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**PERTURBATION THEORY
FOR HAMILTONIANS OF ANDERSON
AND HUBBARD TYPE. II. ANALOG
OF GENERAL STATISTICAL
WICK THEOREM FOR DIAGONAL
OPERATORS, CORRELATION FUNCTIONS**

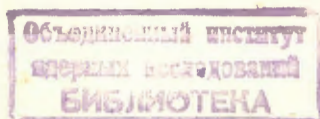
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**ЛАБОРАТОРИЯ
ТЕОРЕТИЧЕСКОЙ ФИЗИКИ**

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B. Westwański *

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1. Introduction

In the preceding paper ^{/1/} (referred to as I hereafter) we described the realization of the general statistical Wick theorem (GSWT) for transverse operators ^{/2/} in the case when unperturbed Hamiltonian contains the Coulomb interaction of the Anderson ^{/3/} and Hubbard ^{/4/} models. According to this realization the diagrammatic representation of transverse and diagonal operators was obtained.

In this paper in § 2 we formulate an analog of GSWT for diagonal operators. In § 3 representation of the most general interaction is presented. In § 4 the Anderson interaction and Hubbard "interaction" in diagonal representation are discussed. Finally, in § 5 the diagrams leading to the Scalapino result ^{/5/} are found; the first order of the irreducible polarization part for the one-particle electron Green function is calculated within the Hubbard model.

2. GSWT for Diagonal Operators

After applying multiply (I.25) the average on the left-hand side of (I.25) becomes a sum of the products of FGF's and averages of products of diagonal operators (I.21) only

$$\langle Q_1^a(r_1) Q_2^b(r_2) \dots Q_{n-1}^{x-1}(r_{n-1}) Q_n^x(r_n) \rangle_0. \quad (1)$$

The operators Q_j^Y are of course "time" independent. However, "times" t_j in (1) should be left to distinguish the operators Q_j^Y , when all lattice site indices are equal. Such a situation occurs in the Anderson model due to (I.9) and (I.11). In the following (for simplicity) we omit "times" in (1). If average (I.25) appears in linked cluster expansion of any quantity then it is simply so-called connected average corresponding to connected diagrams ^{/6/}. Such diagrams may be built up from the following mutually connected parts: 1) interaction lines, 2) transverse FGF's (I.23) (corresponding to transverse operators (I.19a - 20a)), 3) diagonal FGF's (corresponding to diagonal operators (I.21)), which are analogous to "vertex blocks" in ^{/7/} and "semi-invariants" employed in the drone-fermion perturbation method" ^{/8/}. The first point is realized automatically, second one by applying (I.25) (from now on we call it GSWT for transverse operators) and the third- by applying GSWT for diagonal operators (because it has a similar form to (I.25)). The latter type parts are produced if commutators (or anticommutators) of transverse operators are not c-numbers.

To formulate GSWT for diagonal operators (I.21) we have to know their averages. Denoting by Z_0 the partition function (I.1-2) with $V=0$ we can write

$$Z_0 = \prod_{\kappa} Z_{0\kappa};$$

$$Z_{0\kappa} = \text{Tr} \exp[-\beta(E_{\kappa+} Q_{\kappa}^{13} + E_{\kappa-} Q_{\kappa}^{12} + U_{\kappa} Q_{\kappa}^{13} Q_{\kappa}^{12})], \quad (2)$$

where $E_{\kappa\pm}$, U_{κ} are the same parameters as in (I.4-5) and indices are to be omitted after calculations. From (I.15-17), (I.21) and (2) it follows that:

$$Z_{0\kappa} = \exp[-\beta(E_{\kappa+} + E_{\kappa-} + U_{\kappa})] + \exp[-\beta E_{\kappa-}] + \exp[-\beta E_{\kappa+}] + 1, \quad (3)$$

$$\langle Q_{\kappa}^{13} \rangle_0 = Z_0^{-1} D_{\kappa}^{13} Z_0 = Z_{0\kappa}^{-1} \{ \exp[-\beta(E_{\kappa+} + E_{\kappa-} + U_{\kappa})] + \exp[-\beta E_{\kappa-}] \}, \quad (4)$$

$$\langle Q_{\kappa}^{34} \rangle_0 = Z_0^{-1} (1 + D_{\kappa}^{34}) Z_0 = Z_{0\kappa}^{-1} \{ \exp[-\beta E_{\kappa(\pm)}] + 1 \},$$

$$\langle Q_{\kappa}^{23} \rangle_0 = Z_0^{-1} D_{\kappa}^{23} Z_0 = Z_{0\kappa}^{-1} \{ \exp[-\beta E_{\kappa+}] - \exp[-\beta E_{\kappa-}] \},$$

$$\langle Q_{\kappa}^{14} \rangle_0 = Z_0^{-1} (1 + D_{\kappa}^{14}) Z_0 = Z_{0\kappa}^{-1} \{ 1 - \exp[-\beta(E_{\kappa+} + E_{\kappa-} + U_{\kappa})] \},$$

