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SOME QUANTITATIVE RESULTS OF THE STATIONARY BEHAVIOUR OF A SOLID STATE ANTI-STOKES RAMAN OSCILLATOR

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Introduction

It is well known [1] that polaritons are very suitable for realizing tunable Raman lasers. In the simplest case the laser wave may excite only one Stokes mode and one infrared polariton mode. A good description of the stationary behaviour of such a Raman laser can be given within the framework of the theory of an optical parametrical oscillator investigated by Graham et al. in detail [2-4].

In an earlier paper [5] we have investigated some other but more complicated Raman oscillators and have presented some general results of the threshold behaviour and of the influence of phase fluctuations of the pump wave on the line widths of the Raman modes. We have based our theoretical considerations on the theory of an optical parametrical oscillator.

In the present paper we will give in detail some numerical results of the behaviour of an anti-Stokes Raman oscillator. Such an oscillator is of high interest because it will be possible to tune two neighbouring frequencies simultaneously. The results will contain the threshold value, the frequency shifts, and the steady photon and polariton numbers. An analysis of the phase fluctuations and the corresponding line shapes will be given in a later paper. Furthermore, it should be noted that the results found by us will be valid also for other laser oscillators. It is only important that there must be four nonlinear coupled (the nonlinear coupling is assumed to be the lowest order coupling) light modes of different frequencies.

Theory

Before giving the numerical results it will be useful to repeat some important relations deduced in [5]. The steady state behaviour of infrared, Stokes, laser, and anti-Stokes mode in the given order is described by oscillations of the frequencies $\omega-\Delta\Omega$, $\omega_{-1}-\Delta\Omega_{-1}$, ω_0, $\omega_1-\Delta\Omega_1$ with the conditions

$$\Delta\Omega = -\Delta\Omega_{-1}\,, \quad \Delta\Omega_1 = \Delta\omega - \Delta\Omega_{-1} \tag{1}$$

between the frequency shifts $\Delta\Omega$, $\Delta\Omega_{-1}$, $\Delta\Omega_1$ caused by the nonlinear coupling. In such a treatment the unperturbed frequencies of the considered modes are assumed to satisfy the conditions $\omega_0-\omega_{-1}-\omega=0$ and $\omega_1-\omega_0-\omega=\Delta\omega$ ($\Delta\omega\neq 0$ is supposed because of the dispersion of the substance). Employing Eq. (1) and

$$C = -\frac{2i\,|T_2|^2(\kappa_{-1}-2i\Delta\Omega_{-1})}{(\kappa+2i\Delta\Omega)(\kappa+2i\Delta\Omega_1)} + \frac{2i\,|T_1|^2}{\kappa+2i\,\Delta\Omega} \tag{2}$$

(κ, κ_{-1}, κ_0, κ_1 - damping parameters, T_1 - coupling constant between laser, Stokes, and infrared mode, T_2 - coupling constant between anti-Stokes, laser, and infrared mode) we obtain the frequency shifts $\Delta\Omega$, $\Delta\Omega_{-1}$, $\Delta\Omega_1$ by solving the equation

$$\kappa_{-1}\, Re\, C = 2\Delta\Omega_{-1}\, Im\, C\,, \tag{3}$$

where $Re\,C$ means the real part and $Im\,C$ the imaginary part of C After calculating the frequency shifts we get in an easy manner the threshold condition

$$\frac{4|\hat{F}_0|^2}{\kappa_0^2} \geq N_0 = \frac{\kappa_{-1}}{2Im\,C}\;. \tag{4}$$

The steady state photon number of the laser mode N_0 therefore is identical with the threshold value. Further on, the amplitude $\hat{F}_0$ in Eq. (4) can be easily related to the incident laser flux. Other values we are interested in are the ratios of the steady state photon numbers of Stokes to anti-Stokes mode and of Stokes to infrared mode that can be found to be

$$\frac{N_{-1}}{N} = \frac{4\,|T_1|^2 N_0}{\kappa_{-1}^2 + 4\Delta\Omega_{-1}^2} \,, \qquad (5)$$

$$\frac{N_{-1}}{N_1} = \frac{|T_1|^2(\kappa_1^2 + 4\Delta\Omega_1^2)}{|T_2|^2(\kappa_{-1}^2 + 4\Delta\Omega_{-1}^2)} \,. \qquad (6)$$

