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ON THE POSITIVE ENERGY BOUND STATES OF NONLOCAL SEPARABLE POTENTIALS

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1. Introduction

One of the interesting features of the nonlocal separable potentials is the possibility of obtaining with their help the so-called "bound states with positive energy" (PEBS). In the case of local potentials such states can appear in the many-channel problems, when in one of the closed channels there exists a bound state. When the interaction between the channels is switched off, this state corresponds in the open channels to "bound state embedded in the continuum" [1-3]. After switching on the interactions between the different channels this state becomes resonance in the open channels. The situation with nonlocal potentials differs from that of local ones. In the case of nonlocal potentials one can obtain PEBS even in one-channel case [4]. This fact means that for a given positive energy besides the usual solution of the Schroedinger-equation with the asymptotic behaviour

$$\Psi(k,r) \xrightarrow[r \to \infty]{} A \sin[kr + \delta(k)]/r \tag{1}$$

there exists another solution which asymptotically decreases as $r \to \infty$,

$$\Psi(k,r) \xrightarrow[r \to \infty]{} 0 . \tag{2}$$

If one uses the usual definition of the phase shift $\delta(k)$ according to which $\delta(k)$ is a monotonic function of the energy (relative momenta) then these PEB states have to be counted in the Levinson's theorem, namely

$$\delta(k=0)-\delta(k=\infty)=(n+n')\pi , \tag{3}$$

where the number of the usual (negative energy) bound states is denoted by n and that of PEB states by n' /4-8/. (There exists another definition of the phase shift at the PEBS, see ref. /9/.)

For convenience, in the present paper we investigate only those PEB states which are obtained with the help of one-term nonlocal separable potentials

$$V(\vec{k},\vec{k}') = -\lambda g(k)\, g(k') . \tag{4}$$

According to the results of Martin and Gourdin /4,5/, one obtains a PEBS with the help of such a potential when the equations

$$g(k) = 0 \tag{5}$$

and

$$A(k) \equiv 1 - \lambda P \int \frac{2\mu g^2(p)\, dp}{p^2 - k^2} = 0 \tag{6}$$

are fulfilled simultaneously for a given k (here P denotes the principal value and μ stands for the reduced mass).

In 1968 Tabakin proposed a one-term separable potential of such a kind /10/ for the description of the $p-n$ scattering up to laboratory energy of 400 MeV and he claimed that this potential is convenient for the description of the attraction and repulsion in the two-nucleon interaction. Afterwards it became clear that this potential is not good for the description of the three-nucleon systems, it gives an additional three-body bound state below the energy of -300 MeV /11-13/. Now it is well known that the main reason for the failure of the Tabakin's one-term potential in the description of the three-nucleon systems lies in the fact that the two-body bound state obtained by this potential is not a 1s-state (deuteron) but a 2s-state /14/. It was pointed out, too, by Bolsterli /9/ and the Levinger's group /14/ that due to another (non-strong) interactions (e.g., due to the weak, electromagnetic ones) this PEBS probably appears as a resonance in the scattering cross section. Really, if one changes a little bit the interaction parameters (e.g., λ) then a resonance is obtained instead of the PEBS /4,5,12/. It is easy to see that a similar thing happens when one adds another interaction, e.g.,local one, to the original one. We want to show below that by means of coupling the channel in which there exists a PEBS to another one the PEBS becomes a resonance, too.

