ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА

E4 - 6901

N.M.Plakida

DYSON EQUATION FOR HEISENBERG FERROMAGNET

1973

ЛАБОРАТОРИЯ
ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

E4 - 6901

N.M.Plakida

DYSON EQUATION FOR HEISENBERG FERROMAGNET

Submitted *to Physics Letters*

There is a large number of approximate calculations for the Heisenberg model of a ferromagnet, based on the two-time Green functions /1/ in the literature (see, e.g. /2/), the validity of which is usually difficult to examine. At the same time a diagram method for the spin operators /3/ permits, in principle, to evaluate various contributions to the Green functions.

In the present paper an exact Dyson equation for the two-time Green function will be obtained and the correspondence between some approximate decoupling for the self-energy part and the calculations in the diagram method /3/ will be established. The method of the irreducible Green functions, proposed for the phonon Green functions in our recent papers /4/, which is close to the methods in /5, 6/, will be used.

Consider a ferromagnet which can be described by the Heisenberg Hamiltonian $H = -\frac{1}{2} \sum_{i \neq j} J_{ij} S_i S_j$, $J_{ij} = J(r_i - r_j)$ and write down the equation of motion for the two-time commutator type Green function $G_{ik}(t-t') = \langle\langle S_i^+(t); S_j^-(t') \rangle\rangle$ in the form:

$$\omega G_{ik}(\omega) = 2 \langle S^z \rangle \delta_{ik} + \sum_j J_{ij} \langle\langle S_i^+ S_j^z - S_j^+ S_i^z \mid S_k^- \rangle\rangle_\omega \tag{1}$$

where the usual notation is used /1/. Now we define the irreducible part (ir) of the Green function in (1) by the identity:

$$\langle\langle B_{ij}^{(ir)} \mid S_k^- \rangle\rangle = \langle\langle (S_i^+ S_j^z - S_j^+ S_i^z) - (A_{ij} S_i^+ - A_{ji} S_j^+) \mid S_k^- \rangle\rangle \tag{2}$$

where the coefficients A_{ij} are defined by the condition:

$$\langle [B_{ij}^{(ir)}, S_k^-] \rangle = 0 . \tag{3}$$

Performing the commutation in (3) with the help of (2) and equating the terms at the equal δ -functions δ_{ik} and δ_{jk} we get $(i \neq j)$:

$$A_{ij} = A_{ji} = (2 < S_i^z S_j^z > + < S_i^- S_j^+ >)/2 < S^z > . \qquad (4)$$

The irreducible Green function $\ll B_{ij}^{(ir)}(t); S_k^-(t') \gg$ in (2) is obtained by differentiating it with respect to the second time t' [4,5] and by introducing the irreducible part for the right-hand side operators at time t' by analogy with (2), (3). Then defining the ''zero order'' Green function in the Hartree-Fock approximation by the equation [7] :

$$\omega G^{\circ}_{ik}(\omega) = 2 < S^z > \delta_{ik} + \sum_j J_{ij} A_{ij} \{ G^{\circ}_{ik}(\omega) - G^{\circ}_{jk}(\omega) \}, \qquad (5)$$

(1) can be written in the matrix form as $G = G^{\circ} + G^{\circ} P G^{\circ}, P = P_{j\ell}(\omega)$. From the Dyson equation in the form $G = G^{\circ} + G^{\circ} \Pi G$ we get the equation $P = \Pi + \Pi G^{\circ} P$ from which it follows that the self-energy operator $\Pi_{j\ell}(\omega)$ is defined by the proper part of the operator $P_{j\ell}(\omega)$:

$$\Pi_{ik}(\omega) = P_{ik}^{(p)}(\omega) = < 2S^z >^{-2} \sum_{j\ell} J_{ij} J_{\ell k} \ll B_{ij}^{(ir)} | B_{\ell k}^{+(ir)} \gg_{\omega}^{(p)} \qquad (6)$$

where the proper (p) part of the irreducible Green function has no parts connected by one G° -line.