Results

The numerical results presented in Figs. 1-4 refer to a middle infrared damping parameter $\kappa = 6\kappa_{-1}$ and to the most important case of equal squares of the coupling constants $|T_1|^2 = |T_2|^2$. For convenience the abbreviation β is used for the ratio κ_1/κ_{-1}.

In Fig. 1 the ratio of the threshold values of the anti-Stokes Raman oscillator considered in this paper to an adequate Stokes Raman oscillator (optical parametrical oscillator with a steady state laser photon number n_0 that is based on the assumption that the resonance condition $\omega_0 - \omega_{-1} - \omega = 0$ is fulfilled) versus $\Delta\omega/\kappa_{-1}$ is plotted. In contrast to large $\Delta\omega/\kappa_{-1}$ (obviously large values $\Delta\omega/\kappa_{-1}$ lead to the well known results on an optical parametrical oscillator as can be seen easily) for small values $\Delta\omega/\kappa_{-1}$ a very sensitive dependence of the threshold value as a function of β is found. So Fig. 1 shows that for a given (small) $\Delta\omega/\kappa_{-1}$ the threshold value first of all rapidly increases with β. After reaching a maximum the threshold value decreases with increasing β. It is to be noted that for equal ratios κ_1/κ_{-1} and κ_{-1}/κ_1 the increase of the threshold in the region $\kappa_1 < \kappa_{-1}$ is essentially more rapid than the decrease in the region $\kappa_1 > \kappa_{-1}$. In exceedingly increasing the posi-

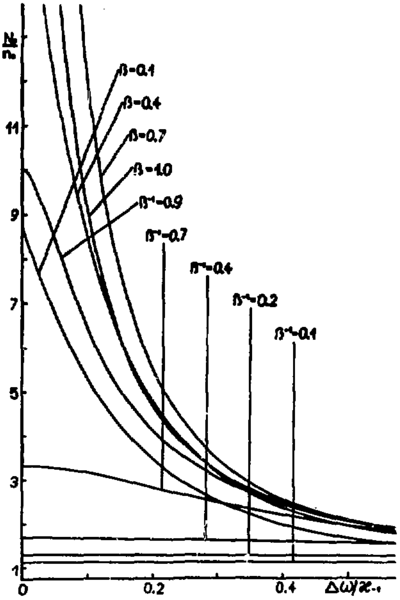

Fig. 1. The steady state threshold behaviour (relatively to an adequate optical parametrical oscillator) of an anti-Stokes Raman oscillator is shown for the fixed infrared damping parameter $\kappa = 6\kappa_{-1}$ and for various ratios $\beta = \kappa_1/\kappa_{-1}$ as a function of $\Delta\omega / \kappa_{-1}$.

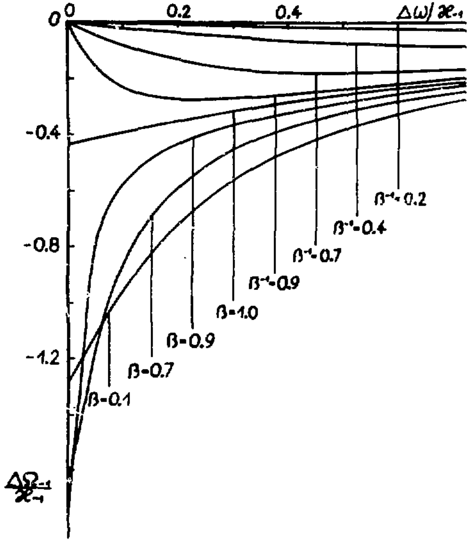

Fig. 2. The frequency shift $\Delta\Omega_{-1}/\kappa_{-1}$ of the Stokes mode is shown for the fixed infrared damping parameter $\kappa=6\kappa_{-1}$ and for various ratios $\beta=\kappa_1/\kappa_{-1}$ as a function of $\Delta\omega/\kappa_{-1}$.

