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1. Introduction

Non-adiabatic effects in nuclear rotational motion have been known for a long time. They show up in different properties of rotational bands such as :

(1) the deviations of energies and transition probabilities[1] from the prediction of the rotational model[2],

(2) the change of nuclear charge radii as observed in Mössbauer[3] and muonic[4] isomer shifts.

These deviations indicate a change of the intrinsic structure in the rotating nucleus. Besides the direct perturbation of the nuclear orbits by the Coriolis force, there arise a change of the average nuclear potential (stretching effect) and a change of the pairing potential (Coriolis antipairing effect). Microscopic calculations of these dynamics[5-7] showed the right trends, but usually gave too large values for the non-adiabatic quantities.

In this work, second order cranking theory is used as in the work of Marshalek[7], but instead of the pairing-plus-quadrupole force Migdal's effective particle-hole (ph) and particle-particle (pp) interactions[8] are applied. The results of the cranking calculation in first order for moments of inertia and gyromagnetic ratios have already been published[9]. In second order, changes of mean square radii have been obtained in almost quantitative agreement with Mössbauer and muonic isomer shifts[10]. Here, we present the corresponding results for the changes of the quadrupole moments. After a short review of the theory, the results for mass and charge quadrupole moments are discussed and compared with the experimental data.

2. Theory

The rotating nucleus is described by the cranking single particle density

$$\varrho^{(\omega)} = \varrho^{(0)} + \omega \varrho^{(1)} + \omega^2 \varrho^{(2)} + \dots$$

with the angular velocity

$$\omega = \frac{\sqrt{I(I+1)}}{\mathcal{J}} \; .$$

The moment of inertia

$$\mathcal{J} = \mathrm{Tr}\{ J_x \varrho^{(1)} \} \qquad (1)$$

and magnetic properties of the rotational states are determined from $\varrho^{(1)}$. In second order, the change of the quadrupole moment

$$\delta\langle Q \rangle = \omega^2 \, \mathrm{Tr}\{ Q \varrho^{(2)} \} , \qquad (2)$$

and the change of the mean square charge radius

$$\delta\langle r_p^2 \rangle = \frac{\omega^2}{Z} \, \mathrm{Tr}\{ r_p^2 \varrho^{(2)} \} \qquad (3)$$

are obtained (Z the number of protons). The equations for $\varrho^{(1)}$ and $\varrho^{(2)}$, developed in ref.[10], have the form

$$\rho^{(1)}_{\nu_1\nu_2} = \frac{\left(\eta^{(-)}_{\nu_1\nu_2}\right)^2}{E_{\nu_1}+E_{\nu_2}} \left[(J_x)_{\nu_1\nu_2} - \sum_{\nu_5\nu_6} F^{ph}_{\nu_1\nu_6,\nu_2\nu_5}\, \rho^{(1)}_{\nu_5\nu_6} \right] - \frac{\eta^{(-)}_{\nu_1\nu_2}\,\xi^{(-)}_{\nu_1\nu_2}}{E_{\nu_1}+E_{\nu_2}} \sum_{\nu_5\nu_6} F^{pp}_{\bar{\nu}_1\nu_2,\bar{\nu}_5\nu_6} \frac{\xi^{(-)}_{\nu_5\nu_6}}{\eta^{(-)}_{\nu_5\nu_6}} \rho^{(1)}_{\nu_5\nu_6} \qquad (4)$$

$$\rho^{(2)}_{\nu_1\nu_2} = \rho^{(2)}_{\nu_1\nu_2}[inh] - \frac{\left(\eta^{(+)}_{\nu_1\nu_2}\right)^2}{E_{\nu_1}+E_{\nu_2}} \left[\mu^{(2)}_{\nu_1\nu_2} + \sum_{\nu_5\nu_6} F^{ph}_{\nu_1\nu_6\nu_2\nu_5}\, \rho^{(2)}_{\nu_5\nu_6} \right] - \eta^{(+)}_{\nu_1\nu_2}\,\xi^{(+)}_{\nu_1\nu_2} \sum_{\nu_5\nu_6} F^{pp}_{\bar{\nu}_1\nu_2\bar{\nu}_5\nu_6} \frac{\xi^{(+)}_{\nu_5\nu_6}}{\eta^{(+)}_{\nu_5\nu_6}} \rho^{(2)}_{\nu_5\nu_6} \qquad (5)$$

