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A MODEL FOR THE DESCRIPTION OF THE FRAGMENTATION OF SINGLE- AND MANY-PARTICLE STATES OVER THE LEVELS OF AN ODD SPHERICAL NUCLEUS

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Модель для описания фрагментации одночастичных и многочастичных состояний по уровням нечетного сферического ядра

В работе исследована модель для описания структуры состояний промежуточных энергий возбуждения и высоковозбужденных состояний в нечетных сферических ядрах. Процесс фрагментации, ответственный за усложнение структуры состояний с ростом энергии возбуждения, описывается с помощью взаимодействия квазичастиц с фононами. Получены основные уравнения модели. Предложен метод вычисления плотности высоковозбужденных состояний в нечетных сферических ядрах.

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A Model for the Description of the Fragmentation of Single- and Many-Particle States Over the Levels of an Odd Spherical Nucleus

A model for the description of the structure of intermediate excitation energy states and highly excited states in odd mass spherical nuclei is considered.The process of fragmentation,that is responsible for the complication of the state structure with increasing excitation energy, is described by means of the interaction of quasiparticles with different type phonons. Basic equations of the model suggested are derived. A method of calculating the density of highly excited states in odd-mass spherical nuclei is proposed.

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The process of fragmentation allows one to understand the complication of the structure of states with increasing excitation energy and to describe highly excited states in terms of quasiparticles and phonons. By the fragmentation we mean the distribution of the strength of single- and many-particle states over nuclear levels. The fragmentation starts to become apparent in low-lying states of odd-mass nuclei (see[1,2]). In refs.[3,4] it is shown that the quasiparticle-phonon interaction is of much importance in the process of fragmentation, and a model for the description of fragmentation in odd mass deformed nuclei is formulated. In the present paper this model is generalized to the case of odd mass spherical nuclei.

When formulating the model we follow the semi-microscopic approach (see ref.[5]), assuming the spherical nucleus Hamiltonian to have the form

$$H = H_{sp} + H_{pair} + H_{\lambda} + H_{\sigma\lambda} \,. \qquad (1)$$

In the expression (1), by H_{sp} we mean the single-particle part of the Hamiltonian, i.e. the average field taken, for example, in the form of the Saxon-Woods potential. The quantity H_{pair} is a residual interaction leading to superconducting pairing correlations, H_{λ} stands for the multipole-multipole residual interaction, $H_{\sigma\lambda}$ for the spin-multipole - spin-multipole residual interaction.

The main starting points of our further considerations are as follows: We assume that the structure of highly excited states of odd mass spherical nuclei is, to a large extent, defined by the interaction of quasiparticles with the phonons of an even-even core. By phonons we imply here not only collective excitations of the type 2_1^+ and 3_1^-, but generally all the excitations which are a superposition of two-quasiparticle states. The energy of phonons and their other characteristics may be taken from neighbouring even-even nuclei since the odd quasiparticle little affects their properties. Thus, the constants entering the determination of residual forces turn out to be fixed in the description of the particular features of even-even nuclei.

The multipole and spin-multipole forces can conveniently be chosen in such a way that they generate phonons with different momenta and parities. In the quasi-boson approximation the production and annihilation operators commute with one another. The 0^+ states of an even-even nucleus are described in terms of pairing vibrational phonons the existence of which is due to the part H_{pair}.

After performing the u, v Bogolubov transformation and making a transition to the phonon operators, the Hamiltonian (1) takes on the form

$$H=\sum_{jm}\varepsilon_j\alpha^+_{jm}\alpha_{jm}-\sum_{\tau}\frac{1}{2G_\tau}\sum_{ii'}\frac{1}{\sqrt{\Phi'(\omega_i)\,\Phi'(\omega_{i'})}}\times$$

$$\times\left[(\sqrt{T_{i'}}+\sqrt{P_{i'}})\Omega^+_{i'}+(\sqrt{T_{i'}}-\sqrt{P_{i'}})\Omega_{i'}\right]\cdot\left[(\sqrt{T_i}+\sqrt{P_i})\Omega_i+(\sqrt{T_i}-\sqrt{P_i})\Omega^+_i\right]+$$

