СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА

E4 - 7487

B.Westwański

PERTURBATION THEORY FOR HAMILTONIANS OF ANDERSON AND HUBBARD TYPE. II. ANALOG OF GENERAL STATISTICAL WICK THEOREM FOR DIAGONAL OPERATORS, CORRELATION FUNCTIONS

1973

ЛАБОРАТОРИЯ
ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

E4 - 7487

B.Westwański *

PERTURBATION THEORY FOR HAMILTONIANS OF ANDERSON AND HUBBARD TYPE. II. ANALOG OF GENERAL STATISTICAL WICK THEOREM FOR DIAGONAL OPERATORS, CORRELATION FUNCTIONS

* On leave of absence from Instytut Fizyki, Uniwersytet Śląski, Katowice, Poland

1. Introduction

In the preceding paper [1] (referred to as I hereafter) we described the realization of the general statistical Wick theorem (GSWT) for transverse operators [2] in the case when unperturbed Hamiltonian contains the Coulomb interaction of the Anderson [3] and Hubbard [4] models. According to this realization the diagrammatic representation of transverse and diagonal operators was obtained.

In this paper in § 2 we formulate an analog of GSWT for diagonal operators. In §3 representation of the most general interaction is presented. In § 4 the Anderson interaction and Hubbard ''interaction'' in diagonal representation are discussed. Finally, in § 5 the diagrams leading to the Scalapino result [5] are found; the first order of the irreducible polarization part for the one-particle electron Green function is calculated within the Hubbard model.

2. GSWT for Diagonal Operators

After applying multiply (I.25) the average on the left-hand side of (I.25) becomes a sum of the products of FGF's and averages of products of diagonal operators (I.21) only

$$\langle Q_1^a(r_1)\, Q_2^b(r_2) \ldots Q_{n-1}^{x-1}(r_{n-1})\, Q_n^x(r_n)\rangle_0 \, . \qquad (1)$$

The operators Q^y_j are of course "time" independent. However, "times" t_j in (1) should be left to distinguish the operators Q's, when all lattice site indices are equal. Such a situation occurs in the Anderson model due to (I.9) and (I.11). In the following (for simplicity) we omit "times" in (1). If average (I.25) appears in linked cluster expansion of any quantity then it is simply so-called connected average corresponding to connected diagrams /6/. Such diagrams may be built up from the following mutually connected parts: 1) interaction lines, 2) transverse FGF's (I.23) (corresponding to transverse operators (I.19a - 20a)), 3) diagonal FGF's (corresponding to diagonal operators (I.21)), which are analogous to "vertex blocks" in /7/ and "semi-invariants" employed in the drone-fermion perturbation method" /8/. The first point is realized automatically, second one by applying (I.25) (from now on we call it GSWT for transverse operators) and the third- by applying GSWT for diagonal operators (because it has a similar form to (I.25)). The latter type parts are produced if commutators (or anticommutators) of transverse operators are not c -numbers.

To formulate GSWT for diagonal operators (I.21) we have to know their averages. Denoting by Z_0 the partition function (I.1-2) with $V=0$ we can write

$$Z_0 = \prod_\kappa Z_{0\kappa} ;$$

$$Z_{0\kappa} = \mathrm{Tr}\exp[-\beta(E_{\kappa+} Q^{13}_\kappa + E_{\kappa-} Q^{12}_\kappa + U_\kappa Q^{13}_\kappa Q^{12}_\kappa)], \qquad (2)$$

where $E_{\kappa\pm}$, U_κ are the same parameters as in (I.4-5) and indices are to be omitted after calculations. From (I.15-17), (I.21) and (2) it follows that:

$$Z_{0\kappa} = \exp[-\beta(E_{\kappa+} + E_{\kappa-} + U_\kappa)] + \exp[-\beta E_{\kappa-}] + \exp[-\beta E_{\kappa+}] + 1 , \qquad (3)$$

$$<Q^{\binom{12}{13}}_\kappa>_0 = Z_0^{-1} D^{\binom{12}{13}}_\kappa Z_0 = Z^{-1}_{0\kappa}\{\exp[-\beta(E_{\kappa+} + E_{\kappa-} + U_\kappa)] + \exp[-\beta E_{\kappa(\mp)}]\},$$

$$<Q^{\binom{34}{24}}_\kappa>_0 = Z_0^{-1}(1 + D^{\binom{34}{24}}_\kappa) Z_0 = Z^{-1}_{0\kappa}\{\exp[-\beta E_{\kappa(\pm)}] + 1\}, \qquad (4)$$

$$<Q^{23}_\kappa>_0 = Z_0^{-1} D^{23}_\kappa Z_0 = Z^{-1}_{0\kappa}\{\exp[-\beta E_{\kappa+}] - \exp[-\beta E_{\kappa-}]\},$$

$$<Q^{14}_\kappa>_0 = Z_0^{-1}(1 + D^{14}_\kappa) Z_0 = Z^{-1}_{0\kappa}\{1 - \exp[-\beta(E_{\kappa+} + E_{\kappa-} + U_\kappa)]\},$$

where

$$D^{ij}_\kappa \equiv D(Q^{ij}_\kappa) ;$$

$$D^{\binom{12}{13}}_\kappa \equiv \partial/\partial(-\beta E_{\kappa(\mp)}) , \quad D^{\binom{34}{24}}_\kappa \equiv - D^{\binom{12}{13}}_\kappa \qquad (5)$$

$$D^{\binom{23}{14}}_\kappa \equiv (\pm)\ D^{13}_\kappa - D^{12}_\kappa .$$

Let us consider first simple examples of the reduction of averages (1):

