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SPIN-PHONON INTERACTION IN THE HEISENBERG-FERROMAGNET

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The influence of the spin-phonon interactions on the system of noninteracting [1,2] or interacting [3] spin waves at low temperatures was treated using an interaction Hamiltonian linear in the phonon field and quadratic in the spin operators.

Spin operators were represented through Bose operators using the Halstein-Primakoff approximation [4], and assuming the coupling to be weak, a perturbation expansion was performed. In a certain approximation it was possible to get the mass operator and calculate attenuation [1] and renormalization of the spin wave energy and magnetization [2].

In this note we shall show that it is possible to get more exactly the expression for the mass operator.

Besides the standard use of the weak coupling between spin and phonon subsystem, calculations will be performed in the linear approximation of the magnon concentration $c = \frac{1}{N} \sum_{\vec{q}} \langle B^{+}_{\vec{q}} B_{\vec{q}} \rangle_0$ which is the small quantity $(\sim T^{3/2})$ in the vicinity of the null temperature.

The total Hamiltonian of the system will be written [2]

$$\mathcal{H} = \mathcal{H}_S + \mathcal{H}_\ell + \mathcal{H}_{int} \,, \tag{1}$$

$$\mathcal{H}_S = -\mu H \sum_{\vec{\ell}} S^z_{\vec{\ell}} - \frac{1}{2} \sum_{\vec{\ell} \neq \vec{m}} J(\vec{\ell} - \vec{m}) \vec{S}_{\vec{\ell}} \vec{S}_{\vec{m}} \,, \tag{1a}$$

$$\mathcal{H}_\ell = \sum_{\vec{\ell}} \frac{P^2_{\vec{\ell}}}{2M} + \frac{1}{2} \sum_{\vec{\ell}, \vec{m}} \vec{U}_{\vec{\ell}} \hat{C}(\vec{\ell}, \vec{m}) \vec{U}_{\vec{m}} \,, \tag{1b}$$

$$\mathcal{H}_{int} = -\frac{1}{2}\sum_{\vec{\ell}\neq\vec{m}} [\vec{U}_{\vec{\ell}} - \vec{U}_{\vec{m}}]\vec{\nabla}_{\vec{\ell}} J(\vec{\ell}-\vec{m})\vec{S}_{\vec{\ell}}\cdot\vec{S}_{\vec{m}} \quad . \qquad (1c)$$

Here $\mathcal{H}_S$ is the Heisenberg Hamiltonian, $\mathcal{H}_\ell$ is the lattice Hamiltonian in the harmonic approximation and $\mathcal{H}_{int}$ is the interaction Hamiltonian which represents the first term of the exchange integral expansion in the powers of the displacement.

For simplicity, in further calculation, we shall investigate only the case of spin $S=1/2$ and use the boson representation of the Pauli operators /5/

$$S^+_{\vec{\ell}} = B_{\vec{\ell}} - B^+_{\vec{\ell}} B_{\vec{\ell}} B_{\vec{\ell}} \; ,$$

$$S^-_{\vec{\ell}} = B^+_{\vec{\ell}} - B^+_{\vec{\ell}} B^+_{\vec{\ell}} B_{\vec{\ell}} \; , \qquad (2)$$

$$S^z_{\vec{\ell}} = \frac{1}{2} - B^+_{\vec{\ell}} B_{\vec{\ell}} + B^+_{\vec{\ell}} B^+_{\vec{\ell}} B_{\vec{\ell}} B_{\vec{\ell}} \quad .$$

