ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА

E4 - 6989

V.K.Lukyanov, A.I.Titov

EXCITATION OF NUCLEI NEAR THE COULOMB BARRIER

1973

ЛАБОРАТОРИЯ
ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

E4 - 6989

V.K.Lukyanov, A.I.Titov

EXCITATION OF NUCLEI NEAR THE COULOMB BARRIER

Submitted to ЯФ

Лукьянов В.К., Титов А.И. **E4 - 6989**

Возбуждение ядер частицами околоарьерной энергией

Для исследования возбуждения ядер с энергией в районе кулоновского барьера используется квазиклассический вариант метода связанных каналов. Вероятность возбуждения низколежащих коллективных состояний оказывается очень чувствительной к изменению параметров ядерных потенциалов и к величинам вероятностей электромагнитных переходов. Проведено сравнение с результатами расчетов по теории возмущений и имеющимися экспериментальными данными.

Препринт Объединенного института ядерных исследований.
Дубна, 1973

Lukyanov V.K., Titov A.I. **E4 - 6989**

Excitation of Nuclei Near the Coulomb Barrier

The coupled channel method together with the trajectory description of the projectile movement are used in investigating the excitation of nuclei near the Coulomb barrier. It was found out that the excitation probabilities are very sensitive to the change of nuclear potential parameters and electromagnetic transition probabilities. The numerical calculations are in a good comparison with existing experimental data.

Preprint. Joint Institute for Nuclear Research.
Dubna, 1973

1. Introduction.

With approaching of the heavy ion energy the Coulomb barrier, the nuclear excitation mechanism of a target-nucleus /1,2/ is added to the usual Coulomb one /3/. This leads to an interesting interference phenomena in the energy (near $E \sim U_B$) and angular (near $\vartheta \approx \vartheta_c = 2\arcsin\left[\frac{U_B}{2E-U_B}\right]$)distributions of inelastic scattering particles, which depend very sensitively on the nuclear interaction potential. The first theoretical investigations of this effect in the framework of the perturbation theory were carried out long ago/3-6/. In the works /3,5,6/ an attention was accented on the near-barrier region, where the nuclear forces only begin to arise, and therefore the typical variations from the smooth Coulomb excitation curves should appear in the inelastic cross-sections. Semi-classical calculations of inelastic scattering of the above-barrier particles have been given in ref./4/. Later /7,8/ the first order perturbation method was extended by means of the accounting the higher step excitations, which play an important role in the case of deformed nuclei (the so-called multiple near-barrier excitation).

Recently the experimental measurements in the near-barrier region were done with the oxyden /9/ and lithium ions/10/, scattered on the spherical nuclei ^{58}Ni and ^{120}Sn. Indeed, the interference effect was found out and the data /9/ were interpreted successfully in the framework of the perturbation method /11/. The purpose of this work is to give a detailed analysis and the comparisons of the heavy ion inelastic scattering just on the spherical nuclei, taking into account some specific features of this case in comparison with the case of deformed nuclei which has been investigated earlier in ref./7/. The consideration is in the framework of the strong coupled channel method. Also we give some methodical analysis with the aim of understanding the main physical details of this phenomena. Namely, the role of the higher step excitations, the dependence on the nuclear state energies and the corresponding transition probabilities $B(E\lambda)$, an influence of the nuclear potential parameter variations, etc., are under consideration.

2. Excitation probabilities

The heavy ion scattering process can be investigated in the framework of the semi-classical approximation because of the relation $\bar{\lambda}/R \ll 1$. For inelastic scattering this method consists of two steps. The first is the calculation of the classical trajectory $\vec{r}=\vec{r}(t)$ of projectiles in the field of an average Coulomb and the nuclear potential $\overline{U}(r)$. Then the function $\vec{r}(t)$ is inserted into the residual interaction $U_{int}(\vec{r}\xi)$, which therefore depends on the time and the internal nuclear coordinates ξ. This interaction plays a role of external field which leads to the nuclear excitation. Thus the nuclear wave function satisfies the following equation:

$$i\hbar\frac{\partial\psi(\xi t)}{\partial t}=\left[H_o(\xi)+U_{int}(\vec{r}(t)\xi)\right]\psi(\xi t). \qquad (1)$$

