar{H}_{0x} U_x + C$ becomes of the type / for S = 1 /

$$H_{0x} = -\Delta S^z_x - \bar{D} S^{z\,2}_x . \qquad /3/$$

Expressing the scalar product $\vec{S}_x \cdot \vec{S}_{x'}$ in terms of components $S^{\pm}, S^z$ we can rewrite the Hamiltonian /1/ in the form

$$\bar{H} = \bar{H}_0 + \bar{V} + C , \qquad /4/$$

where zero-order Hamiltonian $\bar{H}_0$ is given by /2/ with

$$\bar{H}_{0x} = -\bar{h} S^z_x - \bar{D} S^{z\,2}_x - \tfrac{1}{2} \bar{E} \left( S^{-\,2}_x + S^{+\,2}_x \right) , \qquad /5/$$

the interaction term :

$$\bar{V} = -\tfrac{1}{2} \sum_{x \neq x'} \Big\{ \tfrac{3}{2} K_{xx'} \hat{S}^{\bar{z}}_x \hat{S}^{\bar{z}}_{x'} + \left( J_{xx'} - \tfrac{1}{2} K_{xx'} \right) \hat{S}^z_x \hat{S}^z_{x'} +$$

$$+ \tfrac{1}{2} K_{xx'} \hat{S}^{\bar{2}}_x \hat{S}^{2}_{+x'} + J_{xx'} \left[ S^-_x S^z_x \left( S^-_{x'} S^z_{x'} \right)^+ + \left( -S^z_x S^-_x \right) \left( -S^z_{x'} S^-_{x'} \right)^+ \right] +$$

$$+ \left( J_{xx'} - K_{xx'} \right) \left[ S^-_x S^z_x \left( -S^z_{x'} S^-_{x'} \right)^+ + \left( -S^z_x S^-_x \right) \left( S^-_{x'} S^z_{x'} \right)^+ \right] \Big\} \qquad /6/$$

and constant $\bar{C}$ is equal to

$$\bar{C} = \sum_{\varkappa \neq \varkappa'} \{ \tfrac{3}{4} K_{\varkappa\varkappa'} \langle S^2_{\varkappa} \rangle_o \langle S^2_{\varkappa'} \rangle_o + \tfrac{1}{2} ( J_{\varkappa\varkappa'} - \tfrac{1}{2} K_{\varkappa\varkappa'} ) \langle S^z_{\varkappa} \rangle_o \langle S^z_{\varkappa'} \rangle_o +$$

$$+ \tfrac{1}{4} K_{\varkappa\varkappa'} \langle S^2_{-\varkappa} \rangle_o \langle S^2_{+\varkappa'} \rangle_o - \tfrac{1}{4} S(S+1) K_{\varkappa\varkappa'} \} .$$

In the following this constant in /4/ will be dropped. The effective parameters $\bar{h}$ , $\bar{D}$ , $\bar{E}$ depend on the mean fields $(J_o - \frac{1}{2} K_o) m_o$ $\frac{3}{2} K_o Q_o$ , $\frac{1}{4} K_o P_o$ , corresponding to the order parameters $m = \langle S^z \rangle$ , $Q = \langle S^{z2} - \frac{1}{3} S(S+1) \rangle$ , $P = \langle S^2_- + S^2_+ \rangle = \langle S^{x2} - S^{y2} \rangle$ /7/

in the following way :

$$\bar{h} = h + (J_o - \tfrac{1}{2} K_o) m_o , \bar{D} = D + \tfrac{3}{2} K_o Q_o , \bar{E} = E + \tfrac{1}{4} K_o P_o \qquad /8/$$

