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V.G.Soloviev

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In refs.[1-4] a new approach to studing the structure of highly excited states has been proposed. In the framework of this approach, the study of highly excited states proceeds along two lines: general semi-microscopic description based on the operator form of the wave function of a highly excited state and numerical calculations on the basis of the models taking into account quasiparticle-phonon interactions. In refs.[2,5], in the framework of the model taking into account interactions leading to superconducting pairing correlations and multipole forces, the complication of the structure of nuclear states with increasing excitation energy has been studied and the density of nonrotational states in ^{239}U has been calculated up to an excitation energy of 6 MeV. In the present paper we suggest a more general model describing interactions of quasiparticles with phonons in which, in addition to interactions leading to superconducting pairing correlations, we take into account multipole and spin-multipole forces.

1. The Hamiltonian of the Model

We formulate the model in the framework of the semi-microscopic approach (ref.[6]) for the case of a deformed odd mass nucleus. A generalization to the case of a spherical nucleus will be made elsewhere. The Hamiltonian describing interactions between nucleons in the nucleus is written in the form

$$H = H_{av} + H_{pair} + H_{Q} + H_{\sigma} , \qquad (1)$$

where H_{av} is the average nuclear field of the neutron and proton systems, H_{pair} interactions leading to superconducting pairing correlations, H_{Q} multipole-multipole interaction, H_{σ} spin-multipole - spin-multipole interaction.

We consider the interactions of quasi-particles with phonons in a deformed nucleus with an odd number of nucleons[6]. In the odd nucleus there is one quasi-particle in addition to quasi-particles and phonons of the even-even nucleus. The presence of this quasi-particle results in a slight change of phonons. One usually neglects this change and assumes that in an odd A nucleus phonons are indetical with those in an even-even $A-1$ nucleus. The multipole and spin-multipole interaction constants are fixed in defining phonons in even-even nuclei. Therefore in the calculation of the quasi-particle-phonon interaction there is not a single free parameter.

The simultaneous account of multipole and spin-multipole interactions leads to complicated secular equations for determining the phonon energy in even-even nuclei, ref.[6]. However, an admixture of spin-multipole forces effects little even low-lying quadrupole and octupole states. Therefore, usually one-phonon states with $K^{\pi}=0^{+},2^{+}$ and with $K^{\pi}=0^{-},1^{-},2^{-},3^{-}$ (K is the projection of the angular momentum on the nuclear symmetry axis) are described without introducing spin-multipole forces. On the other hand, the structure of the $K^{\pi}=1^{+}$ state is noticeably affected[7] by the spin-multipole interaction, while the influence of multipole forces is negligible. The role of phonons with $\lambda=1$ and $K^{\pi}=1^{+}$ may turn out to be important when explaining particular features of $M1$ transitions from neutron resonances.

Our mathematical apparatus is capable of using phonons. Therefore we calculate $\lambda\geq 4$ phonons, though the majority of such states are not collective. However, we commit no error since when the root of the secular equation for the phonon energy approaches the pole the wave function of the one-phonon state turns to the wave function of the two-quasi-particle state. For the sake of definiteness phonons with $\lambda\geq 4$ are calculated with multipole forces.

Thus, we use in the model spin-spin interactions with $\lambda=1$ for describing $K^{\pi}=1^{+}$ states and multipole-multipole interactions with $\lambda\geq 2$ for describing states with $K^{\pi}=0^{+}$, 2^{+}, 3^{+}, 4^{+},..., and with $K^{\pi}=0^{-}$, 1^{-}, 2^{-},...

An admixture of $\lambda \geq 4$ phonons to phonons with $\lambda = 1,2$ and 3 makes the spin space to two quasi-particles wider. Thus, if we take into account phonons with $\lambda = 1,2,3,...8$ then the spin and particles of two quasi-particles run over all the values from 0^- to 7^- and from 0^+ to 8^+. It should be noted that in spherical nuclei, using multipole interaction it is possible to describe only states with $I^\pi = \lambda^{(-1)^\lambda}$ and to enlarge the spin space for two quasi-particles it is necessary to introduce spin-multipole interactions with many λ values.