By means of (2) and the usual Fourier transformation of the boson, displacement and momentum operators we get the following effective boson Hamiltonian

$$H_S = \sum_{\vec{k}_1} \lambda_{\vec{k}_1} B^+_{\vec{k}_1} B_{\vec{k}_1} + \frac{1}{N}\sum_{\vec{k}_1\vec{k}_2\vec{k}_3} \Big(-\Delta + \frac{1}{2} J_{\vec{k}_1} + \frac{1}{2} J_{\vec{k}_3} - \qquad (3a)$$

$$- \frac{1}{2} J_{\vec{k}_1 - \vec{k}_3} \Big) B^+_{\vec{k}_1} B^+_{\vec{k}_2} B_{\vec{k}_3} B_{\vec{k}_1+\vec{k}_2-\vec{k}_3} \; ,$$

$$H_\ell = \sum_{\vec{k}_1 j} \hbar\, w_{\vec{k}_1 j}\, a^+_{\vec{k}_1 j} a_{\vec{k}_1 j} \; ,$$

$$H_{int} = \frac{1}{2N^{1/2}} \sum_{\vec{k}_1,\vec{k}_2,j} F^{(j)}_{\vec{k}_1\vec{k}_2} B_{\vec{k}_2} B_{\vec{k}_2-\vec{k}_1} (a_{\vec{k}_1 j} + a^+_{\vec{k}_1 j}) + \qquad (3b)$$

$$+ \frac{1}{2N^{3/2}} \sum_{\vec{k}_1\vec{k}_2\vec{k}_3\vec{k}_4 j} \Phi^{(j)}_{\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4} B^+_{\vec{k}_2} B^+_{\vec{k}_3} B_{\vec{k}_4} B_{\vec{k}_2+\vec{k}_3-\vec{k}_1-\vec{k}_4} (a_{\vec{k}_1 j} + a_{\vec{k}_1 j}),$$

where

$$J_{\vec{k}} = \sum_{\vec{\ell}} J(\vec{\ell})\, e^{i\vec{k}\vec{\ell}} \quad , \qquad (4)$$

$$\Delta = \mu H + \frac{1}{2} J_0 \;, \tag{5}$$

$$\lambda_{\vec{k}} = \mu H + \frac{1}{2} (J_0 - J_{\vec{k}}) , \tag{6}$$

$$F^{(j)}_{\vec{k}_1 \vec{k}_2} = i \left(\frac{\hbar}{2M w_{\vec{k}_1 j}} \right)^{1/2} [(\vec{k}_2 , \vec{\ell}_{\vec{k}_1 j}) J_{\vec{k}_2} - (\vec{k}_2 - \vec{k}_1 , \vec{\ell}_{\vec{k}_1 j}) J_{\vec{k}_2 - \vec{k}_1} - (\vec{k}_1 , \vec{\ell}_{\vec{k}_1 j}) J_{\vec{k}_1}] , \tag{7}$$

$$\begin{aligned} \Phi^{(j)}_{\vec{k}_1 , \vec{k}_2 , \vec{k}_3 , \vec{k}_4} = i \left(\frac{\hbar}{2M w_{\vec{k}_1 j}} \right)^{1/2} [& (\vec{k}_1 , \vec{\ell}_{\vec{k}_1 j}) J_{\vec{k}_1} + (\vec{k}_1 + \vec{k}_4 - \vec{k}_2 , \vec{\ell}_{\vec{k}_1 j}) J_{\vec{k}_1 + \vec{k}_4 - \vec{k}_2} + \\ & + (\vec{k}_2 - \vec{k}_4 , \vec{\ell}_{\vec{k}_1 j}) J_{\vec{k}_2 - \vec{k}_4} + (\vec{k}_2 - \vec{k}_1 , \vec{\ell}_{\vec{k}_1 j}) J_{\vec{k}_2 - \vec{k}_1} + \\ & + (\vec{k}_2 + \vec{k}_3 - \vec{k}_1 - \vec{k}_4 , \vec{\ell}_{\vec{k}_1 j}) \times J_{\vec{k}_2 + \vec{k}_3 - \vec{k}_1 - \vec{k}_4} - (\vec{k}_2 , \vec{\ell}_{\vec{k}_1 j}) J_{\vec{k}_2} - \\ & - (\vec{k}_2 + \vec{k}_3 - \vec{k}_4 , \vec{\ell}_{\vec{k}_1 j}) J_{\vec{k}_2 + \vec{k}_3 - \vec{k}_4} \end{aligned} \tag{8}$$

$\vec{\ell}_{\vec{k} j}$ is the unit phonon polarization vector, $w_{\vec{k} j}$ is the phonon frequency in the harmonic approximation, $a^{+}_{\vec{k} j}$ and $a_{\vec{k} j}$ are the phonon creation and annihilation operators.