It is natural to express the solution as an expansion on the nuclear wave functions of the Hamiltonian without an external field

$$\psi(\xi t)=\sum_{\nu} a_{\nu}(t)\, e^{-i(E_\nu/\hbar)t}\,|\nu\rangle, \qquad (2)$$

$$H_o|\nu\rangle = E_\nu|\nu\rangle. \qquad (3)$$

Then the coupled channel equations are obtained

$$i\hbar\frac{\partial a_\nu}{\partial t}=\sum_{\nu'} e^{i\omega_{\nu\nu'}t}\langle\nu|U_{int}|\nu'\rangle a_{\nu'}, \qquad (4)$$

where

$$\omega_{\nu\nu'}=(E_\nu-E_{\nu'})/\hbar. \qquad (5)$$

Coefficients a_ν are the probability amplitudes of finding the nucleus in the states $|\nu\rangle$ at the moment t, so the result of scattering is determined by the amplitudes $a_\nu(t=+\infty)$. Before scattering $(t=-\infty)$, the nucleus is in the ground state $|0\rangle$, therefore

$$a_\nu(t=-\infty)=\delta_{\nu 0}. \qquad (6)$$

This is the boundary condition when solving the eqs. (4).

In the case of the phonon excitations of even nuclei $|\nu\rangle=|nJM\rangle$, where n is the phonon number, J is the state momentum and M - stands for its projection. Bearing in mind that the ground state $|000\rangle$ is the zero phonon and momentum state we write down the excitation probability of the (nJ)- nuclear level

$$P_{nJ}=\sum_M |a_{nJM}(t=+\infty)|^2. \qquad (7)$$

In the first order the amplitudes are expressed as follows

$$P_{nJ}^{p.th.}=\sum_M\left|\frac{1}{\hbar}\int_{-\infty}^{+\infty}dt\, e^{i(E_{nJ}/\hbar)t}\langle nJM|U_{int}|000\rangle\right|^2 \qquad (8)$$

Neglecting the change of trajectory due to the inelastic recoil effect, the excitation probability is expressed in terms of the elastic and inelastic cross sections in a simple way /2/

$$P_{nJ}=\frac{d\sigma_{inel}(\vartheta, E-E_{nJ}, nJ)}{d\sigma_{el}(\vartheta, E, 00)}. \qquad (9)$$

Thus in practice it is necessary to solve the system (4) with the boundary conditions (6), or to calculate the perturbation theory integral in eq. (8). In both cases we should write down an obvious expression for the interaction potential matrix elements.

As a rule for this aim we obtain the average $\overline{U}$ and interaction U_{int} potentials with the help of the multiple expansion of the whole interaction potential

$$U(\vec{r}) = Z_1 e \int \frac{\rho_c(\vec{x}\xi)}{|\vec{r}-\vec{x}|} d\vec{x} - U_0 f_V(\vec{r}\xi) - i W_0 f_W(\vec{r}\xi), \tag{10}$$

where the nuclear charge density is

$$\rho_c = \frac{3Z_2 e}{4\pi R_c^3} \theta(\mathcal{R}(\vec{x}\xi) - x) \tag{11}$$

($\theta(x)=1$ for $x>0$ and $\theta(x)=0$ for $x<0$)

and radial dependence of the nuclear potential is

$$f(\vec{r}\xi) = \left[1 + \exp\frac{r - \mathcal{R}(\vec{r}\xi)}{b}\right]^{-1} \tag{12}$$

$$\mathcal{R} = R + \Delta \qquad \Delta = R \sum_{\lambda\mu} \xi_{\lambda\mu} Y_{\lambda\mu} \tag{13}$$

$$R_c = r_{oc} A_2^{1/3} \qquad R_V = r_{oV}(A_1^{1/3} + A_2^{1/3}) \qquad R_W = r_{oW}(A_1^{1/3} + A_2^{1/3}). \tag{14}$$

Carrying out the excitation probability calculations one should know the projectile trajectory. The latter is determined by an average potential which is a zero term in expanding eq. (10) on $\Delta (\ll R)$. Note, that an imaginary potential must be accounted only in the cross section calculations. Therefore there appears the "absorbtion factor", the result of many other outgoing channels, which are obviously not taken into account. If one suggests as usual, that the imaginary potentials of the elastic and inelastic channels to be equal, the corresponding cross sections would have the same absorbtive factors, the fact which does not change the ratio of elastic and inelastic

cross sections. Thus, using eq. (9) the excitation probabilities are expressed directly by the experimental cross sections.