$$+\frac{1}{2\sqrt{2}}\sum_{\tau}\frac{1}{G_\tau}\sum_{ji}\frac{u^{(+)}_{jj}}{\sqrt{\Phi'(\omega_i)}}\left\{\left[(\sqrt{T_i}+\sqrt{P_i})\Omega^+_i+(\sqrt{T_i}-\sqrt{P_i})\Omega_i\right]B(j)+h.c.\right\}-$$

$$-\frac{1}{4}\sum_{\lambda\mu ii'}\frac{1}{\sqrt{Y(\lambda i)\,Y(\lambda i')}}\sum_{j_1j_2}\frac{\left(f^{\lambda}_{j_1j_2}u^{(+)}_{j_1j_2}\right)^2(\varepsilon_{j_1}+\varepsilon_{j_2})}{(\varepsilon_{j_1}+\varepsilon_{j_2})^2-\omega^2_{\lambda i}}\left[Q^+_{\lambda-\mu i'}+(-)^{\lambda-\mu}Q_{\lambda\mu i'}\right]\left[Q^+_{\lambda\mu i}(-)^{\lambda-\mu}+Q_{\lambda-\mu i}\right]-$$

$$-\frac{1}{2\sqrt{2}}\sum_{\lambda\mu i}\sum_{j_1j_2}\frac{f^{\lambda}_{j_1j_2}v^{(-)}_{j_1j_2}}{\sqrt{Y(\lambda i)}}\left\{\left(Q^+_{\lambda\mu i}(-)^{\lambda-\mu}+Q_{\lambda-\mu i}\right)B(j_1j_2\lambda-\mu)+h.c.\right\}-$$

$$-\frac{1}{4}\sum_{LMi'i}\frac{1}{\sqrt{Z(Li)\,Z(Li')}}\sum_{j_1j_2}\frac{\left(\tilde{f}^{L}_{j_1j_2}u^{(-)}_{j_1j_2}\right)^2(\varepsilon_{j_1}+\varepsilon_{j_2})}{(\varepsilon_{j_1}+\varepsilon_{j_2})^2-\omega^2_{Li}}\left[\Delta^+_{L-Mi'}+(-)^{L-M}\Delta_{LMi'}\right]\left[\Delta^+_{LMi}(-)^{L-M}+\Delta_{L-Mi}\right]-$$

$$-\frac{1}{2\sqrt{2}}\sum_{LMi}\sum_{j_1j_2}\frac{\tilde{f}^{L}_{j_1j_2}v^{(+)}_{j_1j_2}}{\sqrt{Z(Li)}}\left\{\left(\Delta^+_{LMi}(-)^{L-M}+\Delta_{L-Mi}\right)B(j_1j_2L-M)+h.c.\right\}\qquad(2)$$

Here the following notation is used: G_τ the pairing constants for the neutron and proton systems, which are determined in ref. /6/ from the nuclear mass differences, α^+_{jm} and α_{jm} the creation and annihilation operators for a quasiparticle with quantum numbers $n\ell j\equiv j$ and the momentum projection m ; ε_j - the quasiparticle energy; $Q^+_{\lambda\mu i}$, $Q_{\lambda\mu i}$; Δ^+_{LMi}, Δ_{LMi}, Ω^+_i, Ω_i - the creation and annihilation operators of a multipole, spin-multipole and pairing phonon, respectively; λ and L the phonon moments and μ, M their projections. The index i labels the numbers of phonons. By $\omega_{\lambda i}$, ω_{Li} and ω_i we denote the phonon energies

$$B(j_1j_2\lambda\mu)=\sum_{m_1m_2}\langle j_1m_1j_2m_2|\lambda\mu\rangle(-)^{j_2+m_2}\alpha^+_{j_1m_1}\alpha_{j_2-m_2}$$

$\langle j_1m_1j_2m_2|\lambda\mu\rangle$ is the Clebsh-Gordon coefficient. Here and in what follows when determining different coefficients of the vector sums, we follow A.R.Edmonds /7/.