2. The Two-Channel Formalism

There exist different papers which are dealing with two-channel problems using nonlocal separable potentials /15-19/. In these papers the wave function is given by a two-component vector,

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} \tag{7}$$

and the two-body potential is given by a 2x2 matrix,

$$V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \tag{8}$$

which has the following matrix elements in the momentum representation:

$$<\vec{k}_i \mid V_{ij} \mid \vec{k}_j> = -\lambda_{ij}\, g_i(k_i)\, g_j(k_j), \tag{9}$$

where k_i is the relative momenta in the i -th channel. (Of course, $\lambda_{12} = \lambda_{21}$.) In this case the cross sections can be given by the expression

$$\sigma_{i\to j}(k_i^2) = \frac{4\pi^3}{k_i^2} \mid F_{ij}(k_i)\, T_{ij}(k_i) \mid^2, \tag{10}$$

where

$$F_{ij}(k_i) = -4\pi\sqrt{\mu_i\,\mu_j\,k_i\,k_j}\;g_i(k_i)g_j(k_j), \tag{11a}$$

$$T_{ij}(k_i) = \frac{1}{D(k_i)} \times \begin{cases} [\lambda_{ii} - dJ_l(k_l)] & \text{for } i = j, \text{ (here } l \neq i) \\ \lambda_{12} & \text{for } i \neq j, \end{cases} \tag{11b}$$

$$d = \lambda_{11}\lambda_{22} - \lambda_{12}^2, \tag{12}$$

$$D(k_i) = 1 - \lambda_{11}J_1(k_1) - \lambda_{22}J_2(k_2) + d\,J_1(k_1)J_2(k_2), \tag{13}$$

$$J_i(k_i) = 2\mu_i \int \frac{g_i^2(p)\,dp}{p^2 - k_i^2 - i\epsilon}. \tag{14}$$

Here $\mu_i = \frac{m_{i1}\,m_{i2}}{m_{i1} + m_{i2}}$ denotes the reduced mass in the i-th channel, and we have used the units $\hbar = c = 1$.

Let us assume that the first channel is open and investigate the transition amplitude $\mathcal{T}_{11}(k_1)$. Let us assume further that $J_1(k_1)$ and $J_2(k_2)$ are given by

$$J_l(k_l) = A_l + i\,B_l, \qquad l = 1,2. \tag{15}$$

Here the quantities A_l, B_l depend, of course, on k_l, but for simplicity we do not indicate this dependence. (For the real formfactors $g_l(k_l)$ we have $B_l \equiv 0$ when the l-th channel is closed at the given energy.)

After inserting expressions (13) and (15) into eq. (11b) we obtain the following expression for $\mathcal{T}_{11}(k_1) = F_{11}(k_1)\,T_{11}(k_1)$:

$$\mathcal{T}_{11}(k_1) = \frac{\Gamma_1 + i\Gamma_2}{(1 - \lambda_{11}A_1) - \Delta + \frac{i}{2}\Gamma}\,F_{11}(k_1) \tag{16}$$

with

$$\Gamma_1 = \lambda_{11} - dA_2 , \tag{17}$$

$$\Gamma_2 = -dB_2 , \tag{18}$$

$$\Delta = \lambda_{22} A_2 - d(A_1 A_2 - B_1 B_2), \tag{19}$$

$$\Gamma = -2[\lambda_{11} B_1 + \lambda_{22} B_2 - d(A_1 B_2 + A_2 B_1)]. \tag{20}$$

In the case when in the first channel we have a PEBS at $k_1 = k_0$ then, due to the fact that $1 - \lambda_{11} A_1$ depends linearly on k_1 at $k_1 \approx k_0$ and it becomes zero at $k_1 = k_0$ in the neighbourhood of $k_1 = k_0$ the matrix-elements $\mathcal{T}_{11}(k_1)$ can be written down as follows:

$$\mathcal{T}_{11}(k_1 \approx k_0) = \frac{\Gamma_1 + i\Gamma_2}{(k_1 - k_0) - \Delta + \frac{i}{2}\Gamma} F_{11}(k_1) . \tag{21}$$