where

$$D_{\kappa}^{ij} \equiv D(Q_{\kappa}^{ij});$$

$$D_{\kappa}^{13} \equiv \partial/\partial(-\beta E_{\kappa(\mp)}), \quad D_{\kappa}^{34} \equiv -D_{\kappa}^{13}; \quad (5)$$

$$D_{\kappa}^{14} \equiv (\pm) D_{\kappa}^{13} - D_{\kappa}^{12}.$$

Let us consider first simple examples of the reduction of averages (1):

$$\langle Q_k^{13} Q_{\ell}^{12} \rangle_0 = Z_0^{-1} D_k^{13} [Z_0 \langle Q_{\ell}^{12} \rangle_0] = \quad (6)$$

$$1. \quad = \langle Q_k^{13} \rangle_0 \langle Q_{\ell}^{12} \rangle_0 + D_k^{13} \langle Q_{\ell}^{12} \rangle_0,$$

where the second term is the cumulant average,

$$\langle Q_k^{13} Q_{\ell}^{12} \rangle_{0c} = \delta_{\ell,k} D_{\ell}^{13} \langle Q_{\ell}^{12} \rangle_0 = \delta_{k,\ell} D_k^{13} \langle Q_k^{12} \rangle_0. \quad (6a)$$

$$\begin{aligned}
2. \quad \langle Q_k^{13} Q_\ell^{34} \rangle_0 &= Z_0^{-1} D_k^{13} [Z_0 \langle Q_\ell^{34} \rangle_0] = \\
&= \langle Q_k^{13} \rangle_0 \langle Q_\ell^{34} \rangle_0 + D_k^{13} \langle Q_\ell^{34} \rangle_0,
\end{aligned} \tag{7}$$

where the cumulant average

$$\begin{aligned}
\langle Q_k^{13} Q_\ell^{34} \rangle_{0c} &= D_k^{13} \langle Q_\ell^{34} \rangle_0 = \delta_{k,\ell} D_\ell^{13} \langle Q_\ell^{34} \rangle_0 = \\
&= D_\ell^{34} \langle Q_k^{13} \rangle_0 = \delta_{\ell,k} D_k^{34} \langle Q_k^{13} \rangle_0,
\end{aligned} \tag{7a}$$

$$\begin{aligned}
3. \quad \langle Q_k^{12} Q_\ell^{13} Q_p^{12} \rangle_0 &= \langle Q_k^{12} \rangle_0 \langle Q_\ell^{13} \rangle_0 \langle Q_p^{12} \rangle_0 + \\
&(D_k^{12} \langle Q_\ell^{13} \rangle_0) \langle Q_p^{12} \rangle_0 + \langle Q_\ell^{13} \rangle_0 D_k^{12} \langle Q_p^{12} \rangle_0 + \\
&+ \langle Q_k^{12} \rangle_0 D_\ell^{13} \langle Q_p^{12} \rangle_0 + D_k^{12} D_\ell^{13} \langle Q_p^{12} \rangle_0,
\end{aligned} \tag{8}$$

where the last term is the cumulant average

$$\begin{aligned}
\langle Q_k^{12} Q_\ell^{13} Q_p^{12} \rangle_{0c} &= D_k^{12} D_\ell^{13} \langle Q_p^{12} \rangle_0 = \\
&= D_p^{12} D_k^{12} \langle Q_\ell^{13} \rangle_0 = D_\ell^{13} D_p^{12} \langle Q_k^{12} \rangle_0.
\end{aligned} \tag{8a}$$

Each of the possibilities (8a) represents then the product of two Kronecker symbols and second-order differential operator acting on the average of one operator. For example the first possibility in (8a) gives

$$\delta_{k,\ell} \delta_{\ell,p} D_p^{12} D_p^{13} \langle Q_p^{12} \rangle_0. \tag{8b}$$

From these examples (6a, 7a, 8a-b) we can conclude,

that the differential operators (5) play a similar role in the reduction as FGF's (I.23), therefore due to this analogy we will call the above differential operators diagonal FGF's. Furthermore we can generalize the results (6-8b) to an arbitrary type of the average (1) in the form (similar to (I.25)) of GSWT (for diagonal operators)

$$\begin{aligned}
\langle Q_1^a Q_2^b \dots Q_{n-1}^{x-1} Q_n^x \rangle_0 &= \langle Q_1^a Q_2^b \dots Q_n^x \rangle_0 + \\
&+ \langle Q_1^a Q_2^b Q_3^c \dots Q_n^x \rangle_0 + \langle Q_1^a Q_2^b Q_3^c \dots Q_n^x \rangle_0 + \dots + \\
&+ \langle Q_1^a Q_2^b \dots Q_{n-1}^{x-1} Q_n^x \rangle_0 + \langle Q_1^a Q_2^b \dots Q_{n-1}^{x-1} Q_n^x \rangle_0,
\end{aligned} \tag{9}$$

where

$$Q_1^a = \langle Q_1^a \rangle_0,$$

$$Q_1^a Q_2^b \dots Q_{n-1}^{x-1} Q_n^x = Q_1^a Q_n^x Q_2^b \dots Q_{n-1}^{x-1}$$

with

$$Q_1^a Q_n^x = \delta_{1,n} D_n^a \langle Q_n^x \rangle_0 = \delta_{n,1} D_1^x \langle Q_1^a \rangle_0 \tag{9a}$$