In the region $E \sim U_B$ external Coulomb potential plays an important role and the "tail" of the nuclear potential should be added. Therefore the main projectile path is the Coulomb one and only near the nuclear surface the trajectory is partly distorted. This distortion can be taken into account/4/, by renormalization of the Coulomb trajectory, representing an average potential near the nucleus as follows

$$U(r) = \frac{Z_1 Z_2 e^2}{r} + (A + B/r + C/r^2)\theta(r_m + 2\delta - r)\theta(r - r_m). \tag{15}$$

The coefficients A, B, C are determined by fitting eq. (15) to eq. (10) at $\Delta=0$ in the region of the main contribution of nuclear forces:

$$r_m \le r \le r_m + 2\delta \quad \left(r_m = a(1 + \sin^{-1}\vartheta/2) \approx R\,;\ \delta \ll R\,;\ a = \frac{Z_1 Z_2 e^2}{2E}\right).$$

Then the trajectory will be characterized by an excentricity, connected with an observed scattering angle ϑ by the following expression

$$\epsilon_r = \sqrt{\sin^{-2}(\vartheta/2) + C/(E_r a_r^2)}, \tag{16}$$

where a half distance of the nearest approach is

$$a_r = Z_1 Z_2 e_r^2 / 2E_r \tag{17}$$

and

$$E_r = E - A \qquad e_r^2 = e^2 + B/Z_1 Z_2. \tag{18}$$

Out of the nuclear forces $A = B = C = 0$ and therefore the trajectory becomes pure Coulomb one with an excentricity $\epsilon = \sin^{-1}(\vartheta/2)$.

The potential U_{int} is obtained as the first term of an expansion eq. (10) on $\Delta(\ll R)$. Representing it in the form

$$U_{int} = \sum_{\lambda\mu} V_{\lambda\mu}(r,\xi)\, Y^{*}_{\lambda\mu}(\hat{r}) \qquad (19)$$

and parametrizing the trajectory in the coordinate system with the scattering plane xy and a symmetry axis along x /1/ it is possible to carry out a final expression for the coupled channel system (4):

$$i\hbar \frac{\partial a_{nJM}}{\partial \omega} = \sum_{n'J'M'} a_{n'J'M'}\, e^{i\omega_{nJ;n'J'} a_r(\epsilon_r sh\omega + \omega)/v_r}$$

$$\frac{r}{v_r} \langle J'\lambda M'\mu | JM\rangle \frac{\langle J \| \mathcal{M}_\lambda \| J'\rangle}{\sqrt{2J+1}} V_\lambda(r)\, Y^{*}_{\lambda\mu}\left(\frac{\pi}{2}, \varphi\right), \qquad (20)$$

where r and φ are expressed through the integration parameter ω as follows

$$r = a_r(\epsilon_r ch\omega + 1)$$
$$\cos\varphi = \epsilon_r^{-1} - a_r(\epsilon_r^{-1} - \epsilon_r)/r \qquad (21)$$

and the multiple part of an interaction is

$$V_\lambda(r) = V^{C}_\lambda(r) + V^{N}_\lambda(r). \qquad (22)$$

Here the Coulomb and nuclear terms are

$$V^{C}_\lambda = \frac{4\pi Z_1 e}{(2\lambda+1)\, r^{\lambda+1}} \qquad (23)$$

$$V^{N}_\lambda = -\left[U_0 R \frac{\partial f_v}{\partial R_v} + i W_0 R \frac{\partial f_w}{\partial R_w}\right] \frac{\langle J \| \xi_\lambda \| J'\rangle}{\langle J \| \mathcal{M}_\lambda \| J'\rangle} =$$

$$= -\left[U_0 R \frac{\partial f_v}{\partial R_v} + i W_0 R \frac{\partial f_w}{\partial R_w}\right] \frac{4\pi}{3 Z_2 e R_c^{\lambda}} \;; \qquad (R = r_{0v} A_2^{1/3}). \qquad (24)$$

3. Discussion of results

The dependence of the excitation probability P_{nJ} on the nuclear structure parameters and on the geometry parameters, and also a comparison of theoretical predictions with experimental data have been done on the basis of numerical solution of the coupled channel equations (20) and the perturbation theory calculations of eq.(8).