Here $m_o$ , $Q_o$ , $P_o$ are the averages / denoted by $\langle \cdots \rangle_o$ / /7/ over the unperturbed Hamiltonian /2,5/ and $J_o = \sum_{\varkappa'} J_{\varkappa\varkappa'}$ , $K_o = \sum_{\varkappa'} K_{\varkappa\varkappa'}$ . The operators $\hat{S}^2_{\varkappa}$ , $\hat{S}^z_{\varkappa}$ , $\hat{S}^2_{-\varkappa}$ , $\hat{S}^2_{+\varkappa}$ in /6/ are of the type

$$\hat{A}_{\varkappa} = A_{\varkappa} - \langle A \rangle_o . \qquad /9/$$

Such a procedure has been done for operators, which have the non-zero diagonal part in the representation, where $\bar{H}_{o\varkappa}$ in /5/ is diagonal /from now on all further considerations will be valid for S = 1/

3. Interaction in the standard-basis operators/ in diagonal representation /

The unitary transformation $U_{\varkappa}$ diagonalizing the subsystem Hamiltonian /5/ has the form of the 3x3 matrix

$$U_x = \begin{pmatrix} a & 0 & -b \\ 0 & 1 & 0 \\ b & 0 & a \end{pmatrix}$$

where $\quad a^2 = (\Delta + \bar{h})/2\Delta \;, \quad b^2 = (\Delta - \bar{h})/2\Delta \,, \; \Delta^2 = \bar{h}^2 + \bar{E}^2 \,, \; a^2 - b^2 \equiv c \,.$

Then

$$U_x^+ \bar{H}_{ox} U_x \equiv H_{ox} = -\Delta Q^{13} - \bar{D} Q_x^{\bar{13}} \,, \qquad /10/$$

$$U_x^+ S_x^z U_x = c\, Q_x^{13} - ab(S_x^{13} + S_x^{\overset{+}{13}}) \,, \quad U_x^+ \overset{2}{S_x^z} U_x = Q_x^{\bar{13}} \,,$$

$$U_x^+ \overset{2}{S_x^-} U_x = 2ab\, Q_x^{13} + a^2 S_x^{13} - b^2 S^{\overset{+}{13}} \,,$$

$$U_x^+ S_x^- S_x^z U_x = a\, S_x^{12} - b\, S_x^{\overset{+}{23}} \,, \quad U_x^+ (-S_x^z S_x^-) U_x = b S^{\overset{+}{12}} + a\, S_x^{23} \,, \qquad /11/$$

where the operators $Q^{13}$, $Q^{\bar{13}}$, $S^{13}$, $S^{12}$, $S^{23}$ are the spin one operators $S^z$, $\overset{2}{S^z}$, $\overset{2}{S^-}$, $S^- S^z$, $-S^z S^-$, respectively, in the new representation. The role of the operators $S^{12}$, $S^{23}$, $S^{13}$ as the transition ones between energy levels

$$H_{ox}'^{1} = -(\Delta + \bar{D}) \,, \quad H_{ox}'^{2} = 0 \,, \quad H_{ox}'^{3} = \Delta - \bar{D} \qquad /11'/$$

of $H_{ox}$ in /10/ was stressed in /15, 16, 24–25/ .

The operator /4/, under the unitary transformation $U \equiv \prod_x U_x$, takes the form

$$H = H_o + V \,, \qquad /12/$$

where $H_o$ is the sum as in /2/, when $\bar{H}_{ox} \to H_{ox}$, given by /10/ and the interaction $\bar{V}$ in /4/ according to /11/ is equal to

$$V = -\sum_{x \neq x'} V_{\overset{+}{ij}, \bar{pq}}(x', x)\, I_x^{pq}\, I_{x'}^{\overset{+}{ij}} \;;$$
$$ij, pq = \{ 1, 2, 13, \overset{+}{13}, 12, 1\bar{2}, 23, 2\overset{+}{3} \}$$

$$I^{1} = \hat{Q}^{1\bar{3}} \equiv \boxed{13}\ , \quad I^{2} = \hat{Q}^{13} \equiv \boxed{13}\ , \quad I^{ij} = S^{ij} \equiv \textcircled{ij}\,. \qquad /13/$$