$B(j)=(2j+1)^{1/2}B(jj00)$; $f^{\lambda}_{j_1j_2}$ and $\tilde{f}^{L}_{j_1j_2}$ are the reduced matrix elements of the multipole and spin-multipole operators, respectively. The phonon energies are determined from the following equations:

a) for multipole phonons

$$\frac{2\lambda+1}{\varkappa_\lambda} = \sum_{j_1 j_2} \frac{\left(f^{\lambda}_{j_1 j_2} u^{(+)}_{j_1 j_2}\right)^2 (\varepsilon_{j_1}+\varepsilon_{j_2})}{(\varepsilon_{j_1}+\varepsilon_{j_2})^2-\omega^2_{\lambda i}} \equiv X(\omega_{\lambda i})$$

b) for spin-multipole phonons:

$$\frac{2L+1}{\varkappa_L} = \sum_{j_1 j_2} \frac{\left(\tilde{f}^{L}_{j_1 j_2} u^{(-)}_{j_1 j_2}\right)^2 (\varepsilon_{j_1}+\varepsilon_{j_2})}{(\varepsilon_{j_1}+\varepsilon_{j_2})^2-\omega^2_{L i}} \equiv \tilde{X}(\omega_{L i})$$

c) for pairing vibrational phonons:

$$\left\{\sum_j \frac{(u_j^2-v_j^2)2\varepsilon_j}{4\varepsilon_j^2-\omega_i^2} - \frac{1}{G_\tau}\right\}\cdot\left\{\sum_j \frac{2\varepsilon_j}{4\varepsilon_j^2-\omega_i^2} - \frac{1}{G_\tau}\right\} -$$

$$-\omega_i^2\left\{\sum_j \frac{u_j^2-v_j^2}{4\varepsilon_j^2-\omega_i^2}\right\}^2 \equiv T_i P_i - W_i^2 \equiv \Phi(\omega_i) = 0 \ .$$

Further

$$Y(\lambda i) = \frac{1}{2}\frac{\partial}{\partial\omega} X(\omega) \qquad \text{for } \omega=\omega_{\lambda i}$$

$$Z(Li) = \frac{1}{2}\frac{\partial}{\partial\omega} \tilde{X}(\omega) \qquad \text{for } \omega=\omega_{L i}$$

$$u^{(\pm)}_{j_1 j_2} = u_{j_1} v_{j_2} \pm u_{j_2} v_{j_1} \ , \quad v^{(\pm)}_{j_1 j_2} = u_{j_1} u_{j_2} \pm v_{j_1} v_{j_2}$$

In what follows we do not distinguish the notation of different phonons which are generated by different forces and everywhere write the corresponding creation and annihilation operators as $Q^{+}_{\lambda\mu i}, Q_{\lambda\mu i}$

Using the Hamiltonian (2) we consider the problem of finding the energy and wave function of the highly excited state. The wave function is sought in the form

$$\Psi_\nu(JM) = C_J^\nu \Big\{ \alpha^{+}_{JM} + \sum_{\lambda i j} D^{\lambda i}_j(J\nu) \sum_{m\mu} \langle \lambda\mu jm | JM\rangle \alpha^{+}_{jm} Q^{+}_{\lambda\mu i} +$$

$$+ \sum_{\substack{\lambda_1 i_1 \lambda_2 i_2 \\ jI}} F^{\lambda_1 i_1 \lambda_2 i_2}_{jI}(J\nu) \sum_{\substack{\mu_1 \mu_2 \\ m M'}} \langle \lambda_1\mu_1 \lambda_2\mu_2 | IM'\rangle \langle IM' jm | JM\rangle \alpha^{+}_{jm} Q^{+}_{\lambda_1\mu_1 i_1} Q^{+}_{\lambda_2\mu_2 i_2} + \qquad (3)$$