3.1. Dependence on nuclear structure parameters

The nuclear excitation probability P_{nJ} depends on two values, which are determined by the nuclear structure. The first value is the excited state energy E_{nJ}, equal to the transfer energy ΔE, which determines the dependence of P_{nJ} on the impact parameter

$$\xi = \frac{ak}{2} \frac{\Delta E}{E}. \qquad (25)$$

The second one is the transition $E\lambda$ -probability

$$B_{J\to J'}(E\lambda) = \frac{1}{2J+1} |\langle J \| \mathcal{M}_\lambda \| J'\rangle|^2. \qquad (26)$$

Application of the calculation methods depends mainly on magnitudes of these two parameters. Indeed, for the "small" $B(E\lambda)$ it is possible to use the perturbation method (8). However it is necessary to investigate, which of the magnitudes $B(E\lambda)$ may be called "small" ones, and what is the typical dependence of P_{nJ} on the nuclear state energy. Figures 1 and 2a show the results of the calculations of excitation probabilities in the framework of the perturbation theory and the coupled channel method for two levels (the ground

0^+ and one-phonon 2^+), related to the corresponding exact amplitudes. The phonon states have been considered to be equidistant, and in the exact calculations the degenerated two-phonon triplet $0^+2^+4^+$ has been taken into account with the corresponding probability transition $B_{1\to2}(E2)=\frac{2}{\sqrt{5}}B_{0\to1}(E2)$. The parameters are given in Table.

Two regions of the $B_{0\to1}(E2)$ magnitudes, corresponding to spherical and deformed nuclei, have been investigated. Naturally, in the latter case our calculations, based on phonon states consideration, can give only qualitative conclusions. Results are the following.

1. As is seen from Fig. 1, the perturbation theory cannot be used for spherical nuclei with $B(E2)>0.2$ (errors become more than 20%). And of course for all the deformed nuclei, where perturbation theory probabilities get off the exact value more than one order. Then, in these cases it is impossible to restrict ourselves only to two low-lying nuclear states in the coupled channel calculations.

2. Fig 2a. shows that the violations of perturbation calculations from exact ones are more significant at low energies of 2^+ level ($E_2<1$ MeV), than at large energies ($E_2\gg 2\div3$ MeV).The corresponding ratios of probabilities $P_{2^+}^{p.th.}/P_{2^+}^{c.c.}$ are displayed on different sides of the unity. But sometimes there exists a case, when the experimental magnitudes $B(E2)$ and E_{2^+} accidently occur just in the region, where this ratio is equal to 1 and the perturbation theory calculations coincide with an exact one.

3. Fig.3b shows, that with the increase of ΔE there is an "attenuation" of P_{2^+} -probabilities in the Coulomb excitation region $E<U_B$ and the corresponding increase of P_{2^+} in the nuclear excitation region $E\gg U_B$. This is due to a suppression factor under integrals (8) (or (20)) $\exp i(\Delta E t/\hbar)$, which develops itself only at $t\neq0$ and oscillates very quickly with the increase of ΔE . The magnitudes $t\neq0$ correspond to large distances on the projectile trajectory, far from the target nucleus, where the main contribution into an excitation is from the Coulomb potential only.

3.2 Dependence on nuclear potential parameters.

The behaviour of excitation probabilities P_{2^+} , depending on one of the nuclear potential parameters, is given in Figs. 3-5. Firstly, it occured that the position and depth of an interference minima are determined by r_{0V}, U_0 and b_V , but the main dependence is on the radius parameter. The dependence of excitation probability P_{2^+} on the imaginary potential parameters is weak and they play the role mainly at large scattering angles or energies, far from the interference minima. Their magnitudes are determined by experiments with a smaller accuracy, because the trajectory description is almost out of its application when the energy is above the Coulomb barrier. The general behaviour of curves supports an obvious conclusion that an extension of the nuclear force region via increase of r_{0V}, U_0 and b_V parameters leads to the nuclear excitation at larger distances from the target nucleus. Therefore the variations on smooth Coulomb excitation curves should be exhibited at smaller energies and scattering angles.