From the given boson Hamiltonian (3) the equation for the Green function $\langle\langle S^{+}_{\vec{k}} (t) | S^{-}_{\vec{k}} (t') \rangle\rangle$ /6/ where

$$S^{-}_{\vec{k}} = B^{+}_{\vec{k}} - \frac{1}{N} \sum_{\vec{q}_1 , \vec{q}_2} B^{+}_{\vec{k} - \vec{q}_1 + \vec{q}_2} B^{+}_{\vec{q}_1} B_{\vec{q}_2} \;, \tag{9}$$

takes the form

$$E \langle\langle S^{+}_{\vec{k}} | S^{-}_{\vec{k}} \rangle\rangle = \frac{i}{2\pi} \langle [S^{+}_{\vec{k}} , S^{-}_{\vec{k}}] \rangle + \lambda_{\vec{k}} \langle\langle S^{+}_{\vec{k}} | S^{-}_{\vec{k}} \rangle\rangle +$$

$$+ \frac{1}{2N} \sum_{\vec{q}_1 , \vec{q}_2} (J_{\vec{k} - \vec{q}_1 + \vec{q}_2} + J_{\vec{q}_1} - J_{\vec{k} - \vec{q}_1} - J_{\vec{q}_1 - \vec{q}_2}) \langle\langle B^{+}_{\vec{q}_2} B_{\vec{q}_1} B_{\vec{k} - \vec{q}_1 + \vec{q}_2} | S^{-}_{\vec{k}} \rangle\rangle +$$

$$+\frac{1}{2N^{1/2}}\sum_{\vec q_1 j} F^{(j)}_{\vec q_1 \vec k} \ll B_{\vec k-\vec q_1}\left(a_{\vec q_1 j}+a^{+}_{-\vec q_1 j}\right)|S^{-}_{\vec k}\gg +$$

$$+\frac{1}{2N^{3/2}}\sum_{\vec q_1,\vec q_2,\vec q_3, j}\Big[\Phi^{(j)}_{\vec q_1,\vec k,\vec q_2,\vec q_3}+\Phi^{(j)}_{\vec q_1,\vec q_2,\vec k,\vec q_3}-F^{(j)}_{\vec q_1,\vec q_1+\vec q_3}-$$

$$-F^{(j)}_{\vec q_1 \vec k-\vec q_3+\vec q_2}+F^{(j)}_{\vec q_1 \vec q_2}-\frac{1}{N}\sum_{\vec p}\Big(\Phi^{(j)}_{\vec q_1,\vec k-\vec p+\vec q_2,\vec p,\vec q_3}+ \tag{10}$$

$$+\Phi^{(j)}_{\vec q_1,\vec p,\vec k-\vec p+\vec q_2,\vec q_3}\Big)\Big]\times \ll B^{+}_{\vec q_2} B_{\vec q_3} B_{\vec k-\vec q_3+\vec q_2-\vec q_1}\times$$

$$\times\left(a_{\vec q_1 j}+a^{+}_{-\vec q_1 j}\right)|S^{-}_{\vec k}\gg .$$

The linear approximation in c implies taking into account only the Green functions of the form $\ll B^{+}BB|B^{+}\gg$ and $\ll B|B^{+}B^{+}B\gg$ in eq. (10).

After the decoupling

$$\ll B^{+}_{\vec q_2} B_{\vec q_1} B_{\vec k-\vec q_1+\vec q_2}|B^{+}_{\vec k}\gg = \langle B^{+}_{\vec q_2} B_{\vec q}\rangle_0 \ll B_{\vec k}|B^{+}_{\vec k}\gg\left(\delta_{\vec q_1,\vec q_2}+\delta_{\vec q_1,\vec k}\right)$$