$$+ \sum_{\substack{\lambda_1 i_1 \lambda_2 i_2 \\ \lambda_3 i_3 j I_1 I_2}} R^{\lambda_1 i_1 \lambda_2 i_2 \lambda_3 i_3}_{jI_1I_2}(J\nu) \sum_{\substack{\mu_1 \mu_2 m \\ \mu_3 M_1 M_2}} \langle \lambda_1\mu_1 \lambda_2\mu_2 | I_1 M_1\rangle \langle I_1 M_1 \lambda_3\mu_3 | I_2 M_2\rangle \langle I_2 M_2 jm | JM\rangle \alpha^{+}_{jm} \times$$

$$\times Q^{+}_{\lambda_1\mu_1 i_1} Q^{+}_{\lambda_2\mu_2 i_2} Q^{+}_{\lambda_3\mu_3 i_3} \Big\} \Psi_0$$

where Ψ_0 is the wave function of the ground state of an even-even nucleus, ν is the number of the state. The normalization condition for the wave function reads:

$$[C_J^\nu]^2 \Big\{ 1 + \sum_{\lambda j i} [D^{\lambda i}_j(J\nu)]^2 + 2\sum_{\substack{\lambda_1 i_1 \lambda_2 i_2 \\ jI}} [F^{\lambda_1 i_1 \lambda_2 i_2}_{jI}(J\nu)]^2 + 6\sum_{\substack{\lambda_1 i_1 \lambda_2 i_2 \\ \lambda_3 i_3 j I_1 I_2}} [R^{\lambda_1 i_1 \lambda_2 i_2 \lambda_3 i_3}_{jI_1I_2}(J\nu)]^2 \Big\} = 1 \qquad (4)$$

In just the same way as in ref.[4] we calculate the average value of(2)over the state (3), and, on the basis of the variational principle, find the equations that determine the energy $\eta_{J\nu}$ of the state $J\nu$ and the coefficients C,D,F and R. When calculating different matrix elements of the Hamiltonian we put $[B(j_1 j_2 \lambda\mu) Q^{+}_{\lambda'\mu' i'}] = 0$. This condition implies that we have neglected the terms of the type $\sum f^{\lambda}_{j_1 j_2} \times \langle Q_{\lambda'\mu' i'} B(j_1 j_2 \lambda\mu) Q^{+}_{\lambda''\mu'' i''}\rangle$ containing the sum of the products of three different matrix elements. These terms are small compared to those which we have taken into account. It worth noting that in further studies of the model the effect of the rejected terms on the quantities $\eta_{J\nu}$, C, D, F and R should be studied.

The calculations for the wave function coefficients (3) and the state energy have resulted in a system of nonlinear equations. We have transformed it in such a manner that there remained only the following equations linking only $\eta_{J\nu}$ and $F_{jI}^{\lambda_1 i_1 \lambda_2 i_2}(J\nu)$:

$$\mathcal{E}_J - \eta_{J\nu} - \frac{1}{2}\sum_{\lambda_1 i_1 j_1} \frac{[\Gamma(Jj_1\lambda_1 i_1)]^2}{\mathcal{E}_{j_1}+\omega_{\lambda_1 i_1}-\eta_{J\nu}} + \tag{5}$$

$$+\sum_{\substack{\lambda_1 i_1 \lambda_2 i_2\\ j_1 j_2}} \frac{\Gamma(Jj_1\lambda_1 i_1)\Gamma(j_1 j_2 \lambda_2 i_2)}{\mathcal{E}_{j_1}+\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}-\eta_{J\nu}} \sum_{I} F_{j_2 I}^{\lambda_1 i_1 \lambda_2 i_2}(J\nu)\,[(2I+1)(2j_1+1)]^{1/2} \begin{Bmatrix} \lambda_1 & \lambda_2 & I \\ j_2 & J & j_1 \end{Bmatrix} = 0$$

$$F_{j_2 I}^{\lambda_1 i_1 \lambda_2 i_2}(J\nu)\left\{\mathcal{E}_{j_2}+\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}-\sum_{j_3\lambda_3 i_3}\frac{[\Gamma(j_2 j_3\lambda_3 i_3)]^2}{\mathcal{E}_{j_3}+\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\omega_{\lambda_3 i_3}-\eta_{J\nu}}-\eta_{J\nu}\right\}-$$