The non-zero pair interactions in /13/ are :

$$V^{+\ -}_{12,\,12}(x',x) = V^{+\ -}_{23,\,23}(x',x) = \tfrac{1}{2} J_{xx'}\ , \quad V^{+\ -}_{1,\,1}(x',x) = \tfrac{3}{4} K_{xx'}\ ,$$

$$V^{+\ -}_{\left(\substack{12,\,\overset{+}{12} \\ 23,\,\overset{+}{23}}\right)}(x',x) = V^{+\ -}_{\left(\substack{\overset{+}{12},\,12 \\ \overset{+}{23},\,23}\right)}(x',x) = (\pm)\tfrac{1}{2}\, a\, b\, (J_{xx'} - K_{xx'})\ ,$$

$$V^{+\ -}_{12,\,23}(x',x) = V^{+\ -}_{23,\,12}(x',x) = \tfrac{1}{2}\, c\, (J_{xx'} - K_{xx'})\ ,$$

$$V^{+\ -}_{2,\,2}(x',x) = \tfrac{1}{2}\left[ c^{2}\left(J_{xx'} - \tfrac{1}{2}K_{xx'}\right) + 2(ab)^{2} K_{xx'} \right],$$

$$V^{+\ -}_{13,\,13}(x',x) = \tfrac{1}{2}\left[ 2(ab)^{2} J_{xx'} + \tfrac{1}{2} c^{2} K_{xx'} \right]\ ,$$

$$V^{+\ -}_{13,\,\overset{+}{13}}(x',x) = V^{+\ -}_{\overset{+}{13},\,13}(x',x) = \tfrac{1}{2}(ab)^{2}\left(J_{xx'} - K_{xx'}\right),$$

$$V^{+\ -}_{2,\,13}(x',x) = V^{+\ -}_{13,\,2}(x',x) = -\,a\,b\,c\left(J_{xx'} - K_{xx'}\right), \qquad /14/$$

Using the graphical convention presented in /27/ , $V$ in /13/ may be rewritten as follows

$$V = \textcircled{12}\sim\sim\textcircled{\overset{+}{12}} + \textcircled{12}\sim\sim\textcircled{\overset{+}{23}} + \textcircled{23}\sim\sim\textcircled{\overset{+}{12}} + \textcircled{23}\sim\sim\textcircled{\overset{+}{23}} +$$

$$+ \textcircled{\overset{+}{12}}\sim\sim\textcircled{\overset{+}{12}} + \textcircled{12}\sim\sim\textcircled{12} + \textcircled{\overset{+}{23}}\sim\sim\textcircled{\overset{+}{23}} + \textcircled{23}\sim\sim\textcircled{23} +$$

$$+ \boxed{13}\sim\sim\boxed{13} + \boxed{13}\sim\sim\textcircled{\overset{+}{13}} + \textcircled{13}\sim\sim\boxed{13} + \textcircled{13}\sim\sim\textcircled{\overset{+}{13}} +$$

$$+ \textcircled{\overset{+}{13}}\sim\sim\textcircled{\overset{+}{13}} + \textcircled{13}\sim\sim\textcircled{13} + \boxed{\overset{+}{13}}\sim\sim\boxed{\overset{+}{13}}\ . \qquad /15/$$