$$\ll B_{\vec k}|B_{\vec k-\vec q_1+\vec q_2} B^{+}_{\vec q_1} B_{\vec q_2}\gg = \langle B^{+}_{\vec q_2} B_{\vec q_2}\rangle_0 \ll B_{\vec k}|B^{+}_{\vec k}\gg\left(\delta_{\vec q_1,\vec q_2}+\delta_{\vec q_1,\vec k}\right) \tag{11}$$

eq. (10) takes the form

$$\left[(E-\lambda_{\vec k})(1-4c)-D_{\vec k}\right]\ll B_{\vec k}|B^{+}_{\vec k}\gg = \frac{i}{2\pi}(1-2c)+\frac{1-4c}{2N^{1/2}}\times$$

$$\times\sum_{\vec q_1 j} F^{(j)}_{\vec q_1 \vec k}\ll B_{\vec k-\vec q_1}\left(a_{\vec q_1 j}+a^{+}_{-\vec q_1 j}\right)|B^{+}_{\vec k}\gg + \tag{12}$$

$$+\frac{1}{2N^{1/2}}\sum_{\vec q_1 j} W^{(j)}_{\vec q_1,\vec k}\ll B_{\vec k-\vec q_1}\left(a_{\vec q_1 j}+a^{+}_{-\vec q_1 j}\right)|B^{+}_{\vec k}\gg ,$$

where

$$\langle B^{+}_{\vec q_2} B_{\vec q_2}\rangle_0=\frac{1}{\exp(\lambda_{\vec k}/kT)-1} \tag{13}$$

$$D_{\vec{k}} = \frac{1}{N} \sum_{\vec{p}} (J_{\vec{k}} + J_{\vec{p}} - J_0 - J_{\vec{k}-\vec{p}}) < B^+_{\vec{p}} B_{\vec{p}} >_0, \tag{14}$$

$$W^{(j)}_{\vec{q}_1,k} = \frac{1}{N} \sum_{\vec{p}} [\Phi^{(j)}_{\vec{q}_1\vec{k},\vec{p},\vec{p}} + \Phi^{(j)}_{\vec{q}_1,\vec{k},\vec{p},\vec{k}-\vec{q}} + \Phi^{(j)}_{\vec{q}_1,\vec{p},\vec{k},\vec{k}-\vec{q}} + \Phi^{(j)}_{\vec{q}_1,\vec{p},\vec{k},\vec{p}} +$$

$$+ 2F^{(j)}_{\vec{q}_1,\vec{k}} - 2F^{(j)}_{\vec{q}_1,\vec{q}_1+\vec{p}} - \frac{1}{N} \sum_{\vec{p}_1} (\Phi^{(j)}_{\vec{q}_1,\vec{k}-\vec{p}_1+\vec{p},\vec{p}_1,\vec{p}} + \Phi^{(j)}_{\vec{q}_1,\vec{p}_1,\vec{k}-\vec{p}_1+\vec{p},\vec{p}} + \tag{15}$$

$$+ \Phi^{(j)}_{\vec{q}_1,\vec{k}-\vec{p}_1+\vec{p},\vec{p}_1,\vec{k}-\vec{q}_1} + \Phi^{(j)}_{\vec{q}_1,\vec{p}_1,\vec{k}-\vec{p}_1+\vec{p},\vec{k}-\vec{q}_1})] \times < B^+_{\vec{p}} B_{\vec{p}} >_0.$$

We find the Green functions $\langle\langle B_{\vec{k}-\vec{q}_1} a_{\vec{q}_1 j} | B^+_{\vec{k}} \rangle\rangle$ and $\langle\langle B_{\vec{k}-\vec{q}_1} a^+_{-\vec{q}_1 j} | B^+_{k} \rangle\rangle$ using the Hamiltonian (1) with the interchange $S^+_{\vec{k}} = B_k$ and $S^-_{\vec{k}} = B^+_{\vec{k}}$ and by means of the decoupling

$$\langle\langle B_{\vec{q}-\vec{k}_1} a_{\vec{k}-\vec{q},j} (a_{\vec{k}_1 j'} + a^+_{-\vec{k}_1 j'}) | B^+_{\vec{k}} \rangle\rangle =$$