(and is equal to (6a) for $Q_1^a = Q_k^{13}, Q_n^x = Q_\ell^{12}$). What the dots in (9) mean we explain by the example of the following term for $n = 5$:

$$\begin{aligned}
\langle Q_1^a Q_2^b Q_3^c Q_4^d Q_5^e \rangle_0 &= \langle Q_1^a Q_2^b Q_3^c Q_4^d Q_5^e \rangle_0 + \\
\langle Q_1^a Q_2^b Q_3^c Q_4^d Q_5^e \rangle_0 &+ \langle Q_1^a Q_2^b Q_3^c Q_4^d Q_5^e \rangle_0,
\end{aligned}$$

where

$$\underbrace{Q_1^a Q_2^b Q_3^c Q_4^d Q_5^e} = \underbrace{Q_1^a Q_3^c Q_2^b Q_4^d Q_5^e};$$

$\underbrace{Q_1^a Q_3^c}$ is to be calculated in the manner (9a),

$$\underbrace{Q_1^a Q_2^b Q_3^c Q_4^d Q_5^e} = \underbrace{Q_1^a Q_3^c Q_5^e} \underbrace{Q_2^b Q_4^d}$$

and the c -function

$$\underbrace{Q_1^a Q_3^c Q_5^e}$$

is of the type of (8a) and is equal to it for $Q_1^a = Q_k^{12}$,

$$Q_3^c = Q_l^{13}, \quad Q_5^e = Q_p^{12}.$$

In general we define the contraction of k -diagonal operators in the manner

$$\begin{aligned} \underbrace{Q_1^a Q_2^b \dots Q_k^x} &= D_1^a D_2^b \dots D_{k-1}^{x-1} \langle Q_k^x \rangle_0 = \\ &= \delta_{1,2} \delta_{2,3} \dots \delta_{k-1,k} D_k^a D_k^b \dots D_k^{x-1} \langle Q_k^x \rangle_0. \end{aligned} \quad (10)$$

Thus it is a $(k-1)$ -fold derivative of average $\langle Q_k^x \rangle_0$. Of course (10) is invariant under the arbitrary permutation of the indices

$$\binom{a}{1}, \binom{b}{2}, \dots, \binom{x-1}{k-1}, \binom{x}{k}.$$

Summarizing all the process of the reduction consisting of two stages (I.25) and (9) we can say that every "time"-

ordered average on the left-hand side of (I.25) at the end becomes a sum of the products of FGF's (I.23), (5) and averages (4); or, in other words, the sum of the products of transverse FGF's (I.23) and the derivatives (5) of averages (4). This sum may be splitted into two parts, one of which corresponds to unconnected diagrams and the second to connected ones. When the average on the left-hand side of (I.25) appears in the linked cluster expansion of some quantity it is equal to the second part.

3. Representation of the "Interaction"

There are a good deal of the interactions in (I.33) as well as FGF's (I.23), (5) on the one hand, and the simple product properties (in Appendix I.A) for the projection type operators in Θ_K , on the other hand. In this situation it seems to be better to represent: all interactions (transverse and diagonal) (I.33) with the help of one way line \sim in the manner

$$\sum_{K \neq K'} T_{KK'}^{ij, \ell k} J_{KK'}^{ij} J_{KK'}^{\ell k} \equiv \textcircled{ij} \sim \textcircled{\ell k}, \quad (11)$$

$$\sum_{K \neq K'} T_{KK'}^{ij, \ell k} Q_{KK'}^{ij} Q_{KK'}^{\ell k} \equiv \boxed{ij} \sim \boxed{\ell k}, \quad (12)$$

where the circle $\textcircled{}$ refers to the transverse operators (I.19-20) and the square $\boxed{}$ - to diagonal ones (I.21); every transverse FGF's (I.23) by means of the labeled directed line as in (I.27), all diagonal FGF's (5) - by undirected line

$$\tau \text{ --- } \tau', \quad (13)$$

where the diagonal operators, standing at its ends with "time" τ and τ' correspondingly determine the kind FGF (5) due to (9-10).