1.

$$<Q^{13}_k Q^{12}_\ell>_0 = Z_0^{-1} D^{13}_k [Z_0 <Q^{12}_\ell>_0] = <Q^{13}_k>_0 <Q^{12}_\ell>_0 + D^{13}_k <Q^{12}_\ell>_0 , \qquad (6)$$

where the second term is the cumulant average,

$$<Q^{13}_k Q^{12}_\ell>_{0c} = \delta_{\ell,k}\ D^{13}_\ell <Q^{12}_\ell>_0 = \delta_{k,\ell}\ D^{12}_k <Q^{13}_k>_0 . \qquad (6a)$$

2. $$\langle Q_k^{13} Q_\ell^{34}\rangle_0 = Z_0^{-1} D_k^{13} [Z_0 \langle Q_\ell^{34}\rangle_0] =$$

$$= \langle Q_k^{13}\rangle_0 \langle Q_\ell^{34}\rangle_0 + D_k^{13} \langle Q_\ell^{34}\rangle_0 \,, \tag{7}$$

where the cumulant average

$$\langle Q_k^{13} Q_\ell^{34}\rangle_{0c} = D_k^{13} \langle Q_\ell^{34}\rangle_0 = \delta_{k,\ell}\, D_\ell^{13} \langle Q_\ell^{34}\rangle_0 =$$

$$= D_\ell^{34} \langle Q_k^{13}\rangle_0 = \delta_{\ell,k}\; D_k^{34} \langle Q_k^{13}\rangle_0 \,, \tag{7a}$$

3. $$\langle Q_k^{12} Q_\ell^{13} Q_p^{12}\rangle_0 = \langle Q_k^{12}\rangle_0 \langle Q_\ell^{13}\rangle_0 \langle Q_p^{12}\rangle_0 +$$

$$(D_k^{12} \langle Q_\ell^{13}\rangle_0) \langle Q_p^{12}\rangle_0 + \langle Q_\ell^{13}\rangle_0\; D_k^{12} \langle Q_p^{12}\rangle_0 +$$

$$+ \langle Q_k^{12}\rangle_0\, D_\ell^{13} \langle Q_p^{12}\rangle_0 + D_k^{12} D_\ell^{13} \langle Q_p^{12}\rangle_0 \,, \tag{8}$$

where the last term is the cumulant average

$$\langle Q_k^{12} Q_\ell^{13} Q_p^{12}\rangle_{0c} = D_k^{12} D_\ell^{13} \langle Q_p^{12}\rangle_0 =$$

$$= D_p^{12} D_k^{12} \langle Q_\ell^{13}\rangle_0 = D_\ell^{13} D_p^{12} \langle Q_k^{12}\rangle_0 \,. \tag{8a}$$

Each of the possibilities (8a) represents then the product of two Kronecker symbols and second-order differential operator acting on the average of one operator. For example the first possibility in (8a) gives

$$\delta_{k,\ell}\;\; \delta_{\ell,p}\;\; D_p^{12} D_p^{13}\; \langle Q_p^{12}\rangle_0 \,. \tag{8b}$$

From these examples (6a, 7a, 8a-b) we can conclude, that the differential operators (5) play a similar role in the reduction as FGF's (I.23), therefore due to this analogy we will call the above differential operators diagonal FGF's. Furthermore we can generalize the results (6-8b) to an arbitrary type of the average (1) in the form (similar to (I.25)) of GSWT (for diagonal operators)

$$\langle Q_1^a Q_2^b \ldots Q_{n-1}^{x-1} Q_n^x\rangle_0 = \langle \underset{|}{Q}{}_1^a Q_2^b \ldots Q_n^x\rangle_0 +$$

$$+ \langle \underset{|}{Q}{}_1^a \underset{\bullet}{Q}{}_2^b Q_3^c \ldots Q_n^x\rangle_0 + \langle \underset{|}{Q}{}_1^a Q_2^b \underset{\bullet}{Q}{}_3^c \ldots Q_n^x\rangle_0 + \ldots + \tag{9}$$

$$+ \langle \underset{|}{Q}{}_1^a Q_2^b \ldots \underset{\bullet}{Q}{}_{n-1}^{x-1} Q_n^x\rangle_0 + \langle \underset{|}{Q}{}_1^a Q_2^b \ldots Q_{n-1}^{x-1} \underset{|}{Q}{}_n^x\rangle_0 \,,$$

where

$$\underset{|}{Q}{}_1^a = \langle Q_1^a\rangle_0 \,,$$

$$\underset{|}{Q}{}_1^a Q_2^b \ldots Q_{n-1}^{x-1} \underset{|}{Q}{}_n^x = \underset{|}{Q}{}_1^a \underset{|}{Q}{}_n^x Q_2^b \ldots Q_{n-1}^{x-1}$$

with

$$\underset{|}{Q}{}_1^a \underset{|}{Q}{}_n^x = \delta_{1,n} D_n^a \langle Q_n^x\rangle_0 = \delta_{n,1} D_1^x \langle Q_1^a\rangle_0 \tag{9a}$$