Inclusion of nuclear forces not only changes the nuclear transition potential, but also draws the trajectory nearer to the nucleus. The given above formulas allow us to account this effect by means of a simple renormalization of the Coulomb trajectory. Fig. 5b gives a comparison of two excitation probabilities P_{2+}, calculated with and without the trajectory distortion. One can see that the inclusion of an average nuclear potential considerably brings the trajectory nearer to the nucleus. In spite of the fact that the form of an interference picture is not changed qualitatively, the effect of the trajectory change should be taken into account in all the cases of quantitative calculations, the fact which has been taken into account in all present calculations.

3.3 Comparisons with the experimental data.

The theoretical calculations and experimental results on the near-barrier excitations in the reactions $^{6}Li + ^{58}Ni$, ^{120}Sn /10/ and $^{16}O + ^{58}Ni$ /9/ are compared in Fig. 6 and 7. One can see, that the calculated curves with the parameters from Table are in a good agreement with experimental data. The reaction $^{16}O + ^{58}Ni$ has been analyzed earlier in the framework of the perturbation theory in ref. /11/. The parameters of the work coinside with ours except for b_v, which occurs here to be smaller on 0.1 fermi. Bearing in mind, that in this case the perturbation theory is well working (see 3.1), this small discrepancy is a consequence of different methods of accounting the nuclear distortion.

Analyzing the obtained results, we note first of all that diffusness parameters b_v in the lithium reactions are found to be about one half as much again as in the reactions with ^{16}O.

This fact correlates with the known result on the "friable" structure of ^{6}Li. Then, all the potentials occur to be shallow ones in comparison with commonly used in the calculations of light particle scattering (n, p, α, etc.). It is possible to interprete this fact in the following way: these parameters characterize mainly the transition potential U_{int} and only the "tail" of the average nuclear potential $\overline{U}$. At any rate this fact is interesting to be analysed as the new data on the near-barrier excitation of nuclei appear.

Received by Publishing Department
on March 14, 1973.

References:

1. K.Alder, A.Bohr, T.Huus, B.Mottelson, A.Winther, Rev. Mod. Phys., 28, 432 (1956).
2. K.Alder, A.Winther, Mat.Fys.Medd.Dan.Vid.Selsk. 32, No.8 (1968)
3. V.K.Lukyanov, Izv. AN SSSR, ser. fis. 28, 1212, 1964.
4. B.N.Kalinkin, J.Grabovsky, Acta Phys.Polon. 29, 283, 1966.
5. C.Leclercq-Willain, Nucl.Reactions Induced by Heavy Ions. Proceedings of the International Conf. Heidelberg, 1969, p.543.
6. M.Samuel, U.Smilansky, Phys.Letters, 18B, 318, 1968.
7. V.K.Lukyanov, K.A.Gridnev, V.S.Zvonov, Phys.Letters 19B, 535, 1969.
8. C.Leclercq-Willain, J.Phys. 32, 833, 1971.
9. F.Videbaek, I.Chernov, P.R.Christensen, E.E.Gross, Phys. Rev. Lett. 28, 1072, 1972.
10. V.I.Chuev, Yu.A.Gluchov, V.I.Manko, B.G.Novatsky, A.A.Ogloblin, S.B.Sakuta, D.N.Stepanov, Phys.Letters 42B, 63, 1972.
11. R.A.Broglia, S.Landowne, A.Winther. Phys.Letters, 40B, 293, 1972.
12. R.A.Broglia, A.Winther, Nucl.Phys. 182, 112, 1972.

TABLE *)

Figure	Reaction	U_o MeV	r_{ov} MeV	b_v fm	W_o MeV	r_{ow} fm	b_w fm
1,2	$^{16}O + ^{58}Ni$	5,0	1,56	0,60	6,0	1,27	0,54
3a	-"-	5,0	1,56		6,0	1,27	0,54
3b	-"-		1,56	0,60	6,0	1,27	0,54
4a	$^{6}Li + ^{58}Ni$	2,0	1,8	1,0	6,0		0,60
4b	-"-	2,0	1,8	1,0		1,25	0,60
5a	-"-	2,5		0,70	6,0	1,5	0,54
5b	-"-	2,5	1,9	0,70	6,0	1,5	0,54
6a	-"-	2,0	1,8	1,0	6,0	1,25	0,65
6b	$^{6}Li + ^{120}Sn$	1,5	1,95	0,50	6,0	1,5	0,60
7a,b	$^{16}O + ^{58}Ni$	3,4	1,56	0,50	6,0	1,27	0,54