4. Representation of Anderson Interaction and Hubbard "Interaction"

As an example of the application of above formalism we consider the interaction (I.11) in Anderson Hamiltonian (I.2), (I.8-11) and the "interaction" (I.12) in Hubbard one. From (I.19) we obtain

$$\begin{aligned} d_+ &= J^{12} + J^{34} \\ d_- &= J^{13} + J^{24} \end{aligned} \quad (14)$$

Taking into account (11), (14) for d-electron operators in the Anderson model and representing in addition k-electron operators in the following way

$$\begin{aligned} c_{k+} (c_{k+}^+) &\text{ by } \ominus (\oplus), \\ c_{k-} (c_{k-}^+) &\text{ by } \square (\boxplus) \end{aligned} \quad (15)$$

we can write the transverse interaction (I.11) as follows

$$\begin{aligned} V &= V_+ + V_-; \\ V_+ &= \sum_k \{ V_{kd}^* (J_d^{12} + J_d^{34}) c_{k+} + V_{kd} c_{k+}^+ (J_d^{12} + J_d^{34}) \} = \\ &\equiv \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4}, \end{aligned} \quad (16)$$

$$\begin{aligned} V_- &= \sum_k \{ V_{kd}^* (J_d^{13} + J_d^{24}) c_{k-} + V_{kd} c_{k-}^+ (J_d^{13} + J_d^{24}) \} = \\ &\equiv \text{Diagram 5} + \text{Diagram 6} + \text{Diagram 7} + \text{Diagram 8} \end{aligned}$$

The "interaction" (I.12) takes similar form

$$V_H = V_{H+} + V_{H-};$$

$$\begin{aligned} V_{H+} &= \sum_{\kappa \neq \kappa'} T_{\kappa, \kappa'} (J_{\kappa}^{12} + J_{\kappa}^{34}) (J_{\kappa'}^{12} + J_{\kappa'}^{34}) = \\ &\equiv \text{Diagram 9} + \text{Diagram 10} + \text{Diagram 11} + \text{Diagram 12}, \\ V_{H-} &= \sum_{\kappa \neq \kappa'} T_{\kappa, \kappa'} (J_{\kappa}^{13} + J_{\kappa}^{24}) (J_{\kappa'}^{13} + J_{\kappa'}^{24}) = \\ &\equiv \text{Diagram 13} + \text{Diagram 14} + \text{Diagram 15} + \text{Diagram 16} \end{aligned} \quad (17)$$

5. Applications

The application of presented diagram method to calculation of the free energy F in the Anderson model is given in Appendix A. For example, first four diagrams in second order in V_{kd} give the Scalapino result (Eq. (7) in ref. /5/). The dominant fourth-order free energy contribution given by Scalapino (Eq. (12) in ref. /5/) comes from diagrams indicated as C(ij, lk) and D(ij, ...) in App. A. Infinite diagram summation in the Anderson model is continued.

In this section first order result will be derived for the transverse Green function in the Hubbard model. The calculations are conveniently carried out by employing a matrix representation and we therefore define a 2x2 matrix Green function $G_{\pm}^{(\pm)}(k, i\lambda_m)$ by

$$G_{\pm}^{(\pm)}(k, i\lambda_m) = \begin{pmatrix} G_{12,12}^{(\pm)}(k, i\lambda_m) & G_{12,34}^{(\pm)}(k, i\lambda_m) \\ G_{13,13}^{(\pm)}(k, i\lambda_m) & G_{13,24}^{(\pm)}(k, i\lambda_m) \\ G_{34,12}^{(\pm)}(k, i\lambda_m) & G_{34,34}^{(\pm)}(k, i\lambda_m) \\ G_{24,13}^{(\pm)}(k, i\lambda_m) & G_{24,24}^{(\pm)}(k, i\lambda_m) \end{pmatrix} \quad (18)$$

where the elements of the matrix are causal Green functions of the transverse Fermi type operators (I.19a) and are defined by

$$G_{ij,fp}^+(k, i\lambda_m) = \langle\langle J_k^{ij}; J_k^{fp} \rangle\rangle \quad (19)$$

with

$$\langle\langle J_k^{ij}; J_k^{fp} \rangle\rangle = \frac{1}{2} \sum_{\kappa} \int_{-\beta}^{\beta} d\tau \exp[-ik(R_{\kappa} - R_{\kappa'}) + i\lambda_m \tau] *$$

$$* \langle T J_{\kappa}^{fp}(0) J_{\kappa}^{ij}(\tau) \rangle.$$

The corresponding matrix for the transverse interaction is

$$\underline{V}(k) = V(k) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad (20)$$

where

$$V(k) = -t(k) = -(\epsilon_k - \epsilon), \quad (21)$$

$t(k)$ is the Fourier transform of $T_{\kappa, \kappa'}$ in (17) and ϵ_{κ} describes the unperturbed band structure /4/.

Denoting by $\Sigma_{/7,9-12/}$ the irreducible polarization part of G we can write the graphical equation

$$G_{(-)}^+(k, i\lambda_m) = \Sigma_{(-)}^+ + \Sigma_{(-)}^+ V(k) G_{(-)}^+(k, i\lambda_m), \quad (22)$$

where Σ_{+} is the 2x2 matrix.