(and is equal to (6a) for $Q_1^a = Q_k^{13}, Q_n^x = Q_\ell^{12}$). What the dots in (9) mean we explain by the example of the following term for $n = 5$:

$$\langle \underset{|}{Q}{}_1^a Q_2^b \underset{\bullet}{Q}{}_3^c Q_4^d Q_5^e\rangle_0 = \langle \underset{|}{Q}{}_1^a Q_2^b \underset{|}{Q}{}_3^c Q_4^d Q_5^e\rangle_0 +$$

$$\langle \underset{|}{Q}{}_1^a Q_2^b \underset{|}{Q}{}_3^c \underset{\bullet}{Q}{}_4^d Q_5^e\rangle_0 + \langle \underset{|}{Q}{}_1^a Q_2^b \underset{|}{Q}{}_3^c Q_4^d \underset{|}{Q}{}_5^e\rangle_0 \,,$$

where

$$\underbrace{Q_1^a Q_2^b} Q_3^c Q_4^d Q_5^e = \underbrace{Q_1^a Q_3^c} Q_2^b Q_4^d Q_5^e;$$

$\underbrace{Q_1^a Q_3^c}$ is to be calculated in the manner (9a),

$$\underbrace{Q_1^a Q_2^b Q_3^c Q_4^d Q_5^e} = \underbrace{Q_1^a Q_3^c Q_5^e} Q_2^b Q_4^d$$

and the c -function

$$\underbrace{Q_1^a \; Q_3^c \; Q_5^e}$$

is of the type of (8a) and is equal to it for $Q_1^a = Q_k^{12}$,

$$Q_3^c = Q_\ell^{13}, \quad Q_5^e = Q_p^{12}.$$

In general we define the contraction of k -diagonal operators in the manner

$$\underbrace{Q_1^a Q_2^b \ldots Q_k^x} = D_1^a D_2^b \ldots D_{k-1}^{x-1} \langle Q_k^x \rangle_0 =$$
$$= \delta_{1,2}\, \delta_{2,3} \ldots \delta_{k-1,k}\; D_k^a D_k^b \ldots D_k^{x-1} \langle Q_k^x \rangle_0 \quad . \tag{10}$$

Thus it is a (k-1)-fold derivative of average $\langle Q_k^x \rangle_0$. Of course (10) is invariant under the arbitrary permutation of the indices

$$\binom{a}{1}, \binom{b}{2}, \ldots, \binom{x-1}{k-1}, \binom{x}{k}.$$

Summarizing all the process of the reduction consisting of two stages (I.25) and (9) we can say that every "time"-

ordered average on the left-hand side of (I.25) at the end becomes a sum of the products of FGF's (I.23), (5) and averages (4); or, in other words, the sum of the products of transverse FGF's (I.23) and the derivatives (5) of averages (4). This sum may be splitted into two parts, one of which corresponds to unconnected diagrams and the second to connected ones. When the average on the left-hand side of (I.25) appears in the linked cluster expansion of some quantity it is equal to the second part.

3. Representation of the "Interaction"

There are a good deal of the interactions in (I.33) as well as FGF's (I.23), (5) on the one hand, and the simple product properties (in Appendix I.A) for the projection type operators in Θ_κ, on the other hand. In this situation it seems to be better to represent all interactions (transverse and diagonal) (I.33) with the help of one wavy line ~~~~ in the manner

$$\sum_{\kappa \neq \kappa'} T_{\kappa\kappa'}^{ij_\kappa, \ell k_{\kappa'}} J_\kappa^{ij_\kappa} J_{\kappa'}^{\ell k_{\kappa'}} \equiv (ij) \sim\sim\sim (\ell k) \;, \tag{11}$$

$$\sum_{\kappa \neq \kappa'} T_{\kappa,\kappa'}^{ij_\kappa, \ell k_{\kappa'}} Q_\kappa^{ij_\kappa} Q_{\kappa'}^{\ell k_{\kappa'}} \equiv \boxed{ij} \sim\sim\sim \boxed{\ell k} \;, \tag{12}$$

where the circle ○ refers to the transverse operators (I.19-20) and the square □ - to diagonal ones (I.21); every transverse FGF's (I.23) by means of the labeled directed line as in (I.27), all diagonal FGF's (5) - by undirected line

$$\tau \;\text{———}\; \tau', \tag{13}$$

where the diagonal operators, standing at its ends with "time" τ and τ' correspondingly determine the kind FGF (5) due to (9-10).