with the inhomogeneous part

$$\rho^{(2)}_{\nu_1\nu_2}[inh] = -\xi^{(+)}_{\nu_1\nu_2} \sum_{\nu_3} \frac{\rho^{(1)}_{\nu_1\nu_3}}{\eta^{(-)}_{\nu_1\nu_3}} \frac{\rho^{(1)}_{\nu_3\nu_2}}{\eta^{(-)}_{\nu_3\nu_2}} - \frac{2\,\eta^{(+)}_{\nu_1\nu_2}}{E_{\nu_1}+E_{\nu_2}} \sum_{\nu_3} \left(\xi^{(-)}_{\nu_1\nu_3} V^{(1)}_{\nu_1\nu_3} - \eta^{(-)}_{\nu_1\nu_3} \Delta^{(1)}_{\nu_1\nu_3} \right) \frac{\rho^{(1)}_{\nu_3\nu_2}}{\eta^{(-)}_{\nu_3\nu_2}} - \frac{\eta^{(+)}_{\nu_1\nu_2}\,\xi^{(+)}_{\nu_1\nu_2}}{E_{\nu_1}+E_{\nu_2}} \sum_{\nu_3\nu_4\nu_5} F^{pp}_{\bar{\nu}_1\nu_2,\bar{\nu}_3\nu_4} \frac{\xi^{(+)}_{\nu_3\nu_4}}{\eta^{(+)}_{\nu_3\nu_4}} \cdot \frac{\rho^{(1)}_{\nu_3\nu_5}}{\eta^{(-)}_{\nu_3\nu_5}} \cdot \frac{\rho^{(1)}_{\nu_5\nu_4}}{\eta^{(-)}_{\nu_5\nu_4}}$$

including the effective fields

$$V^{(1)}_{\nu_1\nu_2} = (J_x)_{\nu_1\nu_2} - \sum_{\nu_5\nu_6} F^{ph}_{\nu_1\nu_6,\nu_2\nu_5}\, \rho^{(1)}_{\nu_5\nu_6} ,$$

$$\Delta^{(1)}_{\nu_1\nu_2} = \sum_{\nu_5\nu_6} F^{pp}_{\bar{\nu}_1\nu_2,\bar{\nu}_5\nu_6} \frac{\xi^{(-)}_{\nu_5\nu_6}}{\eta^{(-)}_{\nu_5\nu_6}} \rho^{(1)}_{\nu_5\nu_6} .$$

The change of the chemical potential

$$\mu^{(2)}_{\nu_1\nu_2} = \delta_{\nu_1\nu_2} \begin{cases} \delta\mu^{P} & \text{, for protons} \\ \delta\mu^{N} & \text{, for neutrons} \end{cases}$$

is determined from the conditions of proton and neutron number conservation

$$\sum_{\text{protons}} \rho^{(2)}_{\nu\nu} = \sum_{\text{neutrons}} \rho^{(2)}_{\nu\nu} = 0 \ .$$

The BCS-quantities are defined as usual

$$E_\nu = \sqrt{(\varepsilon_\nu - \mu)^2 + \Delta_\nu^2} \ , \quad v_\nu^2 = \frac{1}{2}\left(1 - \frac{\varepsilon_\nu - \mu}{E_\nu}\right) \ , \quad u_\nu^2 = 1 - v_\nu^2 \ ,$$

$$\eta^{(\pm)}_{\nu_1\nu_2} = u_{\nu_1} v_{\nu_2} \pm v_{\nu_1} u_{\nu_2} \ , \quad \xi^{(\pm)}_{\nu_1\nu_2} = u_{\nu_1} u_{\nu_2} \mp v_{\nu_1} v_{\nu_2} \ ,$$

but the gap equation

$$\Delta_{\nu_1} = - \sum_{\nu_2} F^{pp}_{\bar{\nu}_1\nu_1, \bar{\nu}_2\nu_2} \frac{\Delta_{\nu_2}}{2E_{\nu_2}}$$

now contains the same pp-interaction F^{pp} as in eqs. (4) and (5). It is chosen for T = 1 pairing