From (14), (18-19) we have

$$\langle\langle d_{k(-)}^+; d_{k(-)}^+ \rangle\rangle \equiv \underline{G}_{(-)}^+(k, i\lambda_m) =$$

$$= G_{(13,13)}^{12,12}(k, i\lambda_m) + G_{(13,24)}^{12,34}(k, i\lambda_m) +$$

$$+ G_{(24,13)}^{34,12}(k, i\lambda_m) + G_{(24,24)}^{34,34}(k, i\lambda_m). \quad (23)$$

According to (20), (22) the solution for Green function (23) takes the form

$$\underline{G}_{(-)}^+ = (\underline{\Sigma}_{(-)}^+ - V(k))^{-1}, \quad (24)$$

where

$$\underline{\Sigma}_{(-)}^+ = \Sigma_{(13,13)}^{12,12} + \Sigma_{(13,24)}^{12,34} + \Sigma_{(24,13)}^{34,12} + \Sigma_{(24,24)}^{34,34}.$$

The components of Σ_{+} are given in Appendix B up to first order (i.e., they involve one internal momentum summation only). The diagrams for the components of Σ_{+} ($\Sigma_{13,13}^{12,12}$, $\Sigma_{13,24}^{12,34}$, $\Sigma_{24,13}^{34,12}$, $\Sigma_{24,24}^{34,34}$) can be obtained according to Appendix I.B from those in App. B by substitution $12 \rightarrow 13$, $34 \rightarrow 24$.

The zeroth-order result for $\underline{\Sigma}_{+}$ due to App. B takes the form /13/, i.e.,

$$\underline{\Sigma}_{+}^{\circ} = \text{Diagram} = \langle Q_{\kappa}^{12} \rangle_0 (i\lambda_m + H_{0\kappa}^{12})^{-1} + \langle Q_{\kappa}^{34} \rangle_0 (i\lambda_m + H_{0\kappa}^{34})^{-1}. \quad (25)$$

When $\underline{\Sigma}_{+}$ in (24) is equal to $\underline{\Sigma}_{+}^{\circ}$, then we obtain, as in /13/, Hubbard I result for G_{+}° /4/.

Effective (transverse) interaction $\underline{V}_{+}^{\circ}(k, i\lambda_m)$ is defined similarly as in /7,9-12/ i.e.,

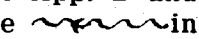
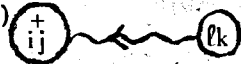
$$\underline{\tilde{V}}_{+}^{\circ}(k, i\lambda_m) \equiv \underline{V}(k) + \underline{V}(k) \underline{G}_{+}^{\circ} \underline{V}(k), \quad (26)$$

where

$$\tilde{V}_{+}^{(-)} = \begin{pmatrix} \approx \begin{matrix} 12,12 \\ (13,13) \end{matrix} & \approx \begin{matrix} 12,34 \\ (13,24) \end{matrix} \\ \approx \begin{matrix} 34,12 \\ (24,13) \end{matrix} & \approx \begin{matrix} 34,34 \\ (24,24) \end{matrix} \end{pmatrix}$$

It is easy to see that all components of $\tilde{V}_{+}^{(-)}$ are equal /13/, i.e.,

$$\tilde{V}_{+}^{(-)} = V(k) \begin{pmatrix} 11 \\ 11 \end{pmatrix} (1 - \sum_{+}^{\circ} V(k))^{-1}. \quad (27)$$

These components are represented in the App. B and App. C with the aid of the directed wavy line  in the manner ($i < j, \ell < k$) 

The renormalized spectrum $\epsilon_{\pm}^{(1)} (\epsilon_{\pm 1} - \epsilon_{\pm 2} = \Delta_{\pm})$ is obtained from the condition $1 + t(k) \sum_{\pm}^{\circ} = 0$ and was given in /13/.

In the approximation given in App. B for Σ_{+} we get

$$\Sigma_{+}^{(1)} = \sum_{i=1}^5 A_i, \quad (28)$$

where A_1 is the sum of the diagrams (B.1)

$$A_1 = (i\lambda_m + H_{0\kappa}^{+12})^{-1} N^{-1} \sum_q \{ \langle Q_{\kappa}^{12} \rangle_0 - n(13) - n(24) + \\ - t(q) [Z_{-1} n(\epsilon_{-1}(q)) + Z_{-2} n(\epsilon_{-2}(q))] + \\ - t(q) \beta U [n(\epsilon_{+1}(q)) - n(\epsilon_{+2}(q))] \Delta_{+}^{-1}(q) D_{\kappa}^{12} \langle Q_{\kappa}^{12} \rangle_0 \}. \quad (29)$$

The sum of diagrams (B.8) gives

$$A_2 = (i\lambda_m + H_{0\kappa}^{+34})^{-1} N^{-1} \sum_q \{ \langle Q_{\kappa}^{34} \rangle_0 + n(13) + n(24) + \\ + t(q) [Z_{-1} n(\epsilon_{-1}(q)) + Z_{-2} n(\epsilon_{-2}(q))] +$$

$$- t(q) \beta U [n(\epsilon_{+1}(q)) - n(\epsilon_{+2}(q))] \Delta_{+}^{-1}(q) D_{\kappa}^{12} \langle Q_{\kappa}^{34} \rangle_0 \}.$$