4. Representation of Anderson Interaction and Hubbard "Interaction"

As an example of the application of above formalism we consider the interaction (I.11) in Anderson Hamiltonian (I.2), (I.8-11) and the "interaction" (I.12) in Hubbard one. From (I.19) we obtain

$$d_+ = J^{12} + J^{34}$$
$$d_- = J^{13} + J^{24}. \qquad (14)$$

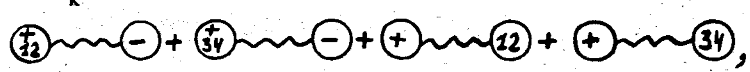
Taking into account (11), (14) for d-electron operators in the Anderson model and representing in addition k-electron operators in the following way

$$c_{k+}\ (c^+_{k+}) \quad \text{by} \quad \ominus \ (\oplus),$$
$$c_{k-}\ (c_{k-}) \quad \text{by} \quad \boxminus \ (\boxplus) \qquad (15)$$

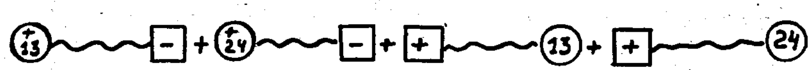
we can write the transverse interaction (I.11) as follows

$$V = V_+ + V_- ;$$

$$V_+ = \sum_k \{ V^*_{kd} (\overset{+}{J}{}^{12}_d + \overset{+}{J}{}^{34}_d) c_{k+} + V_{kd} c^+_{k+} (J^{12}_d + J^{34}_d) \} =$$

≡  (16)

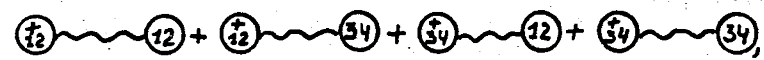
$$V_- = \sum_k \{ V^*_{kd} (\overset{+}{J}{}^{13}_d + \overset{+}{J}{}^{24}_d) c_{k-} + V_{kd} c^+_{k-} (J^{13}_d + J^{24}_d) \} =$$

≡ 

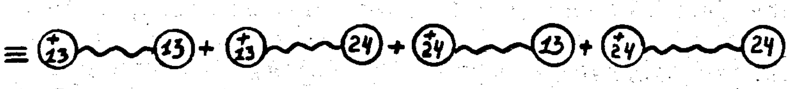
The "interaction" (I.12) takes similar form

$$V_H = V_{H+} + V_{H-} ;$$

$$V_{H+} = \sum_{\kappa \neq \kappa'} T_{\kappa,\kappa'} (\overset{+}{J}{}^{12}_{\kappa} + \overset{+}{J}{}^{34}_{\kappa}) (J^{12}_{\kappa'} + J^{34}_{\kappa'}) =$$

≡ 

$$V_{H-} = \sum_{\kappa \neq \kappa'} T_{\kappa,\kappa'} (\overset{+}{J}{}^{13}_{\kappa} + \overset{+}{J}{}^{24}_{\kappa}) (J^{13}_{\kappa'} + J^{24}_{\kappa'}) = \qquad (17)$$

≡ 

5. Applications

The application of presented diagram method to calculation of the free energy F in the Anderson model is given in Appendix A. For example, first four diagrams in second order in V_{kd} give the Scalapino result (Eq. (7) in ref. /5/). The dominant fourth-order free energy contribution given by Scalapino (Eq. (12) in ref. /5/) comes from diagrams indicated as C(ij, ℓk) and D(ij,...) in App. A. Infinite diagram summation in the Anderson model is continued.

In this section first order result will be derived for the transverse Green function in the Hubbard model. The calculations are conveniently carried out by employing a matrix representation and we therefore define a 2x2 matrix Green function $G_{\substack{+\\(-)}}(k, i\lambda_m)$ by

$$G_{\substack{+\\(-)}}(k, i\lambda_m) = \begin{pmatrix} G_{\substack{12,12\\(13,13)}}(k, i\lambda_m) & G_{\substack{12,34\\(13,24)}}(k, i\lambda_m) \\ G_{\substack{34,12\\(24,13)}}(k, i\lambda_m) & G_{\substack{34,34\\(24,24)}}(k, i\lambda_m) \end{pmatrix} \qquad (18)$$

where the elements of the matrix are causal Green functions of the transverse Fermi type operators (I.19a) and are defined by

$$G_{ij,fp}(k,i\lambda_m) = \langle\langle \overset{+}{J}{}^{ij}_{k} ; J^{fp}_{k} \rangle\rangle . \qquad (19)$$

with

$$\langle\langle \overset{+}{J}{}^{ij}_{k} ; J^{fp}_{k} \rangle\rangle = \frac{1}{2} \sum_{\kappa} \int_{-\beta}^{\beta} d\tau \, \exp[-ik(R_{\kappa} - R_{\kappa'}) + i\lambda_m \tau] \, * $$

$$ * \langle T J^{fp}_{\kappa'}(0) \, \overset{+}{J}{}^{ij}_{\kappa}(\tau) \rangle .$$

The corresponding matrix for the transverse interaction is

$$\underline{V}(k) = V(k) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} , \qquad (20)$$

where

$$V(k) = -t(k) = -(\epsilon_k - \epsilon) , \qquad (21)$$

$t(k)$ is the Fourier transform of $T_{\kappa,\kappa'}$ in (17) and ϵ_{κ} describes the unperturbed band structure /4/.