*) In the excitation probability calculations the reduced transition probabilities were taken as follows

$$B_{0\to 1}(E2) = \begin{cases} 0.07\ e^2b^2 & \text{(Figs. 2b–6a, 7a,b)} \\ 0.3\ e^2b^2 & \text{(Fig. 6b)} \end{cases}$$

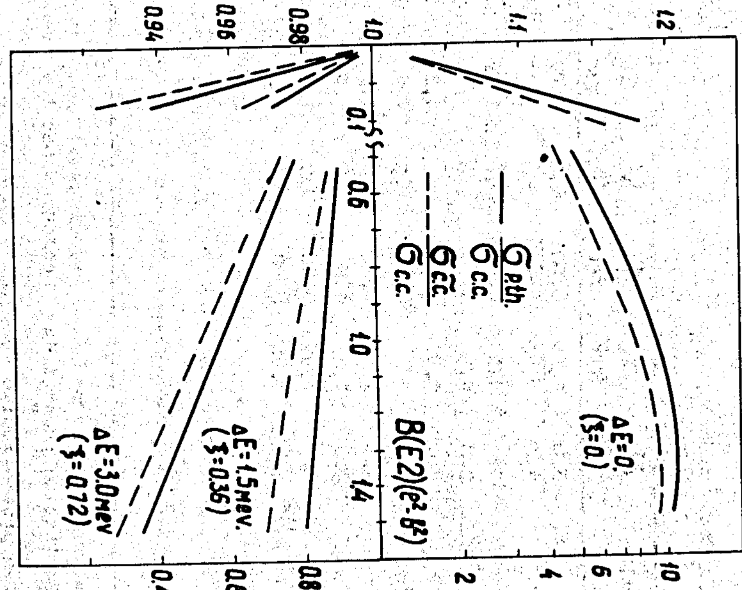

Fig. 1. The investigation of the perturbation theory applicability in describing of excitation cross sections in $^{16}O+^{58}Ni$. The scattering angle and kinetic energy $\vartheta_{cm}=75°$, $E_{lab}=$ 44 MeV.

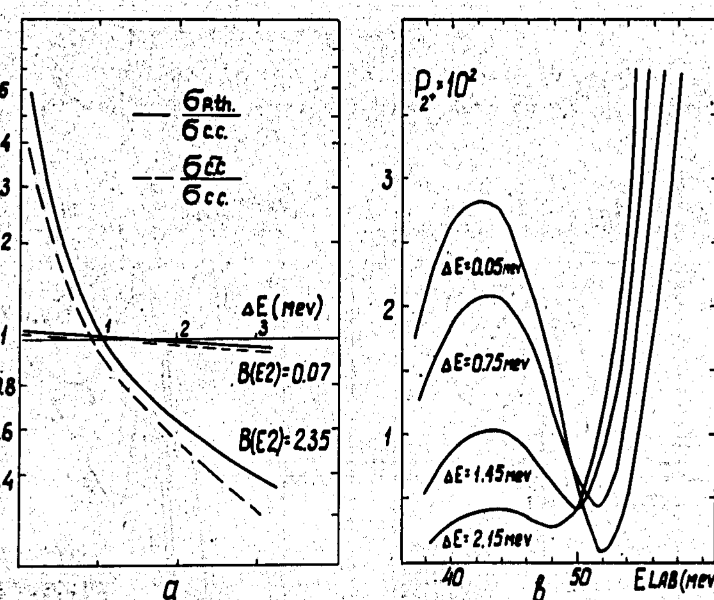

Fig. 2a) The investigation of the perturbation theory applicability. The kinetic energy $E_{lab}=$ 44 MeV.

b) The excitation probability dependence on an energy of the nuclear excitation level ΔE .