The coefficients $Z_{-1}^{(2)}$ are equal to:

$$Z_{-1}^{(2)} = (\pm) \{ \beta U \Delta_{-}^{-1}(q) D_{\kappa}^{13} \langle Q_{\kappa}^{12} \rangle_0 + \\ - \langle Q_{\kappa}^{13} \rangle_0 (\epsilon_{-(1)}^{(2)}(q) - H_{0\kappa}^{+24}) (\epsilon_{-(1)}^{(2)}(q) - H_{0\kappa}^{+13})^{-1} \Delta_{-}^{-1}(q) + \\ - \langle Q_{\kappa}^{24} \rangle_0 (\epsilon_{-(1)}^{(2)}(q) - H_{0\kappa}^{+13}) (\epsilon_{-(1)}^{(2)}(q) - H_{0\kappa}^{+24})^{-1} \Delta_{-}^{-1}(q) \},$$

$n(ij)$ is given in (I.23) and $n(\epsilon_{\pm}^{(1)}) = (\exp[\beta \epsilon_{\pm}^{(1)}] + 1)^{-1}$.

Collecting the diagrams (B.2), (B.4), (B.6) and (B.9) we get

$$A_3 = -D_{\kappa}^{12} \langle Q_{\kappa}^{12} \rangle_0 * \frac{1}{N} \sum_q t(q) U^2 (i\lambda_m + H_{0\kappa}^{+12})^{-1} (i\lambda_m + H_{0\kappa}^{+34})^{-1} * \\ * (i\lambda_m + \epsilon_{+1}(q))^{-1} (i\lambda_m + \epsilon_{+2}(q))^{-1}.$$

The diagrams (B.3), (B.5), (B.7) and (B.10) give two types of contributions A_4 and A_5 :

$$A_4 = [(i\lambda_m + H_{0\kappa}^{+12})^{-1} - (i\lambda_m + H_{0\kappa}^{+34})^{-1}]^2 * \quad (30)$$

$$* N^{-1} \sum_q t(q) \{ Y_{-1} n(\epsilon_{-1}(q)) + Y_{-2} n(\epsilon_{-2}(q)) \},$$

$$A_5 = - [(i\lambda_m + H_{0\kappa}^{+12})^{-1} - (i\lambda_m + H_{0\kappa}^{+34})^{-1}]^2 *$$

$$\begin{aligned}
& * N^{-1} \sum_q t(q) \{ X_{-1} [W_{-1}^{23} (i\lambda_m + \epsilon_{-1}(q) - H_{0\kappa}^{+23})^{-1} + \\
& + W_{-1}^{14} (i\lambda_m + H_{0\kappa}^{+14} - \epsilon_{-1}(q))^{-1}] + \\
& + X_{-2} [W_{-2}^{23} (i\lambda_m + \epsilon_{-2}(q) - H_{0\kappa}^{+23})^{-1} + \\
& + W_{-2}^{14} (i\lambda_m + H_{0\kappa}^{+14} - \epsilon_{-2}(q))^{-1}] \},
\end{aligned} \quad (31)$$



where

$$\begin{aligned}
W_{-\frac{1}{2}}^{23} &= \langle J_{\kappa}^{23} J_{\kappa}^{23} \rangle_0^+ + \langle Q_{\kappa}^{23} \rangle_0 n(\epsilon_{-\frac{1}{2}}(q)), \\
W_{-\frac{1}{2}}^{14} &= \langle J_{\kappa}^{14} J_{\kappa}^{14} \rangle_0^+ + \langle Q_{\kappa}^{14} \rangle_0 n(\epsilon_{-\frac{1}{2}}(q))
\end{aligned}$$

and averages of product of operators J_{κ}^{ij} and J_{κ}^{ij} can be calculated according to App. IA and (I.15-17). The factors Y and X in (30) and (31) have the form

$$\begin{aligned}
Y_{-\frac{1}{2}} &= \left(\frac{-}{+}\right) \langle Q_{\kappa}^{13} \rangle_0 [\epsilon_{-\frac{1}{2}}(q) - H_{0\kappa}^{+24}] \Delta_{-}^{-1}(q) \\
&+ (\pm) \langle Q_{\kappa}^{24} \rangle_0 [\epsilon_{-\frac{1}{2}}(q) - H_{0\kappa}^{+13}] \Delta_{-}^{-1}(q) \\
X_{-\frac{1}{2}} &= \left(\frac{-}{+}\right) [\epsilon_{-\frac{1}{2}}(q) - H_{0\kappa}^{+13}] [\epsilon_{-\frac{1}{2}}(q) - H_{0\kappa}^{+24}] \Delta_{-}^{-1}(q).
\end{aligned}$$

Sum of diagrams in Appendix C gives us average of Q_{κ}^{13} in the first order. The diagrams for $\langle Q_{\kappa}^{12} \rangle$ can be obtained from those (B.1) given in App. B if we replace

 by . Their sum is given by the right-

hand side of eq. (29) without the factor $(i\lambda_m + H_{0\kappa}^{+12})^{-1}$. Replacing Σ_{+} in (24) by $\Sigma_{+}^{(1)}$ given in (28) we get the Green function $\frac{1}{G_{+}^{(1)}}$ in the first order. Analysis of this function is being continued.