Denoting by Σ /7, 9–12/ the irreducible polarization part of G we can write the graphical equation

$$G_{+(-)}(k,i\lambda_m) = \Sigma_{+(-)} + \Sigma_{+(-)} V(k) \, G_{+(-)}(k,i\lambda_m) ; \qquad (22)$$

where Σ_{+} is the 2x2 matrix.

From (14), (18-19) we have

$$\langle\langle d^{+}_{k+(-)} ; d_{k+(-)} \rangle\rangle \equiv \bar{G}_{+(-)}(k,i\lambda_m) =$$

$$= G_{\substack{12,12\\(13,13)}}(k,i\lambda_m) + G_{\substack{12,34\\(13,24)}}(k,i\lambda_m) +$$

$$+ G_{\substack{34,12\\(24,13)}}(k,i\lambda_m) + G_{\substack{34,34\\(24,24)}}(k,i\lambda_m) . \qquad (23)$$

According to (20), (22) the solution for Green function (23) takes the form

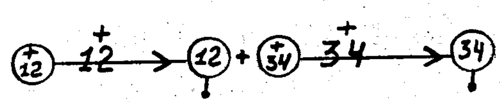
$$\bar{G}_{+(-)} = (\bar{\Sigma}^{-1}_{+(-)} - V(k))^{-1} , \qquad (24)$$

where

$$\bar{\Sigma}_{+(-)} = \Sigma^{12,12\,(13,13)} + \Sigma^{12,34\,(13,24)} + \Sigma^{34,12\,(24,13)} + \Sigma^{34,34\,(24,24)} .$$

The components of Σ_{+} are given in Appendix B up to first order (i.e., they involve one internal momentum summation only). The diagrams for the components of Σ_{-} ($\Sigma^{13,13}, \Sigma^{13,24}, \Sigma^{24,13}, \Sigma^{24,24}$) can be obtained according to Appendix I.B from those in App. B by substitution 12 → 13, 34 → 24.

The zeroth-order result for $\bar{\Sigma}_{+}$ due to App. B takes the form /13/, i.e.,

$$\bar{\Sigma}^{o}_{+} =$$  $$=$$

$$= \langle Q^{12}_{\kappa} \rangle_0 (i\lambda_m + \overset{+}{H}{}'^{12}_{0\kappa})^{-1} + \langle Q^{34}_{\kappa} \rangle_0 (i\lambda_m + \overset{+}{H}{}'^{34}_{0\kappa})^{-1} . \qquad (25)$$

When $\bar{\Sigma}_{+}$ in (24) is equal to $\bar{\Sigma}^{o}_{+}$, then we obtain, as in /13/, Hubbard I result for G_{+} /4/.

Effective (transverse) interaction $\underline{V}_{\pm}(k,i\lambda_m)$ is defined similarly as in /7,9–12/ i.e.,

$$\underline{\tilde{\tilde{V}}}_{+(-)}(k,i\lambda) = \underline{V}(k) + \underline{V}(k) \, G^{o}_{+(-)} \, \underline{V}(k) , \qquad (26)$$

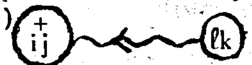
where

$$\overset{\approx}{\underline{V}}_{\substack{+\\(-)}} = \begin{pmatrix} \overset{\approx}{V}{}^{12,12}_{(13,13)} & , & \overset{\approx}{V}{}^{12,34}_{(13,24)} \\ \overset{\approx}{V}{}^{34,12}_{(24,13)} & , & \overset{\approx}{V}{}^{34,34}_{(24,24)} \end{pmatrix}$$

It is easy to see that all components of $\overset{\approx}{V}_{\substack{+\\(-)}}$ are equal /13/, i.e.,

$$\underline{V}_{\substack{+\\(-)}} = V(k)\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}(1-\overline{\Sigma}^{\circ}_{\substack{+\\(-)}}V(k))^{-1} . \qquad (27)$$

These components are represented in the App. B and App. C with the aid of the directed wavy line in the manner $(i<j,\ \ell<k)$



The renormalized spectrum $\epsilon_{\pm\binom{1}{2}}$ $(\epsilon_{\pm 1}-\epsilon_{\pm 2}=\Delta_{\pm})$ is obtained from the condition $1+t(k)\overline{\Sigma}^{\circ}_{\pm}=0$ and was given in /13/.

In the approximation given in App. B for Σ_+ we get

$$\overline{\Sigma}^{(1)}_{+} = \sum_{i=1}^{5} A_i , \qquad (28)$$

where A_1 is the sum of the diagrams (B.1)

$$A_1 = (i\lambda_m + \overset{+}{H}{}'^{12}_{0\kappa})^{-1} N^{-1} \sum_q \{\langle Q^{12}_{\kappa}\rangle_0 - n(\overset{+}{13}) - n(\overset{+}{24}) +$$

$$- t(q)\,[\,Z_{-1}\, n(\epsilon_{-1}(q)) + Z_{-2}\, n(\epsilon_{-2}(q))\,] + \qquad (29)$$

$$- t(q)\,\beta U[n(\epsilon_{+1}(q)) - n(\epsilon_{+2}(q))]\Delta_+^{-1}(q)\, D^{12}_{\kappa} \langle Q^{12}_{\kappa}\rangle_0 \} .$$