$$-\frac{1}{2}\sum_{\substack{\lambda_3 i_3\\ j_1 j_3}}\frac{\Gamma(j_1 j_2\lambda_2 i_2)\Gamma(j_1 j_3\lambda_3 i_3)}{\mathcal{E}_{j_1}+\omega_{\lambda_1 i_1}-\eta_{J\nu}}\sum_{I'} F_{j_3 I'}^{\lambda_1 i_1 \lambda_3 i_3}(J\nu)(2j_1+1)[(2I+1)(2I'+1)]^{1/2}\begin{Bmatrix} j_1 & \lambda_1 & J \\ I & j_2 & \lambda_2 \end{Bmatrix}\begin{Bmatrix} j_1 & \lambda_1 & J \\ I' & j_3 & \lambda_3 \end{Bmatrix}- \tag{6}$$

$$-\frac{1}{2}\sum_{\substack{\lambda_3 i_3\\ j_1 j_3}}\frac{\Gamma(j_1 j_2\lambda_1 i_1)\Gamma(j_1 j_3\lambda_3 i_3)}{\mathcal{E}_{j_1}+\omega_{\lambda_2 i_2}-\eta_{J\nu}}\sum_{I'} F_{j_3 I'}^{\lambda_2 i_2 \lambda_3 i_3}(J\nu)(2j_1+1)[(2I+1)(2I'+1)]^{1/2}(-)^{\lambda_1+\lambda_2+I}\begin{Bmatrix} j_1 & \lambda_2 & J \\ I & j_2 & \lambda_1 \end{Bmatrix}\begin{Bmatrix} j_1 & \lambda_2 & J \\ I' & j_3 & \lambda_3 \end{Bmatrix}-$$

$$-\frac{1}{2}\sum_{\substack{\lambda_3 i_3\\ j_3 j_\nu}}\frac{\Gamma(j_2 j_3\lambda_3 i_3)\Gamma(j_\nu j_3\lambda_2 i_2)}{\mathcal{E}_{j_3}+\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\omega_{\lambda_3 i_3}-\eta_{J\nu}}\sum_{I'} F_{j_\nu I'}^{\lambda_1 i_1 \lambda_3 i_3}(J\nu)[(2I+1)(2I'+1)(2j_2+1)(2j_\nu+1)]^{1/2}\times\begin{Bmatrix} \lambda_1 & \lambda_2 & I \\ \lambda_3 & j_3 & j_2 \\ I' & j_\nu & J \end{Bmatrix}\times$$

$$\times(-)^{\lambda_2+\lambda_3+j_2+j_\nu}-$$

$$-\frac{1}{2}\sum_{\substack{\lambda_3 i_3\\ j_3 j_\nu}}\frac{\Gamma(j_2 j_3\lambda_3 i_3)\Gamma(j_\nu j_3\lambda_1 i_1)}{\mathcal{E}_{j_3}+\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\omega_{\lambda_3 i_3}-\eta_{J\nu}}\sum_{I'} F_{j_\nu I'}^{\lambda_2 i_2 \lambda_3 i_3}(J\nu)[(2I+1)(2I'+1)(2j_2+1)(2j_\nu+1)]^{1/2}\begin{Bmatrix} \lambda_2 & \lambda_1 & I \\ \lambda_3 & j_3 & j_2 \\ I' & j_\nu & J \end{Bmatrix}\times$$

$$\times(-)^{\lambda_2+\lambda_3+j_2+j_\nu+I} =$$

$$=\sum_{j_1}\frac{\Gamma(Jj_1\lambda_1 i_1)\Gamma(j_1 j_2\lambda_2 i_2)}{\mathcal{E}_{j_1}+\omega_{\lambda_1 i_1}-\eta_{J\nu}}[(2j_1+1)(2I+1)]^{1/2}\begin{Bmatrix} j_2 & \lambda_2 & j_1 \\ \lambda_1 & J & I \end{Bmatrix}+$$

$$+\frac{\Gamma(Jj_1\lambda_2 i_2)\Gamma(j_1 j_2\lambda_1 i_1)}{\mathcal{E}_{j_1}+\omega_{\lambda_2 i_2}-\eta_{J\nu}}[(2j_1+1)(2I+1)]^{1/2}\begin{Bmatrix} j_2 & \lambda_1 & j_1 \\ \lambda_2 & J & I \end{Bmatrix}(-)^{\lambda_1+\lambda_2+I}.$$