From this equation one can see that we have a Breit-Wigner expression for the T -matrix with resonance shift (Δ) and resonance width (Γ) depending on k_1. It is easy to obtain the value of these parameters at $k_1 = k_0$ when we have a PEBS in the first channel. For that we have to use the fact that at $k_1 = k_0$ the following equalities hold

$$1 - \lambda_{11} A_1(k_0) = 0 , \qquad B_1(k_0) = 0 . \tag{22}$$

After inserting eq. (22) into eqs. (19) and (20) we obtain

$$\Delta(k_1 = k_0) = \lambda_{12}^2 A_1(k_0) A_2(k_1 = k_0). \tag{23}$$

$$\Gamma(k_1 = k_0) = -2\lambda_{12}^2 A_1(k_0) B_2(k_1 = k_0). \tag{24}$$

3. Calculations, Results and Discussion

For the illustration we show some results of the calculations with the formfactors

$$g_i(k_i) = \frac{1}{\beta_{i1}^2 + k_i^2} - \frac{a_i}{\beta_{i2}^2 + k_i^2}, \qquad \beta_{i2} > \beta_{i1}, \quad a_i > 0 . \tag{25}$$

With the help of such formfactors we obtain a PEBS in the uncoupled i -th channel at $k_i = k_{i0}$ if the equations

$$a_i = \frac{\beta_{i2}^2 + k_{i0}^2}{\beta_{i1}^2 + k_{i0}^2} \tag{26}$$

and

$$2\mu_i \lambda_{ii} \pi^2 = \frac{(\beta_{i1} + \beta_{i2})^2 \beta_{i1} \beta_{i2}}{(a_i \beta_{i1} - \beta_{i2})(a_i - 1)} \tag{27}$$

are fulfilled simultaneously. The pole motion for such a formfactor has been investigated in /20/.

We have fixed the interaction parameters in the following way: in the first channel we have taken both particles with the average nucleon masses ($m_{11} = m_{12} =$ 938.9 MeV), fixed the PEBS almost at the energy where the $^3S_1$ $p-n$ phase ''changes'' its sign (in our calculation $E_{lab}^{PEBS} =$ 259 MeV) and required that in the case of uncoupled channels the low-energy scattering parameters agree with the experimental ones. For our parameters the negative energy bound state is situated at the energy of $E =$ -2.225 MeV and the zero-energy total cross sections $\sigma_t(0) =$ 3 637 mb. In the second channel one of the particles has been taken with the same mass as in the first one ($m_{21} =$ 938.9 MeV) and the mass of the second ''particle'' has been taken different from the other ones. We have repeated the calculations for $m_{22} =$ 1038.9 MeV and $m_{22} =$ 1138.9 MeV. In the first (second) case the second channel is open (closed) at the energy which corresponds to the PEBS. The interaction parameters have been chosen in such a way that neither a bound state nor a resonance appeared in the decoupled second channel. The parameters used throughout this paper are given in table 1.

In fig. 1 we illustrate the dependence of the phase shift $\delta_{11}(k_1)$ on the relative momenta k_1 in the first channel when the interaction between the different channels is switched off $(\lambda_{12} = 0)$. For convenience, we have defined here the phase shift $\delta_{11}(k_1)$ connected with the quantity $\mathfrak{T}_{11}(k_1)$ by the equation

$$\mathfrak{T}_{11}(k_1) = -\frac{1}{\pi} e^{i\delta_{11}(k_1)} \sin \delta_{11}(k_1) \qquad (28)$$

not as a continuous function of k_1, but as those having a discontinuity of $+\pi$ at the PEBS. After switching on the interaction between the channels the PEBS becomes a resonance. One can see in fig. 1 also the phase shift $\delta_{11}(k_1)$ (the real part of $\delta_{11}(k_1)$ above the threshold of the second channel) for two different values of λ_{12} and for the case of m_{22} = 1138.9 MeV. The resonance energy and the resonance width change according to the formulas (19) and (20).