The sum of diagrams (B.8) gives

$$A_2 = (i\lambda_m + \overset{+}{H}{}'^{34}_{0\kappa})^{-1} N^{-1} \sum_q \{\langle Q^{34}_{\kappa}\rangle_0 + n(\overset{+}{13}) + n(\overset{+}{24}) +$$

$$+ t(q)\,[\,Z_{-1}\, n(\epsilon_{-1}(q)) + Z_{-2}\, n(\epsilon_{-2}(q))\,] +$$

$$- t(q)\,\beta U[\,n(\epsilon_{+1}(q)) - n(\epsilon_{+2}(q))]\,\Delta_+^{-1}(q)\, D^{12}_{\kappa} \langle Q^{34}_{\kappa}\rangle_0 \} .$$

The coefficients $Z_{-\binom{1}{2}}$ are equal to:

$$Z_{-\binom{1}{2}} = (\pm)\,\{\beta U \Delta_-^{-1}(q)\, D^{13}_{\kappa}\langle Q^{12}_{\kappa}\rangle_0 +$$

$$-\langle Q^{13}_{\kappa}\rangle_0 (\epsilon_{-\binom{1}{2}}(q) - \overset{+}{H}{}'^{24}_{0\kappa})(\epsilon_{-\binom{1}{2}}(q) - \overset{+}{H}{}'^{13}_{0\kappa})^{-1} \Delta_-^{-1}(q) +$$

$$-\langle Q^{24}_{\kappa}\rangle_0 (\epsilon_{-\binom{1}{2}}(q) - \overset{+}{H}{}'^{13}_{0\kappa})(\epsilon_{-\binom{1}{2}}(q) - \overset{+}{H}{}'^{24}_{0\kappa})^{-1} \Delta_-^{-1}(q)\},$$

$n(\overset{+}{ij})$ is given in (I.23) and $n(\epsilon_{\pm\binom{1}{2}}) = (\exp[\beta\epsilon_{\pm\binom{1}{2}}]+1)^{-1}$.

Collecting the diagrams (B.2), (B.4), (B.6) and (B.9) we get

$$A_3 = -D^{12}_{\kappa}\langle Q^{12}_{\kappa}\rangle_0 * \frac{1}{N}\sum_q t(q)\, U^2 (i\lambda_m + \overset{+}{H}{}'^{12}_{0\kappa})^{-1}(i\lambda_m + \overset{+}{H}{}'^{34}_{0\kappa})^{-1} *$$

$$* (i\lambda_m + \epsilon_{+1}(q))^{-1}(i\lambda_m + \epsilon_{+2}(q))^{-1} .$$

The diagrams (B.3), (B.5), (B.7) and (B.10) give two types of contributions A_4 and A_5 ;

$$A_4 = [\,(i\lambda_m + \overset{+}{H}{}'^{12}_{0\kappa}) \;\; -(i\lambda_m + \overset{+}{H}{}'^{34}_{0\kappa})^{-1}\,]^2 *$$

$$(30)$$

$$* N^{-1} \sum_q t(q)\,\{Y_{-1}\, n(\epsilon_{-1}(q)) + Y_{-2}\, n(\epsilon_{-2}(q))\,\},$$

$$A_5 = -[\,(i\lambda_m + \overset{+}{H}{}'^{12}_{0\kappa})^{-1} - (i\lambda_m + \overset{+}{H}{}'^{34}_{0\kappa})^{-1}\,]^2 *$$

$$* N^{-1}\sum_q t(q)\{X_{-1}[W^{23}_{-1}(i\lambda_m+\epsilon_{-1}(q)-\overset{+}{H}{}'^{23}_{0\kappa})^{-1}+ \qquad (31)$$

$$+W^{14}_{-1}(i\lambda_m+\overset{+}{H}{}'^{14}_{0\kappa}-\epsilon_{-1}(q))^{-1}]+$$

$$+X_{-2}[W^{23}_{-2}(i\lambda_m+\epsilon_{-2}(q)-\overset{+}{H}{}'^{23}_{0\kappa})^{-1}+$$

$$+W^{14}_{-2}(i\lambda_m+\overset{+}{H}{}'^{14}_{0\kappa}-\epsilon_{-2}(q))^{-1}]\},$$

where

$$W^{23}_{-(\frac{1}{2})}=\langle \overset{+}{J}{}^{23}_{\kappa}J^{23}_{\kappa}\rangle_0+\langle Q^{23}_{\kappa}\rangle_0\, n(\epsilon_{-(\frac{1}{2})}(q)).$$