In eqs.(5) and (6) $\Gamma(j_1 j_2 \lambda i)$ denote the following quantities

$\Gamma(j_1 j_2 \lambda i) = \left(\frac{2\lambda+1}{2j_1+1}\right)^{1/2}\frac{1}{\sqrt{Y(\lambda i)}} f_{j_1 j_2}^{\lambda} v_{j_1 j_2}^{(-)}$, provided the phonon λi is a multipole one,

$\Gamma(j_1 j_2 \lambda i) = \left(\frac{2\lambda+1}{2j_1+1}\right)^{1/2}\frac{1}{\sqrt{Z(Li)}} \tilde{f}_{j_1 j_2}^{\lambda} v_{j_1 j_2}^{(+)}$, provided the phonon λi is a spin-multipole one,

$\Gamma(j_1 j_2 \lambda i) = \Gamma(j_1 j_1 0 i) = \frac{1}{G_N} u_{j_1 j_1}\sqrt{\frac{T_i}{\phi'(\omega_i)}}$, provided the phonon λi is a pairing vibrational one.

Now we indicate the expressions for D and R in terms of F and $\eta_{J\nu}$:

$$D_{j_1}^{\lambda_1 i_1}(J\nu) = \frac{1}{\sqrt{2}}\,\frac{\Gamma(Jj_1\lambda_1 i_1)}{\mathcal{E}_{j_1}+\omega_{\lambda_1 i_1}-\eta_{J\nu}} +$$

$$+\frac{\sqrt{2}}{\mathcal{E}_{j_1}+\omega_{\lambda_1 i_1}-\eta_{J\nu}}\sum_{j_2\lambda_2 i_2}\Gamma(j_1 j_2\lambda_2 i_2)\sum_{I} F_{j_2 I}^{\lambda_1 i_1 \lambda_2 i_2}(J\nu)\begin{Bmatrix} \lambda_1 & \lambda_2 & I \\ j_2 & J & j_1 \end{Bmatrix}[(2I+1)(2j_1+1)]^{1/2} \tag{7}$$

$$R_{j_1 I_1 I_2}^{\lambda_1 i_1 \lambda_2 i_2 \lambda_3 i_3}(J\nu) = \frac{1}{3\sqrt{2}} \frac{1}{\mathcal{E}_{j_1}+\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\omega_{\lambda_3 i_3}-\eta_{J\nu}} \times$$

$$\times \left\{ \sum_{j_2 j_3} \Gamma(j_2 j_3 \lambda_3 i_3) F_{j_2 I_1}^{\lambda_1 i_1 \lambda_2 i_2}(J\nu) \left[(2I_2+1)(2j_2+1)\right]^{1/2} (-)^{j_2+J+I_1} \begin{Bmatrix} \lambda_3 & I_1 & I_2 \\ j_3 & J & j_2 \end{Bmatrix} + \right.$$

$$+ \sum_{j_2 j_3} \Gamma(j_2 j_3 \lambda_2 i_2) \sum_{I} F_{j_2 I}^{\lambda_1 i_1 \lambda_3 i_3}(J\nu) \left[(2I_2+1)(2I+1)(2I_1+1)(2j_2+1)\right]^{1/2} (-)^{j_2+J} (-)^{\lambda_2+\lambda_3+I_1} \times$$

$$\times \begin{Bmatrix} \lambda_2 & I & I_2 \\ \lambda_3 & I_1 & \lambda_1 \end{Bmatrix} \begin{Bmatrix} I & \lambda_1 & I_2 \\ j_3 & J & j_2 \end{Bmatrix} + \qquad (8)$$

$$+ \sum_{j_2 j_3} \Gamma(j_2 j_3 \lambda_1 i_1) \sum_{I} F_{j_2 I}^{\lambda_2 i_2 \lambda_3 i_3}(J\nu) \left[(2I_2+1)(2I+1)(2I_1+1)(2j_2+1)\right]^{1/2} (-)^{j_2+J+\lambda_2+\lambda_3} \times$$

$$\left. \times \begin{Bmatrix} \lambda_1 & I & I_2 \\ \lambda_3 & I_1 & \lambda_2 \end{Bmatrix} \begin{Bmatrix} I & \lambda_1 & I_2 \\ j_3 & J & j_2 \end{Bmatrix} \right\}$$