In fig. 2 we illustrate the behaviour of the phase shifts $\delta_{11}(k_1)$ (the real part of $\delta_{11}(k_1)$ above the threshold of the second channel, i.e., above k_1 = 306.41 MeV/c) and $Re[\delta_{22}(k_1)]$ in the vicinity of the momenta, corresponding to the PEBS in the decoupled first channel for the case of m_{22} = 1038.9 MeV. For comparison we show also the energy dependence of the eigenphase shift $\delta_+(k)$ defined for our case as /21/:

$$e^{2i\delta_{\pm}(k)} = \frac{S_{11}+S_{22}}{2} \pm \frac{1}{2}\sqrt{(S_{11} - S_{22})^2 + 4S_{12}^2} \qquad (29)$$

One can see that $Re[\delta_{22}(k_1)]$ changes similarly to the case when there is a "true" bound state in another channel /17/. The "strange" behaviour of $\delta_{11}(k_1)$ can be understood looking at the resonance circle /21/ * (see fig. 3) illustrating the dependence of the diagonal element $S_{11}(k_1)$ of the S-matrix and remembering that $Re[\delta_{11}(k_1)]$ can be obtained from the following expression

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* With the help of the resonance circles the resonance parameters can be obtained with a simple graphical method /22,23/.

Table 1

The interaction parameters

i	β_{i1} (fm^{-1})	β_{i2} (fm^{-1})	λ_{ii} (fm^{-2})	a_i
1	2.67	5.34	3.2233	3.0854
2	2.67	5.34	0.035	3.0854

$$tg\{2Re[\delta_{11}(k_1)]\} = \frac{Im[S_{11}(k_1)]}{Re[S_{11}(k_1)]} . \qquad (30)$$

We have checked the fulfilment of the many-channel Levinson's theorem /24-26/ and we have found that the following expressions hold:

$$\sum_{i=1}^{2} [\delta_{ii}(k_i = 0) - \delta_{ii}(k_i = \infty)] = \pi , \qquad (31a)$$

$$\sum_{i=+,-} [\delta_i(k_1 = 0) - \delta_i(k_1 = \infty)] = \pi . \qquad (31b)$$

We had really one bound state in the composite system of two coupled channels. (Of course, below the threshold of the second channel we have $\delta_{22} = 0$, $\delta_+ = \delta_{11}$, $\delta_- = 0$.).

Finally, we want to stress that our calculations indicate once more that potentials like the Tabakin's one-term separable potential cannot be used for the description of the $p-n$ interaction due to the fact that they contradict the experiments, according to which for the $^3S_1$ phase we have

$$\delta(k = 0) - \delta(k = \infty) = \pi$$

and there is no resonance at the energy where the phase shift for the above-mentioned potential ''changes'' its sign. Maybe, that similar potentials are more useful in describing another (non two-nucleon) problems (see refs. /27,28/).

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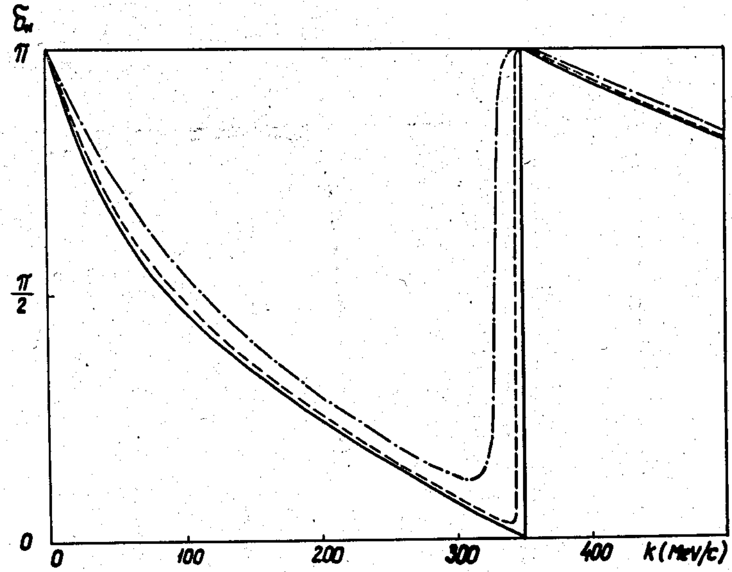

Fig. 1. The dependence of the phase shift $\delta_{11}(k_1)$ on k_1 for $\lambda_{12} = 0$ (solid line), for $\lambda_{12} = 0.3$ fm^{-2} (dashed line) and for $\lambda_{12} = 0.6$ fm^{-2} (dash-dotted line) in the case of $m_{22} = 1138.9$ MeV.