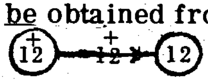
$$W^{14}_{-(\frac{1}{2})}=\langle \overset{+}{J}{}^{14}_{\kappa}J^{14}_{\kappa}\rangle_0+\langle Q^{14}_{\kappa}\rangle_0\, n(\epsilon_{-(\frac{1}{2})}(q))$$

and averages of product of operators $\overset{+}{J}{}^{ij}_{\kappa}$ and J^{ij}_{κ} can be calculated according to App. IA and (I.15-17). The factors Y and X in (30) and (31) have the form

$$Y_{-(\frac{1}{2})}=(\mp)\langle Q^{13}_{\kappa}\rangle_0[\epsilon_{-(\frac{1}{2})}(q)-\overset{+}{H}{}'^{24}_{0\kappa}]\Delta^{-1}_{-}(q)$$

$$+(\pm)\langle Q^{24}_{\kappa}\rangle_0[\epsilon_{-(\frac{1}{2})}(q)-\overset{+}{H}{}'^{13}_{0\kappa}]\Delta^{-1}_{-}(q)$$

$$X_{-(\frac{1}{2})}=(\mp)[\epsilon_{-(\frac{1}{2})}(q)-\overset{+}{H}{}'^{13}_{0\kappa}][\epsilon_{-(\frac{1}{2})}(q)-\overset{+}{H}{}'^{24}_{0\kappa}]\Delta^{-1}_{-}(q).$$

Sum of diagrams in Appendix C gives us average of Q^{13}_{κ} in the first order. The diagrams for $\langle Q^{12}_{\kappa}\rangle$ can be obtained from those (B.1) given in App. B if we replace  by [12]. Their sum is given by the right-

hand side of eq. (29) without the factor $(i\lambda_m+\overset{+}{H}{}'^{12}_{0\kappa})^{-1}$.

Replacing $\overline{\Sigma}_+$ in (24) by $\overline{\Sigma}^{(1)}_+$ given in (28) we get the Green function $\overline{G}^{(1)}_+$ in the first order.

Analysis of this function is being continued.

Acknowledgements

The author would like to express his sincere thanks to A.Pawlikowski for critical reading of the manuscript and helpful comments.

He is also indebted to A.I.Larkin, N.M.Plakida and J.Czerwonko for useful discussions.

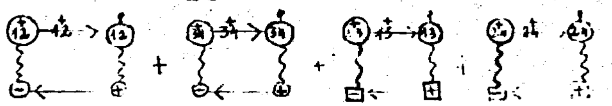
References

1. B.Westwański. JINR Preprint, E4-7486, Dubna, 1973. To be published in Acta Phys. Polonica.
2. B.Westwański and A.Pawlikowski. Phys. Lett.,A, 43, 201 (1973).
3. P.W.Anderson. Phys.Rev., 124, 41 (1961).
4. J.Hubbard. Proc. Roy. Soc., A276, 238 (1963); ibid., A277, 237 (1964); ibid., A281, 401 (1964); ibid., A285, 545 (1965).
5. D.J.Scalapino. Phys.Rev.Lett., 16, 937 (1966).
6. A.A.Abrikosov, L.P.Gorkov and I.E.Dzyaloshinsky. Quantum Field Theoretical Methods in Statistical Physics, 2nd Edn. (Oxford, Pergamon) (1965).
7. F.G.Vaks, A.I.Larkin. S.A.Pikin. Zh. eksp.teor. Fiz., 53, 281 (1967); ibid., 53, 1089 (1967).
8. M.G.Cottam and R.B.Stinchcombe. J.Phys. C.: Solid St.Phys., 3, 2283 (1970).
9. Ju.A.Izyumov and F.A.Kassan-Ogly. Fiz. Metallov y Metalloved., 30, 225 (1970); ibid., 26, 385 (1968).
10. E.M.Pikalev, M.A.Savchenko and J.Solyom. Zh. eksp. teor. Fiz., 55, 1404 (1968).
11. M.G.Cottam and M.J.Jones. J.Phys.C: Solid St.Phys., 6, 1020 (1973)., ibid., 6, 1037 (1973).
12. H.Yamada and S.Takada. Prog.Theor.Phys., 49, 1401 (1973).
13. B.Westwański. JINR Preprint, E4-7315, Dubna, 1973. To be published in Phys.Lett.A.

Received by Publishing Department
on October 9, 1973.

Appendix A. Free Energy Diagrams up to Fourth-Order in V_{kd} for Anderson Model

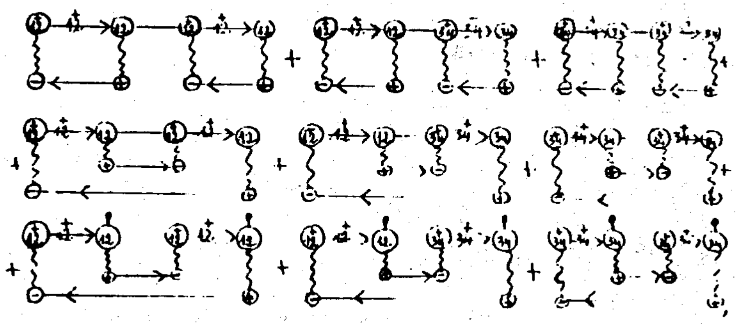
F =



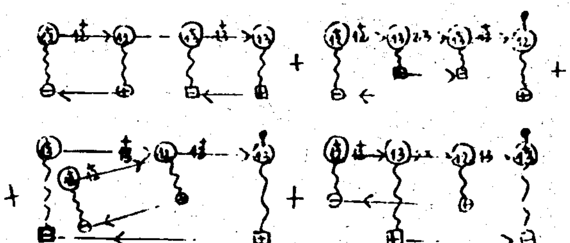
+ B(12,34) + B(13,24) + C(12,13) + C(13,12) + C(12,24) +
+ C(24,12) + C(34,13) + C(13,34) + C(34,24) + C(24,34) +
+ D(12,13,24,34) + D(34,24,13,12) + D(13,12,34,24) +
+ D(24,34,12,13) ,