Equations (5) and (6) take a more simple form if in the wave function (3) we merely restrict ourselves to one- and two-phonon terms, i.e. when $R = 0$. Then the system will have the form

$$\mathcal{E}_J - \eta_{J\nu} - \frac{1}{\sqrt{2}} \sum_{\lambda i j} D_j^{\lambda i}(J\nu) \Gamma(J j \lambda i) = 0 \qquad (9)$$

$$D_{j_1}^{\lambda_1 i_1}(J\nu) \left(\mathcal{E}_{j_1} + \omega_{\lambda_1 i_1} - \eta_{J\nu} - \frac{1}{2} \sum \frac{[\Gamma(j_1 j_2 \lambda_2 i_2)]^2}{\mathcal{E}_{j_2}+\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}-\eta_{J\nu}} \right) -$$

$$- \frac{1}{2} \sum_{j_2 \lambda_2 i_2} \frac{\Gamma(j_1 j_2 \lambda_2 i_2) \Gamma(j_3 j_2 \lambda_1 i_1)}{\mathcal{E}_{j_2}+\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}-\eta_{J\nu}} D_{j_3}^{\lambda_2 i_2}(J\nu) (-)^{\lambda_2+\lambda_1+j_1+j_3} \left[(2j_1+1)(2j_3+1)\right]^{1/2} \begin{Bmatrix} \lambda_2 & j_2 & j_1 \\ \lambda_1 & J & j_3 \end{Bmatrix} = \qquad (10)$$

$$= \frac{1}{\sqrt{2}} \Gamma(J j_1 \lambda_1 i_1)$$

Notice that eq.(9) is valid for arbitrarily complicated wave function (3), i.e. when one takes into account the contribution of not only two but also three-, four- and higher phonon terms.

The solution of eqs.(5) and (6) (as that of eqs.(9) and (10)) is a very troublesome problem. This appears to be practically possible only in the framework of some assumptions. In ref. /4/ an approach is considered according to which in eqs.(5) and (6) there are kept only the so-called coherent terms, i.e. those connected with the squared matrix elements. In this case eq.(10) takes the form

$$D_{j_1}^{\lambda_1 i_1}(J\nu) \left[\mathcal{E}_{j_1} + \omega_{\lambda_1 i_1} - \eta_{J\nu} - \frac{1}{2} \sum_{\lambda_2 i_2 j_2} \frac{[\Gamma(j_1 j_2 \lambda_2 i_2)]^2}{\mathcal{E}_{j_2}+\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}-\eta_{J\nu}} \times \right.$$

$$\left. \times \left(1 - \delta_{\lambda_1 \lambda_2} \delta_{i_1 i_2} (2j_1+1) \begin{Bmatrix} \lambda_1 & j_2 & j_1 \\ \lambda_1 & J & j_1 \end{Bmatrix} \right) \right] = \frac{1}{\sqrt{2}} \Gamma(J j_1 \lambda_1 i_1) . \qquad (11)$$

Inserting (11) in (9) we obtain a secular equation for :

$$\mathcal{E}_J - \eta_{J\nu} = \frac{1}{2} \sum_{j_1 \lambda_1 i_1} \frac{[\Gamma(J j_1 \lambda_1 i_1)]^2}{\mathcal{E}_{j_1}+\omega_{\lambda_1 i_1}-\eta_{J\nu} - \frac{1}{2} \sum_{j_2 \lambda_2 i_2} \frac{[\Gamma(j_1 j_2 \lambda_2 i_2)]^2}{\mathcal{E}_{j_2}+\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}-\eta_{J\nu}} \left\{ 1 - \delta_{\lambda_1 \lambda_2} \delta_{i_1 i_2} \begin{Bmatrix} \lambda_1 & j_1 & j_2 \\ \lambda_1 & j_1 & J \end{Bmatrix} (2j_1+1) \right\}} . \qquad (12)$$

However the neglect of the coherent terms leaves a trance. Among the solutions for eq. (12) there is a number of unphysical "redundant" solutions. The causes of appearance and the character of these "false" roots is discussed in ref. /4/ Here we merely want to stress that a consistent exclusion of such solutions is a very cumbersome problem (even for eqs.(9)

and (11)). This problem is a subject for separate studies.