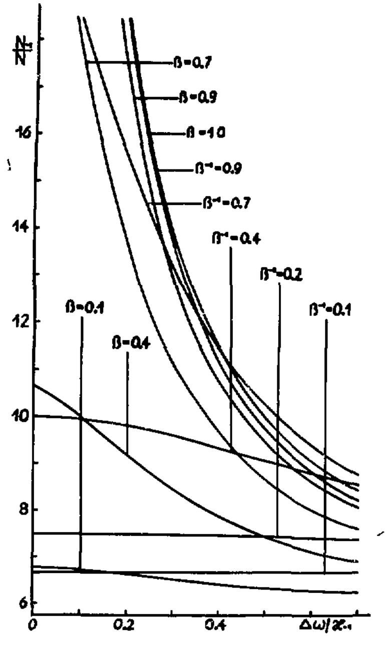

Fig. 3. The ratio of the steady state photon numbers of Stokes to infrared mode is shown for the fixed infrared damping parameter $\kappa = 6\kappa_{-1}$ and for various ratios $\beta = \kappa_1/\kappa_{-1}$ as a function of $\Delta\omega/\kappa_{-1}$.

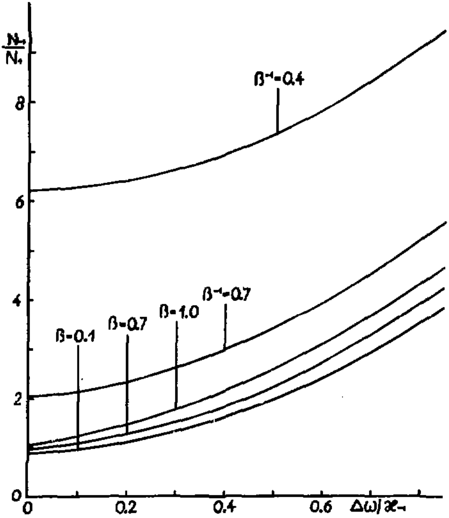

Fig. 4. The ratio of the steady state photon numbers of Stokes to anti-Stokes mode is shown for the fixed infrared damping parameter $\kappa = 6\kappa_{-1}$ and for various ratios $\beta = \kappa_1/\kappa_{-1}$ as a function of $\Delta\omega/\kappa_{-1}$.

tion of the threshold maximum value (as a function of β) shifts to $\kappa_1 = \kappa_{-1}$ for $\Delta\omega/\kappa_{-1} \to 0$. Because of this "resonance" behaviour of the threshold with respect to β the region $\beta^{-1} < 0.9$ is suggested to provide advantageous experiments.

Figure 2 shows the possible frequency shifts $\Delta\Omega_{-1}$ of the Stokes mode for different parameters β. From this figure it is also clearly seen that (for small $\Delta\omega/\kappa_{-1}$ values) different behaviours of the frequency shift $\Delta\Omega_{-1}$ are found by going from $\kappa_1 < \kappa_{-1}$ to $\kappa_1 > \kappa_{-1}$. Especially the frequency shift value $\Delta\Omega_{-1}$ can be relatively large for $\kappa_1 < \kappa_{-1}$.

A similar "resonance" behaviour with respect to β as the threshold shows the ratio of the steady state photon numbers of Stokes to infrared mode. This is plotted in Fig. 3. Hence, the "resonance" behaviour mentioned above means that for $\beta \approx 1$ and unchanged number of Stokes photons the number of available infrared photons rapidly decreases. It is evident due to this fact that there must be the necessity of an increasing threshold value as it is illustrated in Fig. 1. In accordance with the value κ/κ_{-1} assumed to be equal to 6 for large $\Delta\omega/\kappa_{-1}$ the ratio of Stokes to infrared photon numbers goes to the value 6 as it is expected with respect to the results calculated for an optical parametrical oscillator.