$$= (1 + < a^+_{\vec{k}-\vec{q},j} a_{\vec{k}-\vec{q},j} >_0) \langle\langle B_{\vec{k}} | B^+_{\vec{k}} \rangle\rangle \delta_{\vec{k}_1,\vec{q}-\vec{k}} \delta_{j,j'},$$

$$\langle\langle B_{\vec{q}-\vec{k}_1} a^+_{\vec{q}-\vec{k},j} (a_{\vec{k}_1 j} + a_{-\vec{k}_1 j}) | B^+_{\vec{k}} \rangle\rangle = \tag{16}$$

$$= < a^+_{\vec{k}-\vec{q},j} a_{\vec{k}-\vec{q},j} >_0 \langle\langle B_{\vec{k}} | B^+_{\vec{k}} \rangle\rangle \delta_{\vec{k}_1,\vec{q}-\vec{k}} \delta_{jj'}.$$

The interchange of these expressions in eq. (12) gives

$$\{E - \lambda_{\vec{k}} - M_{\vec{k}}(E)\} \langle\langle B_{\vec{k}} | B^+_{\vec{k}} \rangle\rangle = \frac{i}{2\pi}(1 + 2c), \tag{17}$$

where

$$M_{\vec{k}}(E) = D_{\vec{k}} + \frac{1}{4N} \sum_{\vec{q},\vec{k}} | F^{(j)}_{\vec{q},\vec{k}} |^2 \{ \frac{1 + < B^+_{\vec{k}-\vec{q}} \cdot B_{\vec{k}-\vec{q}} >_0 + < a^+_{\vec{q} j} a_{\vec{q} j} >_0}{E - \lambda_{\vec{k}-\vec{q}} - h w_{\vec{q} j}} +$$

$$+\frac{\langle a^{+}_{\vec{q}j}\, a_{\vec{q}j}\rangle_0 - \langle B^{+}_{\vec{k}-\vec{q}}\, B_{\vec{k}-\vec{q}}\rangle_0}{E-\lambda_{\vec{k}-\vec{q}}+hw_{\vec{q}j}}\Big\} + \frac{1}{4N}\sum_{\vec{q}j} W^{(j)}_{\vec{q},\vec{k}}\, F^{(j)*}_{\vec{q},\vec{k}} \times$$

$$\times\Big\{\frac{1+\langle a^{+}_{\vec{q}j}\, a_{\vec{q}j}\rangle_0}{E-\lambda_{\vec{k}-\vec{q}}-hw_{\vec{q}j}} + \frac{\langle a^{+}_{\vec{q}j}\, a_{\vec{q}j}\rangle_0}{E-\lambda_{\vec{k}-\vec{q}}+hw_{\vec{q}j}}\Big\}. \quad (18)$$

The first term in eq. (18) is the Dyson mass operator /7/, the second term is the mass operator of /1,2/ and the third term is obtained by taking into account more exactly kinematical interactions in the H_{int}.

It should be noted that in the approximation $\lambda_k = \mu H + Ia^2 k^2$ the first and the third term in eq. (18) are equal to zero.

In conclusion we would like to point out that the mass operator (18) should be used in finding the expansion of magnetization in the powers of temperature and that the application of the upper methodology to some more realistic Hamiltonian can be of interest in treating kinetic and relaxation processes in the ferromagnetic at low temperatures.

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References

1. E.N.Yakovlev. Fiz.Tver.Tela., 4, 594 (1962).
2. E.Pytte. Ann.Phys. (USA), 32, 377 (1965).
3. V.N.Kashcheev, M.A.Krivoglaz. Fiz.Tver.Tela., 3, 1541 (1961).
4. T.Holstein, H.Primakoff. Phys.Rev., 58, 1098 (1940).
5. V.M.Agranovic, B.S.Tosic. Zh.Exp. i Teor.Fiz., 53, 149 (1967).
6. V.S.Tyablikov. Methods of the Quantum Theory of Magnetism. Plenum Press., N.Y., 1967.
7. F.J.Dyson. Phys.Rev., 102, 1217 (1956).

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