The solution of the Dyson equation by the Fourier transformation can be written in the form

$$G_{ik}(\omega) = N^{-1} \sum_q e^{i\vec{q}(\vec{r}_i - \vec{r}_k)} 2 < S^z > \{ \omega - E_q - 2 < S^z > \Pi_q(\omega) \}^{-1} . \qquad (7)$$

The energy of spin excitations in the Hartree-Fock approximation (5) is given by

$$E_q = N^{-1} \sum_{ij} J_{ij} A_{ij} (1 - e^{i\vec{q}(\vec{r}_i - \vec{r}_j)}) = < S^z > (J_0 - J_q) +$$
$$+ N^{-1} \sum_{q'} (J_{q'} - J_{\vec{q} - \vec{q}'}) \{ K_{q'}^{-+} + 2K_{q'}^{zz} \} / 2 < S^z > , \qquad (8)$$

where J_q, K_q^{-+}, K_q^{zz} are the Fourier components of the exchange energy J_{ij} and the correlation functions $K_{ij}^{-+} = < S_i^- S_j^+ >$, $K_{ij}^{zz} = < (S_i^z - < S^z >)(S_j^z - < S^z >) >$, respectively. Eq. (8) was obtained in the first order theory by decoupling (see e.g. [2,7]) and by diagram method [3], where the first term defines the energy of spin waves in RPA (Tyablikov decoupling [1]), the second and the third ones describe an elastic scattering by spin waves $(\sim K_q^{-+})$ and by the fluctuations of the z-spin components $(\sim K_q^{zz})$. The second and higher order corrections are described by the frequency dependent part of the self-energy in (7) which can be written in the form:

$$2 < S^z > \Pi_q(\omega) = \int_{-\infty}^{\infty} \frac{d\omega'}{\omega - \omega'} (e^{\frac{\omega'}{\theta}} - 1) \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{-i\omega' t} \frac{1}{N} \sum_{ijk\ell} e^{-i\vec{q}(\vec{r}_i - \vec{r}_k)} \times$$
$$\times J_{ij} J_{\ell k} \{ < B_{\ell k}^{+(ir)}(t) B_{ij}^{(ir)} >^{(p)} / 2 < S^z > \} . \qquad (9)$$

It gives rise to the damping of the spin waves which can be evaluated by the decoupling of the irreducible two-time correlation function in (9). For example, an inelastic scattering of the spin waves by the fluctuations of the z-spin components is given from the diagram consideration by the two-time decoupling

$$< \{ S_\ell^z(t) S_k^-(t) \}^{(ir)} \{ S_i^z S_j^+ \}^{(ir)} > \approx K_{\ell i}^{zz}(t) K_{kj}^{-+}(t)$$

which gives the result of [3]:

$$2 < S^z > \Pi_q^{(1)}(\omega) = \frac{1}{N} \sum_{q'} (J_{q'} - J_{\vec{q}' - \vec{q}})^2 \frac{1}{\omega - \epsilon_{q' - q}} K_{q'}^{zz} \qquad (9a)$$

if we use approximations: $K_{ij}^{zz}(t) \approx K_{ij}^{zz}(t = 0)$ and $-Im[\omega - E_q - 2 < S^z > \Pi_q(\omega + i\epsilon)]^{-1} \approx \pi \delta(\omega - \epsilon_q)$ for the calculation $K^{-+}(t)$ by the Green function (7). Higher order corrections can be obtained by introducing some representations for S_i^z operators in (9), which will be discussed elsewhere.

The author thanks Drs. Yu.G.Rudoj and Yu.A.Tserkovnikov for useful discussions.

References

1. S.V.Tyablikov. Methods in the Quantum Theory of Magnetism. (Plenum Press, New York, 1967).
2. Yu.G.Rudoj, Yu.A.Tserkovnikov. Teor. i Mat. Fiz., 14, 102 (1973).
3. V.G.Vaks, A.I.Larkin, S.A.Pikin. Zh. Eksp. i teor. Fiz., 53, 281, 1089 (1967).
   Yu.A.Izyumov, F.A.Kassan-Ogly. Fiz. Met. i Metalloved., 26, 385 (1968), ibid. 30, 225 (1970).
4. N.M.Plakida. Teor. i Mat. Fiz., 5, 147 (1970); ibid. 12, 135 (1972); H.Konwent, N.M.Plakida. Phys.Lett., 37A, 173 (1971).
5. Yu.A.Tserkovnikov. Teor. i Mat. Fiz., 7, 250 (1971).
   M.W.C.Dharmawardana, C.Mavroyannis. Phys.Rev., B1, 1166 (1970).
6. M.Ichiyanagi. J. Phys. Soc. Jap., 32, 604 (1972).
7. V.Mubayi, R.V.Lange. Phys.Rev., 178, 882 (1969).
   R.P.Kenan. Phys.Rev., B1, 3205 (1970).

Received by Publishing Department
on January 19, 1973.