Contrary to the results presented up to now the ratio of steady state photon numbers of Stokes to anti-Stokes mode (appropriate curves are plotted in Fig. 4) always increases with β in a more or less continuous manner. It is seen that for equal ratios κ_1/κ_{-1} and κ_{-1}/κ_1 the increase of N_{-1}/N_1 in the region $\kappa_1 < \kappa_{-1}$ is smaller than in the region $\kappa_1 > \kappa_{-1}$. Furthermore, for experiments the region $\beta^{-1} < 0.4$ seems to be not very useful since the gain of anti-Stokes photons relatively to Stokes photons is expected to be small. It should be noted that generally in the limit $\beta \to \infty$ (similarly to the case $\Delta\omega/\kappa_{-1} \to \infty$) the well known results for an optical parametrical oscillator follow.

In Figs. 5-8 for some specially chosen parameters β the same physical quantities as in Figs. 1-4 versus

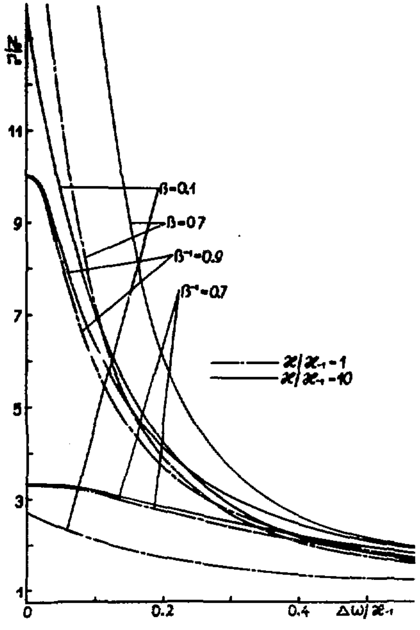

Fig. 5. The steady state threshold behaviour is shown for some specially chosen parameters β and the two infrared damping parameters $\kappa = \kappa_{-1}$ and $\kappa = 10\kappa_{-1}$.

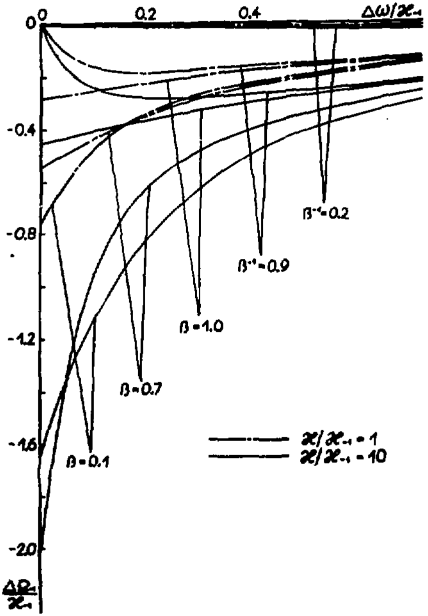

Fig. 6. The frequency shift $\Delta\Omega_{-1}/\kappa_{-1}$ of the Stokes mode is shown for some specially chosen parameters β and the two infrared damping parameters $\kappa = \kappa_{-1}$ and $\kappa = 10\kappa_{-1}$.