We pass to the questions the solution of which within the framework of the model considered is possible already now. The suggested model for the description of highly excited states of spherical atomic nuclei is a concrete realization of a more general approach to this problem which is most completely presented in ref./8/. Instead of the many-quasiparticle wave function used in ref./8/ we consider a many-phonon wave function by replacing a pair of quasiparticles by one phonon with the corresponding momentum and parity. The question arises as to how it is possible to obtain such a phonon, how to try all possible two-quasiparticle states with arbitrary momentum using the language of phonon excitations. To this end it is necessary to introduce residual forces which will generate phonons. In a deformed nucleus a multipole-multipole interaction suffices practically to obtain excitations with needed momenta. While in a spherical nucleus it is necessary to use in addition another forces since the multipole-multipole interaction can generate only such excitations in which $J^{\pi} = \lambda^{(-)^{\lambda}}$. To obtain the remaining excitations use can be made of spin-multipole forces. The spin-multipole forces $\varkappa_L [\vec{\sigma} \times Y_\lambda]_L f(r)$ give rise to phonon excitations with moments $L = \lambda, \lambda \pm 1$ and parity $\pi = (-)^{\lambda}$. However the two-quasiparticle structure of spin-multipole phonons with $J^{\pi} = \lambda^{(-)^{\lambda}}$ essentially differs from that of spin-multipole phonons with the same quantum numbers,- they do not contain diagonal two-quasiparticle components. Therefore the phonons $2^+, 3^-, 4^+$ and so on should be obtained by

means of multipole-multipole forces while the phonons $2^-; 3^+, 4^-$ and so on by means of spin-multipole-spin-multipole forces. As was already mentioned, the 0^+ states are obtained by means of the pairing superfluid interaction. The introduction of new forces does not lead to new constants. These constants are determined by specifying the energy of the lowest phonon with a given J^{π} which in turn is chosen to be equal to the energy of the lowest two-quasiparticle state with the same quantum numbers.

Within the framework of the model, even without solving eqs.(5) and (6) we can calculate the characteristics of the highly excited part of the spectrum of odd-mass spherical nuclei. Here we may use the fact that the number of poles of eqs.(5) and (6) in a certain energy interval $(E, E+\Delta E)$ coincides with the number of the poles of the equations defining the poles of eq.(6) in the same interval. The energies of the latter poles are $\varepsilon_j + \omega_{\lambda i}$, $\varepsilon_j + \omega_{\lambda i} + \omega_{\lambda' i'}$. By trying all such states with given J^{π} in the energy interval $(E, E+\Delta E)$ we may determine the excited state density. This way is just employed to study the state density as a function of the spin, the excitation energy, etc. In most of spherical nuclei the wave function (3) is not sufficiently complicated for the excitation state density near the neutron binding energy to be well described. It is necessary to take into account components with a larger number of phonons. However, in nuclei around closed shells the wave function (3) includes a sufficient number of components for these calculations to be performed.

If we take into account phonons up to large multipole and spin-multipole moments and many roots of secular equations for phonons, then the wave function (3) will contain thousands of different components. Among them several components are expected to be large enough. The wave function (3) is essentially the one of the compound state. In such a way we achieve a strong manifestation of the fragmentation process that is characteristic of the intermediate energy state.

We may hope that the finding of the solutions for eqs. (5) and (6), or simpler eqs. (9) and (10) and their study make it possible to clarify the main features of this process. This in turn will give us a possibility of tracing the way of constructing a theory of highly excited states of atomic nuclei.

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