4. Realization of GSWT for transverse and diagonal operators

From the form of the interaction in /15/ it follows that we have to use in the reduction /25/ six /maximal possible number / free Green functions $G^{ij}$ of Bose type, corresponding to six transverse spin operators $S^{ij}$ /24-25,27/ :

$$G^{ij}(\tau-\tau') = \exp[(\tau-\tau')H'^{ij}_{ox}][\, n(ij)\,\theta(\tau-\tau') + (n(ij)+1)\,\theta(\tau'-\tau)] =$$

$$= \beta^{-1}\sum_{m} \exp[-i\lambda_m(\tau-\tau')]\; G^{ij}(i\lambda_m) \,, \qquad /16/$$

where

$$H'^{ij}_{ox} \equiv H'^{j}_{ox} - H'^{i}_{ox} \,, \quad n(ij) \equiv \left(\exp[\beta H'^{ij}_{ox}] - 1\right)^{-1},$$

$$G^{ij}(i\lambda_m) \equiv \left(i\lambda_m + H'^{ij}_{ox}\right)^{-1} \qquad /17/$$

and $i\lambda_m$ is the imaginary Bose frequency. According to the realization of GSWT /24/ in the case S = 1 /25/ and to the definition of the reduction procedure /27/ we obtain the diagrammatic representation of transverse operators $S^{ij}$ given in Appendix B : two cases- particle and hole - are introduced here / as in /27/ / for convenience. Analogously as it was done in /27/ we get all possible representations for diagonal operators /25/

$$Q^{12} \equiv L^{12} \equiv \boxed{12} \,, \quad Q^{23} \equiv L^{23} \equiv \boxed{23} \,, \quad \hat{Q}^{13} \equiv \tfrac{1}{4}\hat{L}^{13} \equiv \boxed{13}$$

replacing $\xrightarrow{ij}$ (ij) by $\boxed{ij}$ on the figures with two incoming and one outgoing directed lines in the particle case of Appendix B. The hole case / $Q^{ij} = -Q^{ji}$ / of representation may be considered in a similar way . From the table of products in App.A and the commutation relations in /25/ according to /13/ we obtain the diagrammatic representations of $\hat{Q}^{\bar{13}}$ /diagonal operator/ in terms of $\boxed{\bar{3}}$ with one incomming and one outgoing line

$$-\left(\begin{smallmatrix}\overset{+}{lk}\\ lk\end{smallmatrix}\right) \!\!\rightarrow\!\! \overset{(\pm)}{\boxed{\bar{3}}} \left(\begin{smallmatrix}\overset{+}{lk}\\ lk\end{smallmatrix}\right) \!\!\rightarrow\!\!- \quad ,$$

where $lk = 12, 32$ .

There it should be noted, that the above described diagrammatic representation of both the transverse and diagonal operators may be employed in the case of more general interaction than that in /13,15/.

The process of the reduction of averages consists of two stages /27/ .At the first one we are using the transverse operators for the reduction.When they are exhausted,i.e.,the only diagonal operators remain under the symbol of average,then we employ the analog of GSWT for diagonal operators / for all definitions and notations we refer the reader to /27/ and here we point out the new aspects for this case/

$$\langle Q_1^a\, Q_2^b \cdots Q_n^x \rangle_0 = \langle \underset{\bullet}{Q_1^a}\, Q_2^b \cdots Q_n^x \rangle_0 + $$

$$+ \langle \underbrace{Q_1^a\, Q_2^b} \cdots Q_n^x \rangle_0 + \cdots + \langle \underbrace{Q_1^a\, Q_2^b \cdots Q_{n-1}^{x-1}}\, Q_n^x \rangle_0 +$$

$$+ \langle \underbrace{Q_1^a\, Q_2^b \cdots Q_n^x} \rangle_0 , \qquad /48/$$

where the operators $Q$'s run over the set

$$Q^{13}_\varkappa , \quad Q^{\bar{13}}_\varkappa ,$$

$$Q_\varkappa^{\binom{12}{13}} = - Q_\varkappa^{\binom{21}{31}} = Q^{13}_\varkappa + (\pm)3\, Q^{\bar{13}}_\varkappa + (\mp)2 \qquad /49/$$

The zero-order result for the averages of operators /19/ takes the form :

$$\langle Q_x^{(\substack{1\bar{3}\\13})} \rangle_0 = Z_0^{-1} D_x^{(\substack{1\bar{3}\\13})} Z_0 = Z_{0x}^{-1} \{ \exp[\beta(\Delta_x + \bar{D}_x)] + (\pm) \exp[-\beta(\Delta_x - \bar{D}_x)] \}$$