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Appendix A. Free Energy Diagrams up to Fourth-Order in V_{kd} for Anderson Model

$$F = \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} + \text{[Diagram 4]} + \text{[Diagram 5]} + \text{[Diagram 6]} + \text{[Diagram 7]} + \text{[Diagram 8]} + \text{[Diagram 9]} + \text{[Diagram 10]} + \text{[Diagram 11]} + \text{[Diagram 12]} + \text{[Diagram 13]} + \text{[Diagram 14]} + \text{[Diagram 15]} + \text{[Diagram 16]} + \text{[Diagram 17]} + \text{[Diagram 18]} + \text{[Diagram 19]} + \text{[Diagram 20]}$$

where

$$B(12,34) = \text{[Diagram 21]} + \text{[Diagram 22]} + \text{[Diagram 23]} + \text{[Diagram 24]} + \text{[Diagram 25]} + \text{[Diagram 26]} + \text{[Diagram 27]} + \text{[Diagram 28]} + \text{[Diagram 29]} + \text{[Diagram 30]} + \text{[Diagram 31]} + \text{[Diagram 32]} + \text{[Diagram 33]} + \text{[Diagram 34]} + \text{[Diagram 35]} + \text{[Diagram 36]} + \text{[Diagram 37]} + \text{[Diagram 38]} + \text{[Diagram 39]} + \text{[Diagram 40]}$$

$$C(12,13) = \text{[Diagram 41]} + \text{[Diagram 42]} + \text{[Diagram 43]} + \text{[Diagram 44]} + \text{[Diagram 45]} + \text{[Diagram 46]} + \text{[Diagram 47]} + \text{[Diagram 48]} + \text{[Diagram 49]} + \text{[Diagram 50]} + \text{[Diagram 51]} + \text{[Diagram 52]} + \text{[Diagram 53]} + \text{[Diagram 54]} + \text{[Diagram 55]} + \text{[Diagram 56]} + \text{[Diagram 57]} + \text{[Diagram 58]} + \text{[Diagram 59]} + \text{[Diagram 60]}$$

$$D(12,13,24,34) = \text{[Diagram 61]} + \text{[Diagram 62]} + \text{[Diagram 63]} + \text{[Diagram 64]} + \text{[Diagram 65]} + \text{[Diagram 66]} + \text{[Diagram 67]} + \text{[Diagram 68]} + \text{[Diagram 69]} + \text{[Diagram 70]} + \text{[Diagram 71]} + \text{[Diagram 72]} + \text{[Diagram 73]} + \text{[Diagram 74]} + \text{[Diagram 75]} + \text{[Diagram 76]} + \text{[Diagram 77]} + \text{[Diagram 78]} + \text{[Diagram 79]} + \text{[Diagram 80]}$$

Appendix B. Irreducible Polarization Part Components of Σ_+

$$\Sigma^{12,12(1)} = \text{[Diagram 81]} + \text{[Diagram 82]} + \text{[Diagram 83]} + \text{[Diagram 84]} + \text{[Diagram 85]} + \text{[Diagram 86]} + \text{[Diagram 87]} + \text{[Diagram 88]} + \text{[Diagram 89]} + \text{[Diagram 90]} + \text{[Diagram 91]} + \text{[Diagram 92]} + \text{[Diagram 93]} + \text{[Diagram 94]} + \text{[Diagram 95]} + \text{[Diagram 96]} + \text{[Diagram 97]} + \text{[Diagram 98]} + \text{[Diagram 99]} + \text{[Diagram 100]}$$

$$\text{[Diagram 101]} + \text{[Diagram 102]} + \text{[Diagram 103]} + \text{[Diagram 104]} + \text{[Diagram 105]} + \text{[Diagram 106]} + \text{[Diagram 107]} + \text{[Diagram 108]} + \text{[Diagram 109]} + \text{[Diagram 110]} + \text{[Diagram 111]} + \text{[Diagram 112]} + \text{[Diagram 113]} + \text{[Diagram 114]} + \text{[Diagram 115]} + \text{[Diagram 116]} + \text{[Diagram 117]} + \text{[Diagram 118]} + \text{[Diagram 119]} + \text{[Diagram 120]}$$