$$F^{pp} = C \cdot L^{pp} \cdot \delta(\vec{r}_1 - \vec{r}_2)$$

Similarly, the ph-interaction is taken as

$$F^{ph} = C \cdot \{ f_0 + f_0' \tau_1\tau_2 + (g_0 + g_0' \tau_1\tau_2)\sigma_1\sigma_2 \} \ \delta(\vec{r}_1 - \vec{r}_2)$$

where the parameter

$$f_0(\vec{r}) = f_0^{ex} + (f_0^{in} - f_0^{ex}) \frac{\rho(\vec{r})}{\rho(0)}$$

and only this one is chosen density dependent.

The following set of parameters

$$C = \frac{d\varepsilon_F}{dn} = 386 \ \text{MeV} \cdot \text{fm}^3$$

$$L^{pp} = -0.33 \ , \quad g_0 = g_0' = 0.5$$

$$f_0^{ex} = -2 \ , \quad f_0^{in} = 0 \ , \quad f_0' = 0.1$$

is used in the calculations. For details of the single particle basis see ref.[10].

3. Results and Conclusions

Table 1 presents the calculated changes of charge and mass quadrupole moments $\delta\langle Q_p\rangle^{th}$ ($\delta\langle Q_M\rangle^{th}$) for $2^+ \rightarrow 0^+$ transitions. $\delta\langle Q\rangle^{th}$ is determined from the same change of the density $\rho^{(2)}$ as $\delta\langle r_p^2\rangle$ in ref.[10] (see Table 1, column 7). For comparison, the experimental ground state quadrupole moments $\langle Q_p\rangle^{exp}$ [11] are given in column 2. Columns 3-5 display the strong influence of the two residual interactions F^{ph} and F^{pp} on the calculated $\delta\langle Q\rangle$ showing the three cases:

(a) without residual interactions,

(b) only with ph interaction and

(c) results of the full theory.

Note that pairing in the ground state has been taken into account in all three cases. The increase of $\delta\langle Q\rangle$ due to the residual interactions is particularly sharp for soft N = 90 nuclei; for well deformed nuclei only F^{pp} plays a dominant role.

Furthermore, the change of the mass distribution $\frac{1}{A}\delta\langle Q_M\rangle$ is persistently larger than the change of the charge distribution $\frac{1}{Z}\delta\langle Q_p\rangle$ at the beginning of the deformed region thus indicating a different behaviour of protons and neutrons. In order to test the classical relation

$$\delta\langle r_p^2\rangle_Q = \frac{1}{2Z^2}\frac{\langle Q_p\rangle}{\langle r_p^2\rangle}\,\delta\langle Q_p\rangle$$

$$\left(\langle r_p^2\rangle = \frac{3}{5}\cdot 1.2^2\cdot A^{\frac{2}{3}}\right)$$

which follows for a homogeneous liquid drop of sheroidal shape under the assumption of volume conservation, the values $\delta\langle r_p^2\rangle_Q$ are given in column 6. They should be compared with the microscopic $\delta\langle r_p^2\rangle$ in column 7 calculated according to eq.(3). As the $\delta\langle r_p^2\rangle_Q$ are found to be larger than $\delta\langle r_p^2\rangle$ (more than 50% for N = 90 nuclei), we conclude that the assumptions of the liquid drop, which are frequently used, do not hold.

In Table 2, the theory is compared with some experimental results. Direct evidence for $\delta\langle Q_p\rangle$ has been obtained from lifetime measurements [1] in rotational bands using the relation

$$B(E2;\, I\to I-2) = B(E2;\, 2\to 0)\,\frac{\langle I020|(I-2)0\rangle^2}{\langle 2020|00\rangle^2}\left[1+\frac{\alpha}{2}\bigl(I(I+1)+(I-2)(I-1)\bigr)\right]$$

$$\alpha\, I(I+1) = \frac{\delta\langle Q_p(I)\rangle}{\langle Q_p\rangle}\;.$$

Alternatively, the parameter α can be determined from β -band mixing and from Coulomb excitation. In columns 4 and 5, experimental α^{exp} [1] are compared with the theoretical results of this paper. For comparison, Columns 6–9 give also experimental and theoretical results [10] for Mössbauer isomer shifts $\delta\langle r_p^2\rangle$ and muonic isomer shifts δE_{μ}^{is}.