where

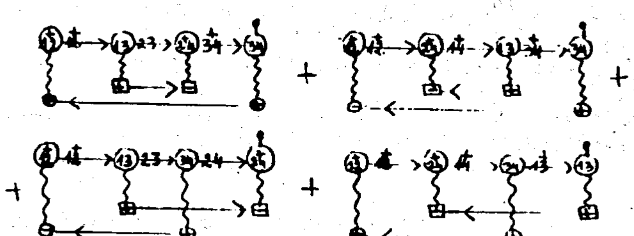
B(12,34) =



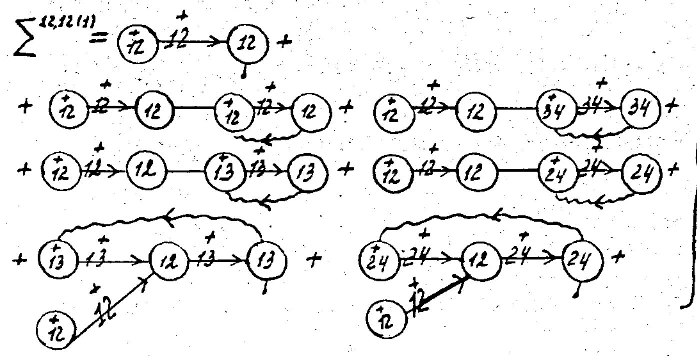
C(12,13) =



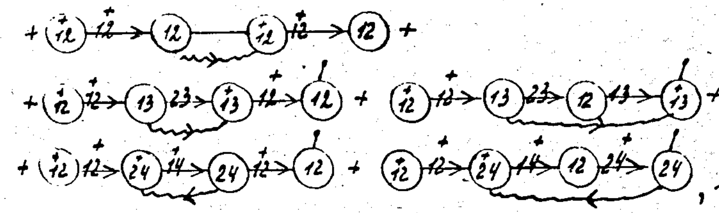
D(12,13,24,34) =



Appendix B. Irreducible Polarization Part Components of $\Sigma_{+}^{(1)}$

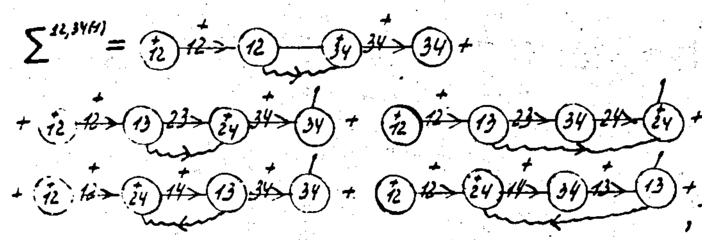


(B.1)



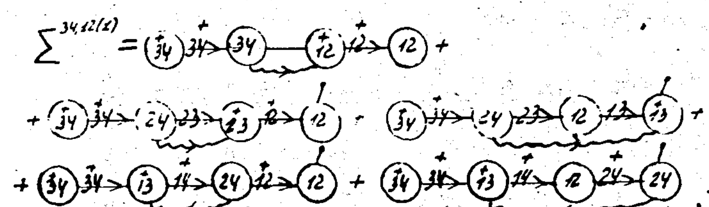
(B.2)

(B.3)



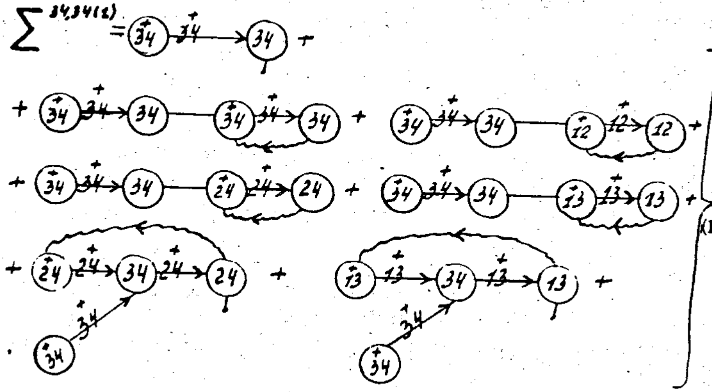
(B.4)

(B.5)

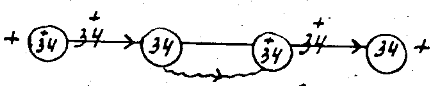


(B.6)

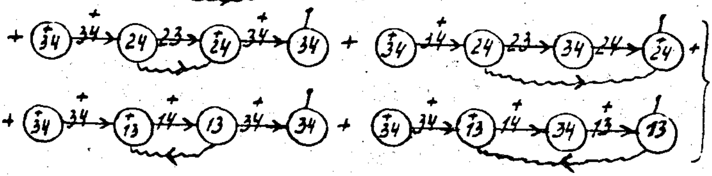
(B.7)



(B.8)



(B.9)



(B.10)

Appendix C. Diagram Representation of Average of Q_κ^{13}