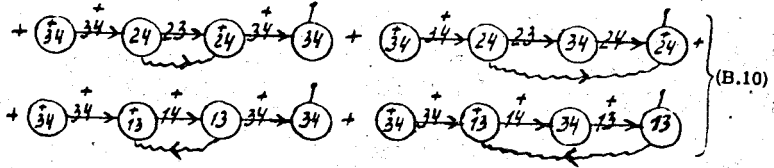
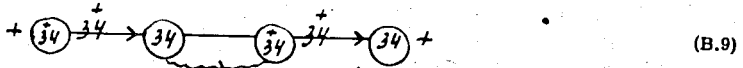
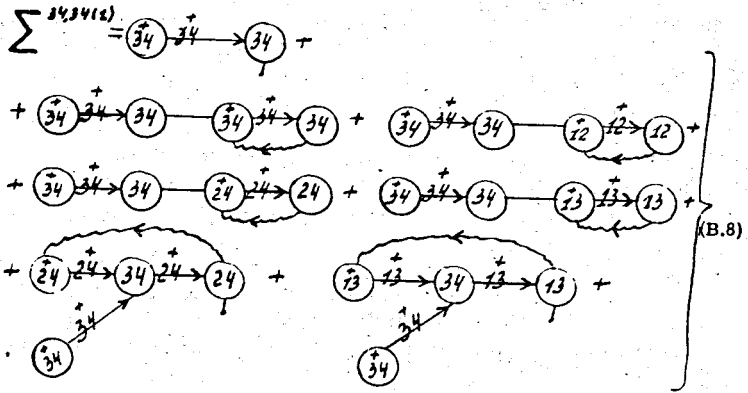
$$\text{[Diagram 121]} + \text{[Diagram 122]} + \text{[Diagram 123]} + \text{[Diagram 124]} + \text{[Diagram 125]} + \text{[Diagram 126]} + \text{[Diagram 127]} + \text{[Diagram 128]} + \text{[Diagram 129]} + \text{[Diagram 130]} + \text{[Diagram 131]} + \text{[Diagram 132]} + \text{[Diagram 133]} + \text{[Diagram 134]} + \text{[Diagram 135]} + \text{[Diagram 136]} + \text{[Diagram 137]} + \text{[Diagram 138]} + \text{[Diagram 139]} + \text{[Diagram 140]}$$

$$\Sigma^{12,34(1)} = \text{[Diagram 141]} + \text{[Diagram 142]} + \text{[Diagram 143]} + \text{[Diagram 144]} + \text{[Diagram 145]} + \text{[Diagram 146]} + \text{[Diagram 147]} + \text{[Diagram 148]} + \text{[Diagram 149]} + \text{[Diagram 150]} + \text{[Diagram 151]} + \text{[Diagram 152]} + \text{[Diagram 153]} + \text{[Diagram 154]} + \text{[Diagram 155]} + \text{[Diagram 156]} + \text{[Diagram 157]} + \text{[Diagram 158]} + \text{[Diagram 159]} + \text{[Diagram 160]}$$

$$\text{[Diagram 161]} + \text{[Diagram 162]} + \text{[Diagram 163]} + \text{[Diagram 164]} + \text{[Diagram 165]} + \text{[Diagram 166]} + \text{[Diagram 167]} + \text{[Diagram 168]} + \text{[Diagram 169]} + \text{[Diagram 170]} + \text{[Diagram 171]} + \text{[Diagram 172]} + \text{[Diagram 173]} + \text{[Diagram 174]} + \text{[Diagram 175]} + \text{[Diagram 176]} + \text{[Diagram 177]} + \text{[Diagram 178]} + \text{[Diagram 179]} + \text{[Diagram 180]}$$

$$\Sigma^{34,12(2)} = \text{[Diagram 181]} + \text{[Diagram 182]} + \text{[Diagram 183]} + \text{[Diagram 184]} + \text{[Diagram 185]} + \text{[Diagram 186]} + \text{[Diagram 187]} + \text{[Diagram 188]} + \text{[Diagram 189]} + \text{[Diagram 190]} + \text{[Diagram 191]} + \text{[Diagram 192]} + \text{[Diagram 193]} + \text{[Diagram 194]} + \text{[Diagram 195]} + \text{[Diagram 196]} + \text{[Diagram 197]} + \text{[Diagram 198]} + \text{[Diagram 199]} + \text{[Diagram 200]}$$

$$\text{[Diagram 201]} + \text{[Diagram 202]} + \text{[Diagram 203]} + \text{[Diagram 204]} + \text{[Diagram 205]} + \text{[Diagram 206]} + \text{[Diagram 207]} + \text{[Diagram 208]} + \text{[Diagram 209]} + \text{[Diagram 210]} + \text{[Diagram 211]} + \text{[Diagram 212]} + \text{[Diagram 213]} + \text{[Diagram 214]} + \text{[Diagram 215]} + \text{[Diagram 216]} + \text{[Diagram 217]} + \text{[Diagram 218]} + \text{[Diagram 219]} + \text{[Diagram 220]}$$



Appendix C. Diagram Representation of Average of Q_K^{13}

$$\langle Q_x^{13} \rangle = \boxed{13} +$$