It is seen that the different experimental quantities are simultaneously described by the theory within the experimental errors.

A discrepancy is obtained only for $\delta\langle r_p^2\rangle$ of Sm^{154}. Since the theoretical value $\delta\langle r_p^2\rangle^{th}$ is found to be rather stable agains shifts of s.p. energies and parameters and since the corresponding α^{th} compares well with α^{exp}, the experimental value $\delta\langle r_p^2\rangle^{exp}$ should be remeasured. Also a measurement of δE_μ^{is} in Sm^{154} would be highly desirable.

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Table 1. Change of Charge and Mass Quadrupole Moments for $2 \rightarrow 0$ Transitions. Comparison of $\delta\langle r_p^2\rangle_Q$ Derived from $\delta\langle Q_p\rangle$ by the Liquid Drop Model with Microscopic $\delta\langle r_p^2\rangle$

Nucleus	$\langle Q_p\rangle^{exp}$ [eb]	$\frac{1}{Z}\,\delta\langle Q_p\rangle^{th}$ $\left(\frac{1}{A}\,\delta\langle Q_M\rangle^{th}\right)$ [10^{-5} b]			$\delta\langle r_p^2\rangle_Q^{th}$ [10^{-3} fm^2]	$\delta\langle r_p^2\rangle^{th}$
	ref. 12	$F^{pp}=F^{ph}=0$	$F^{pp}=0$	full theory		ref.11
$_{60}Nd^{150}$	5.15	4.6 (7.4)	42.6 (57.5)	247.9 (303.4)	43.70	18.40
$_{62}Sm^{152}$	5.93	2.1 (5.7)	22.7 (35.8)	119.7 (159.6)	23.25	14.59
$_{62}SM^{154}$	6.65	0.4 (1.2)	1.5 (2.5)	24.8 (33.0)	5.37	5.55
$_{64}Gd^{154}$	6.15	1.4 (7.2)	26.6 (70.8)	175.5 (390.8)	34.2	21.24
$_{64}Gd^{156}$	6.91	0.5 (1.4)	1.4 (3.0)	11.3 (16.7)	2.44	2.06
$_{64}Gd^{158}$	7.20	0.3 (1.0)	1.3 (2.8)	7.0 (8.7)	1.56	1.48
$_{64}Gd^{160}$	7.43	0.2 (0.5)	0.5 (0.8)	3.9 (2.4)	0.89	0.76
$_{66}Dy^{156}$	6.13	-0.3 (7.4)	8.3 (34.4)	49.1 (120.3)	9.12	-0.89
$_{66}Dy^{158}$	6.77	-0.8 (1.8)	-0.8 (1.2)	3.4 (5.0)	0.70	-1.77
$_{66}Dy^{160}$	7.13	-0.8 (0.3)	-1.2 (-1.2)	0.5 (-1.2)	0.11	-1.48
$_{66}Dy^{162}$	7.18	-0.7 (-0.0)	-1.0 (-0.5)	1.0 (1.2)	0.21	-1.16

	$\langle Q_p \rangle^{exp}$ [eb]	$\frac{1}{Z}\,\delta\langle Q_p\rangle^{th}$ $\left(\frac{1}{A}\,\delta\langle Q_M\rangle^{th}\right)$ [10^{-5} eb]			$\delta\langle r_p^2\rangle_Q^{th}$ [10^{-3} fm^2]	$\delta\langle r_p^2\rangle^{t}$
	ref. /12/	$F^{pp}=F^{ph}=0$	$F^{pp}=0$	full theory		ref. /1
$_{66}Dy^{164}$	7.37	−0.6 (−1.6)	−1.4 (−2.3)	−0.0 (−1.4)	0.00	−0.99
$_{68}Er^{158}$	5.26	5.4 (7.8)	8.3 (19.4)	14.4 (29.6)	2.22	−2.83
Er^{160}	6.52 (4.3)	1.2 (12.1)	5.3 (13.9)	3.6	0.66	−1.38
Er^{162}	7.01	0.6 (0.8)	0.7 (1.4)	1.3 (2.9)	0.26	−0.57
Er^{164}	7.42	0.3 (0.5)	−0.1 (−0.0)	1.4 (2.4)	0.30	−0.21
Er^{166}	7.58	0.2 (−1.0)	−0.4 (−1.0)	1.1 (0.8)	0.24	0.16
Er^{168}	7.60	0.2 (−0.5)	−0.3 (−0.8)	2.8 (3.3)	0.60	0.73
Er^{170}	7.57	0.2 (0.0)	0.0 (−0.3)	4.1 (5.3)	0.86	1.04
Yb^{168}	7.39	1.4 (−0.4)	0.6 (−0.5)	2.2 (1.9)	0.44	0.04
Yb^{170}	7.57	1.3 (0.0)	0.7 (−0.2)	3.6 (4.3)	0.73	0.44
Yb^{172}	7.81	1.2 (0.5)	0.9 (0.2)	3.9 (4.7)	0.82	0.48