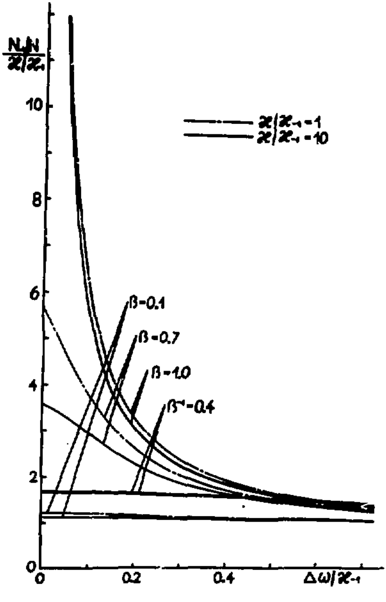

Fig. 7. The ratio of the steady state photon numbers of Stokes to infrared mode is shown for some specially chosen parameters β and the two infrared damping parameters $\kappa = \kappa_{-1}$ and $\kappa = 10\kappa_{-1}$

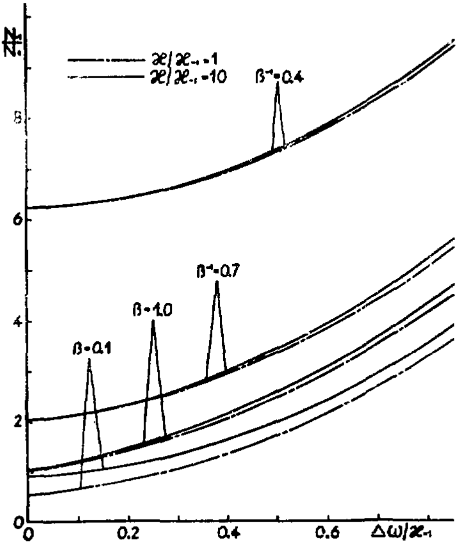

Fig. 8. The ratio of the steady state photon numbers of Stokes to anti-Stokes mode is shown for some specially chosen parameters β and the two infrared damping parameters $\kappa = \kappa_{-1}$ and $\kappa = 10\kappa_{-1}$.

$\Delta\omega/\kappa_{-1}$ are plotted but for the two assumed infrared "boundary" dampings $\kappa = \kappa_{-1}$ and $\kappa = 10\,\kappa_{-1}$ to illustrate the shiftings of the above results due to the dependence of the infrared damping parameter κ. From Figs. 5-8 this κ dependence is shown to be important mainly in the regions of small β and small $\Delta\omega/\kappa_{-1}$.

As it is mentioned above the numerical results presented in Figs. 1-8 are based on the assumption of equal squares of the coupling parameters $|T_1|^2 = |T_2|^2$. Doubtless, the case of different squares $|T_1|^2$ and $|T_2|^2$ also should be of interest because of their frequency dependence. For this reason we also have carried out some calculations by assuming different coupling parameters. For example, the investigation of the threshold behaviour leads to curves similar to those plotted in Fig. 1 and Fig. 5. By supposing $|T_1|^2 < |T_2|^2$ the threshold photon number shifts up to larger values (in comparison with the case $|T_1|^2 = |T_2|^2$ and by using such β values chosen there). In the other case $|T_1|^2 > |T_2|^2$ a lower threshold is found. Furthermore, for $\Delta\omega/\kappa_{-1} \to 0$ the threshold maximum value is found to be at the position $\kappa_1 |T_1|^2 = \kappa_{-1} |T_2|^2$. It is also interesting that for $|T_1|^2 < |T_2|^2$ and $\kappa_1 |T_1|^2 < \kappa_{-1} |T_2|^2$ very small "non allowed" $\Delta\omega/\kappa_{-1}$ regions that include $\Delta\omega/\kappa_{-1} = 0$ can exist. This means that there cannot be found steady state solutions. These regions vanish with increasing $\kappa_1 |T_1|^2$ (relatively to $\kappa_{-1} |T_2|^2$) and do not exist for $\kappa_1 |T_1|^2 > \kappa_{-1} |T_2|^2$.

Hence, in general for an anti-Stokes Raman oscillator the region $\kappa_1 |T_1|^2 > \kappa_{-1} |T_2|^2$ can be proposed to be useful for experiments. To give a more exact description (for example in the critical region $\kappa_1 |T_1|^2 \approx \kappa_{-1} |T_2|^2$) it seems necessary to include higher order interactions into the calculations.

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