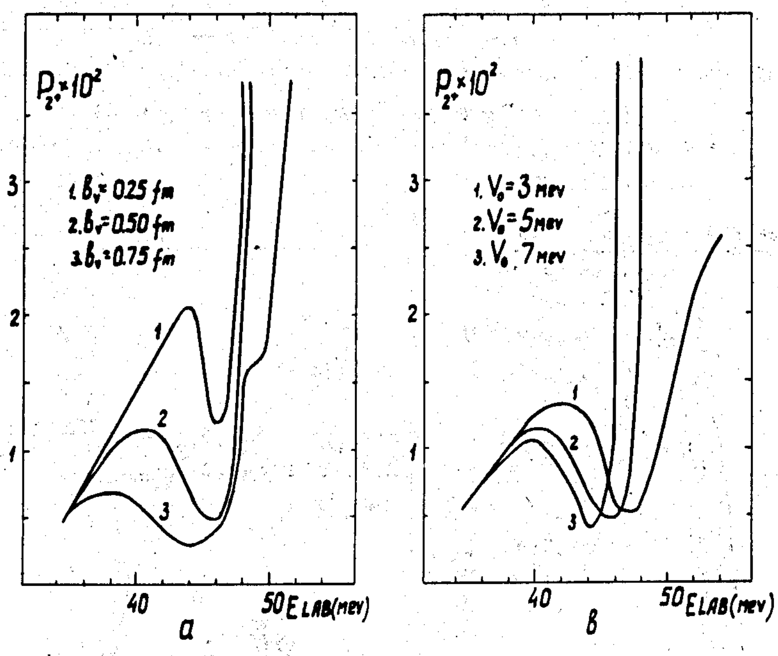

Fig. 3. An excitation probability dependence on the diffusness b_V (a) and depth U_0 (b) parameters in the reaction $^{16}O + ^{58}Ni$ ($E_{lab}=44$ MeV, $\vartheta_{c.m.}=75°$).

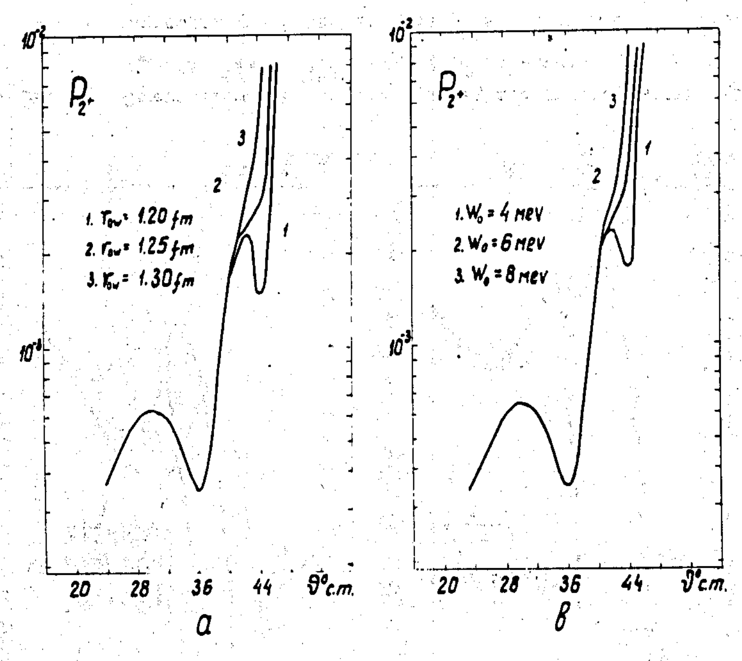

Fig. 4. Dependence on the depth W_0 (b) and radius r_{0W} (a) parameters of an imaginary potential in $^{6}Li + ^{58}Ni$ ($E_{lab}=22.8$ MeV)