/20/

$$\langle G_x^{(\substack{12\\23})} \rangle_0 = Z_0^{-1} (D_x^{(\substack{12\\23})} + (\mp) 2) Z_0 = Z_{0x}^{-1} \cdot 2 (\pm) \{ \exp[\beta((\pm)\Delta_x + \bar{D}_x)] - 1 \}$$

and

$$Z_0 = \prod_x Z_{0x} \quad ;$$

$$Z_{0x} = \mathrm{Tr} \{ \exp[\beta(\Delta_x Q_x^{13} + \bar{D}_x Q_x^{1\bar{3}})] \} =$$

/21/

$$= 1 + \exp[\beta(\Delta_x + \bar{D}_x)] + \exp[-\beta(\Delta_x - \bar{D}_x)] .$$

The differential operators $D_x^{ij}$ /diagonal reductors [27] / corresponding to diagonal operators /19/ are to be chosen in the following way :

$$D_x^{ij} \equiv D(Q_x^{ij}) \quad ;$$

$$D_x^{13} \equiv \partial / \partial(\beta \Delta_x) , \qquad D_x^{1\bar{3}} \equiv \partial / \partial(\beta \bar{D}_x),$$

$$D_x^{(\substack{12\\23})} \equiv D_x^{13} + (\pm) 3 D_x^{1\bar{3}} . \qquad /22/$$

The lattice site indices introduced in /19-22/ are to be omitted [27] in /18/ after calculations.

In contrast to [27] in the interaction /13,15/ there are the operators

$$\hat{Q}_{\varkappa}^{\binom{13}{13}} = Q_{\varkappa}^{\binom{13}{13}} - \langle Q_{13}^{13} \rangle_0 \qquad /23/$$

averages of which vanish ,but it is not so,when differential operators /22/ act on them,i.e., the averages $\langle Q^{\binom{13}{13}} \rangle_0$ in /23/ are to be treated as constants,and this is why the differential operators $D_{\varkappa}^{\binom{13}{13}}$ correspond also to diagonal ones /23/.

5. Conclusions

This work has first of all the methodical meaning.Though the discussion is restricted to spin Hamiltonian constructed from spin one operators, the procedure described in the sec$.2-4 may be easily reformulated for the case of an arbitrary spin.

When the uniaxial anisotropy appears in the Hamiltonian as the additional term to that in /19/, the operators $S^{\pm}$ , $S^{z}$ , $\overset{2}{S^{z}}$ do not form the closed algebra with respect to the commutation relations. The way of getting the corresponding closed algebra is demonstrated in /24-25/ and it is,in general, a Lie algebra of Lie group SU/n/ /28/ , where n = 2S + 1.Of course the operators $S^{\pm}$ , $S^{z}$ can be taken as the generators for Lie algebra of Lie group SU/2/ /28/ .

In contrast to /22-23/ presented here diagrammatic method allows us to construct the diagrams for arbitrary interactions and to arbitrary interaction order according to the rules listed in Appendix B.

In conclusion we would like to point out that in the second part of this paper / reffered to as II / we shall use the presented approach to study the Heiseberg Ferromagnet with uniaxial crystal field.In the limit of the anisotropy constant $D \to 0$ ,the correlation functions- transverse and longitudinal-obtained in II to or -

der $(1/z)^2$ ,where $z$ is the number of spins interacting with any given spin,take the form of those in /19/ .

Acknowledgements

The author would like to express his sincere thanks to N.M. Plakida for critical reading of the manuscript and valuable comments.He is also indebted to A.I.Larkin and A.Pawlikowski for useful discussions.

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Received by Publishing Department
on December 21, 1973.