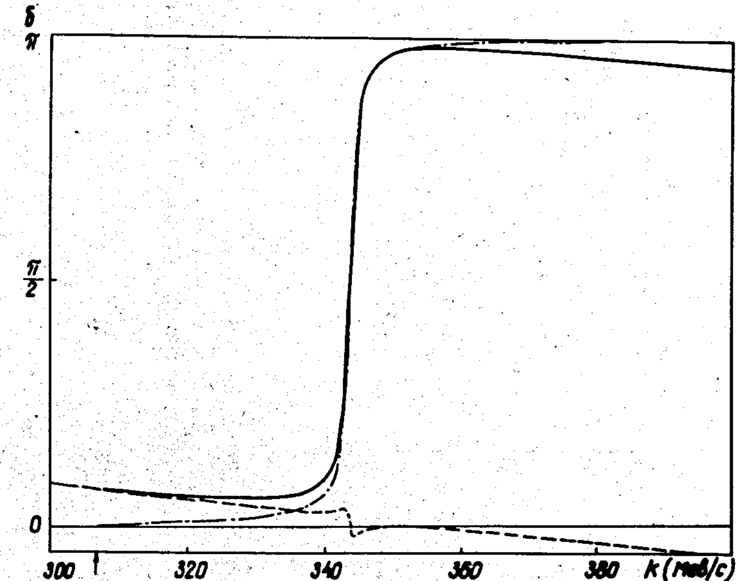

Fig. 2. The dependence of the phase shifts $Re\,[\delta_{11}(k_1)]$ (dashed line), $Re\,[\delta_{22}(k_1)]$ (dash-dotted line) and the eigenphase shift $\delta_+(k_1)$ (solid line) on k_1 for $\lambda_{12} = 0.3\ \mathrm{fm}^{-2}$ and $m_{22} = 1038.9$ MeV. The position of the threshold of the second channel is indicated by arrow.

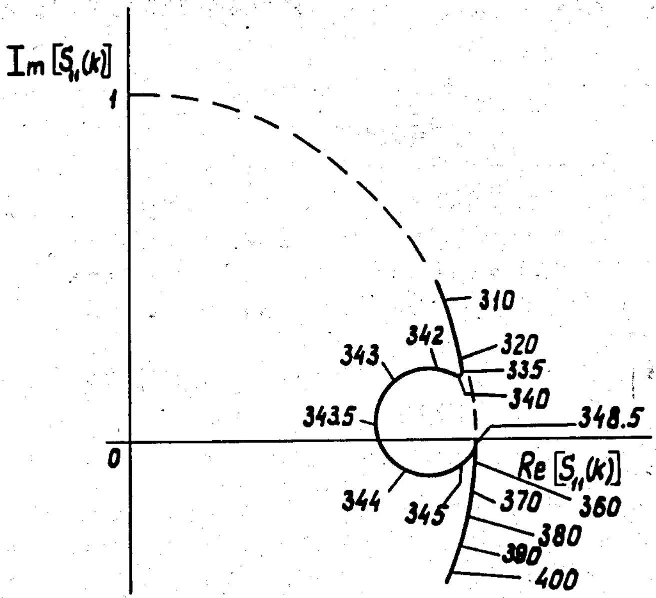

Fig. 3. The resonance circle in the matrix elements $S_{11}(k_1)$. At the trajectory we indicate some momentum values in units MeV/c.