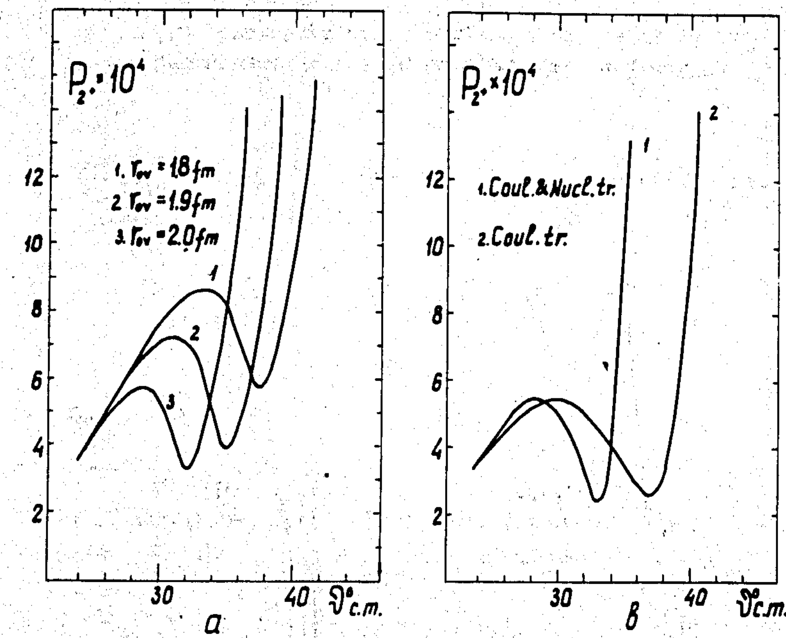

Fig. 5a. Dependence on the real potential radius parameter r_{ov} in $^6Li + ^{58}Ni$ $(E_{lab} = 22.8\ MeV)$.

b. Accounting of the nuclear distortion on the Coulomb trajectory.

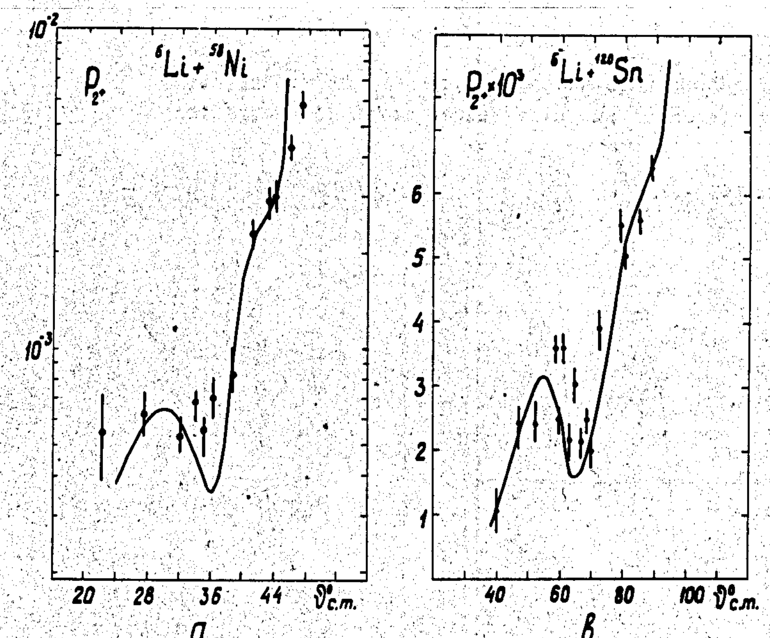

Fig. 6. Comparisons of experimental data and theoretical calculations of P_{2+} for reactions with the lithium ions.

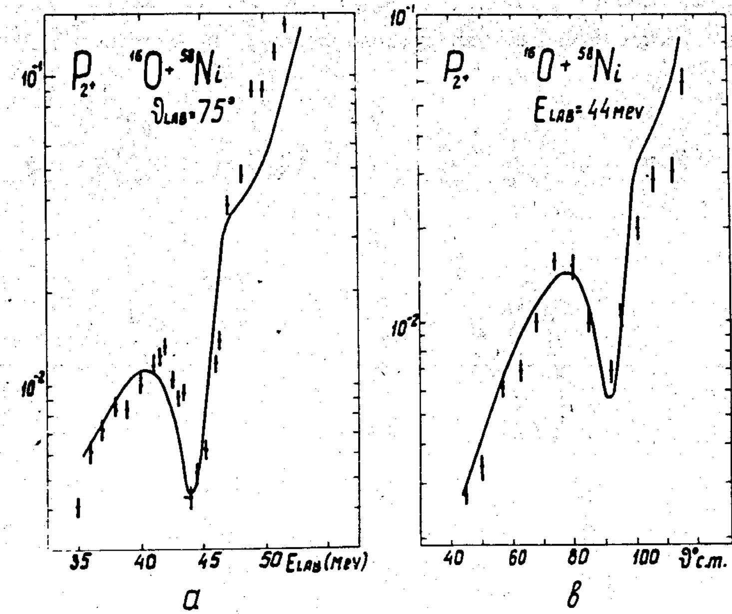

Fig. 7. Comparisons of experimental data and theoretical calculations of P_{2^+} for reactions with the oxydenum ions.