## Appendix A. Products of transverse and diagonal operators

$$S^{12} Q^{12} = -Q^{12} S^{12} = Q^{23} S^{12} = \overset{+}{S}{}^{23} S^{13} = 2 S^{12} Q^{13} = 2 S^{12} Q^{\overline{13}} = 2S^{12},$$

$$S^{23} Q^{23} = -Q^{23} S^{23} = - S^{23} Q^{12} = S^{13} \overset{+}{S}{}^{12} = -2 Q^{13} S^{23} = 2 Q^{\overline{13}} S^{23} = 2 S^{23},$$

$$2 S^{13} Q^{13} = -2 Q^{13} S^{13} = S^{13} Q^{12} = - Q^{23} S^{13} = 2 S^{23} S^{12} =$$

$$= 2 S^{13} Q^{\overline{13}} = 2 Q^{\overline{13}} S^{13} = 2 S^{13},$$

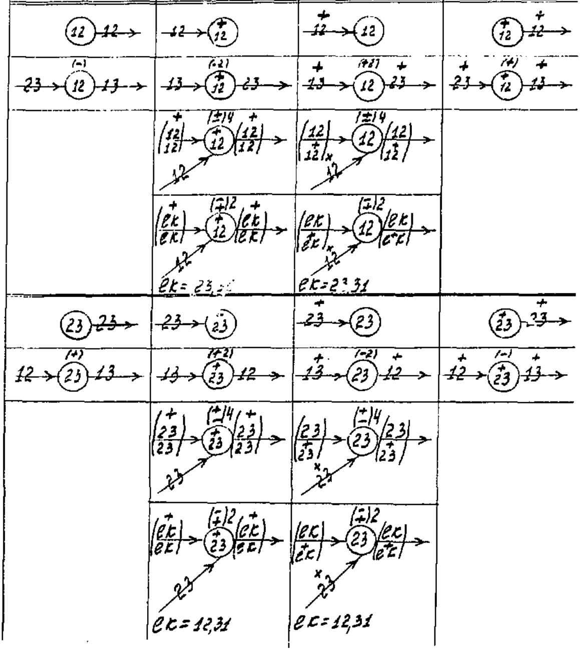
$$2 \overset{+}{S}{}^{12} S^{12} = \overset{+}{S}{}^{13} S^{13} = 4 P^{1} , \quad S^{12} \overset{+}{S}{}^{12} = \overset{+}{S}{}^{23} S^{23} = 2 P^{2} ,$$

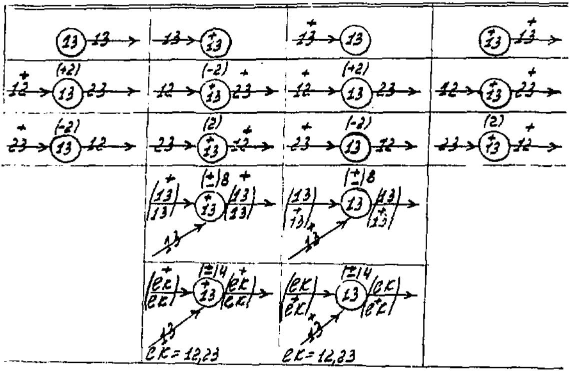
$$2 S^{23} \overset{+}{S}{}^{23} = S^{13} \overset{+}{S}{}^{13} = 4 P^{3} ,$$

where $P^{i}$ is the projection operator on the eigenstate $|i\rangle_x$ of $H_{ox}$ .

Appendix B. Diagrammatic representation of transverse operators

| Particle case | | Hole case | |
|---|---|---|---|
| $S^{ij}$ as the creation operator of „particle ij" | $\overset{+}{S}{}^{ij}$ as the annihilation operator of „particle ij" | $S^{ij}$ as the annihilation operator of „hole ij" | $\overset{+}{S}{}^{ij}$ as the creation operator of „hole ij" |




ek = 12,23
ek = 12,23