	$\langle Q_p\rangle^{exp}$ [eb]	$\frac{1}{Z}\,\delta\langle Q_p\rangle^{th}$ ($\frac{1}{A}\,\delta\langle Q_M\rangle^{th}$) [10^{-5} eb]			$\delta\langle r_p^2\rangle_Q^{th}$ [10^{-3} fm^2]	$\delta\langle r_p^2\rangle^{th}$
	ref. /12/	$F^{pp}=F^{ph}=0$	$F^{pp}=0$	full theory		ref. /11/
Yb^{174}	7.60	1.1 (−0.1)	0.7 (−0.1)	2.4 (1.8)	0.48	0.05
Yb^{176}	7.32	1.2 (−0.3)	0.6 (−0.2)	2.2 (1.1)	0.42	0.13
Hf^{172}	6.65	1.3 (0.6)	1.4 (0.9)	4.5 (7.3)	0.78	0.19
Hf^{174}	6.97	1.2 (0.9)	1.4 (1.1)	4.6 (7.7)	0.83	0.15
Hf^{176}	7.37	1.2 (0.2	1.0 (0.2	3.3 (4.6)	0.62	−0.11
Hf^{178}	6.81	1.3 (−0.2)	0.7 (−0.1)	3.0 (2.7)	0.52	0.25
Hf^{180}	6.83	1.3 (−0.0)	0.8 (−0.0)	5.2 (3.6)	0.90	1.02

Table 2. Comparison of Calculated α -Coefficients, Mössbauer Isomer Shifts $\delta\langle r_p^2\rangle$ and Muonic Isomer Shifts δE_μ^{is} with Experiment.

	$\delta\langle Q_p\rangle^{exp}$ [e·b]	$\delta\langle Q_p\rangle^{th}$ [10^{-3} eb]	$\alpha^{th}\cdot 10^3$	$\alpha^{exp}\cdot 10^3$	$\delta\langle r_p^2\rangle^{th}$ [10^{-3} fm^2]	$\delta\langle r_p^2\rangle^{exp}$	$\delta E_\mu^{is(th)}$ [eV]	$\delta E_\mu^{is(exp)}$
$_{60}Nd^{150}$	5.15[a]	148.6	4.81	4±2[b]	18.4	-	999	840(120)[f] 1050(120)[g]
$_{62}Sm^{152}$	5.93[a]	74.1	2.08	1.9±0.6[c] 2.2±0.6[c]	14.6	20(7)[e] 12(4)[e]	851	770(40)[f] 996(40)[g]
$_{62}Sm^{154}$	6.65[a]	15.4	0.39	0.6±0.6[c]	5.5	1.2(8)[e] 0.75(50)[e]	318	-
$_{64}Gd^{154}$	6.15[a]	112.2	3.04	2.6±1.0[d] 2.7±0.6[c]	21.2	25(5)[e] 16(3)[g]	1258	980(150)[f] 1185(150)[g]
$_{64}Gd^{156}$	6.91[a]	7.2	0.17	0.6±0.6[d] 0.1±0.1[c]	2.1	3.6[e] 2.0[e]	132	0(80)[f] 99(80)[g]

a) ref./11/ b) ref./13/ c) ref./1/ d) ref./12/ e) ref./3/
f) ref./ 4/ g) ref./10/.