Taking into account the secular equations for determining the phonon energy ω_g and ω_n (where by g we denote $\lambda\mu j$ for multipole phonons, by n- we denote $\lambda = 1, \mu$ for spin phonons, j is the number of the root of the secular equation), the appropriate part of the Hamiltonian (1) can be written in the form[6,7]

$$H_M = H_{vq} + H_\sigma^1 , \qquad (2)$$

$$H_{vq} = \sum_q \epsilon(q) B(q,q) - \frac{1}{2} \sum_g \frac{1}{Y_q} \sum_{q,q'} \frac{(f_\sigma^q(q,q') U_{qq'}^{(+)})^2 (\epsilon(q)+\epsilon(q'))}{(\epsilon(q)+\epsilon(q'))^2 - \omega_g^2} Q_g^+ Q_g -$$

$$- \frac{1}{4} \sum_g \frac{1}{\sqrt{Y_g}} \sum_{q,q'} V_{q,q'}^{(-)} \{ (f^g(q,q') B(q,q') + \bar{f}^g(q,q') \bar{B}(q,q'))(Q_g^+ + Q_g) + \text{c.c.} \}, \qquad (3)$$

$$H^1_\sigma = H^1_\sigma(p) + H^1_\sigma(n) \tag{4}$$

$$H^1_\sigma(n) = -\frac{1}{2}\sum_n \frac{1}{Y_n}\sum_{ss'} \frac{(\sigma^{(1)}_{ss'}\, U^{(-)}_{ss'})^2 (\epsilon(S)+\epsilon(S'))}{(\epsilon(S)+\epsilon(S'))^2 - \omega_n^2} Q^+_n Q_n +$$

$$+\frac{1}{4}\sum_n \frac{1}{\sqrt{Y_n}}\sum_{ss'} V^{(+)}_{ss'}\{(\sigma^{(1)}_{ss'}\mathcal{B}(SS') + \bar{\sigma}^{(1)}_{ss'}\bar{\mathcal{B}}(SS'))(Q^+_n + Q_n) + \text{c.c.}\}. \tag{5}$$

Here Q_g , Q_n are the phonon operators, $\epsilon(g)=\sqrt{C^2+(E(g)-\lambda)^2}$ where C is the correlation function, λ chemical potential, $E(g)$ single-particle energy; $V^{(\pm)}_{qq'} = U_q U_{q'} \pm V_q V_{q'}$, , $U^{(\pm)}_{qq'} = U_q V_{q'} \pm U_{q'} V_q$ where U_q , V_q are the Bogolubov transformation coefficients. The set of the quantum numbers characterizing the single-particle level of the everage field is denoted by $(s\sigma)$ for the neutron system and by $(q\sigma)$ for both systems, $\sigma=\pm 1$. The matrix elements of the $\lambda\mu$ multipole moment operators and the spin operators are denoted as [6,7]

$$f^g_\sigma(q_1,q_2) = \begin{cases} f^g(q_1,q_2) & \text{provided } K_1 \pm \mu = K_2 \\ \sigma \bar{f}^g(q_1,q_2) & \text{provided } K_1 + K_2 = \pm\mu, \end{cases} \tag{6}$$

$$\sigma_{\sigma}^{(1)}(q_1,q_2)=\begin{cases} -\sigma\sigma^{(1)}_{q_1,q_2} & \text{provided} \quad K_1 \pm 1 = K_2 \\ \bar{\sigma}^{(1)}_{q_1,q_2} & \text{provided} \quad K_1 + K_2 = \pm 1 \end{cases} \tag{7}$$

where

$$\sigma^{(\mu)}_{qq'} = \langle q+|\sigma_{\mu} + (-)^{\mu}\sigma_{-\mu}|q'+\rangle , \quad \bar{\sigma}^{(\mu)}_{qq'} = \langle q+|\sigma_{\mu} + (-)^{\mu}\sigma_{-\mu}|q'-\rangle ,$$

σ_{μ} are the Pauli matrices. Further

$$B(q,q') = \sum_{\sigma} a^{+}_{q\sigma} a_{q'\sigma} , \qquad \bar{B}(q,q') = \sum_{\sigma} \sigma a^{+}_{q-\sigma} a_{q'\sigma}$$

$$\mathfrak{B}(q,q') = \sum_{\sigma} \sigma a^{+}_{q-\sigma} a_{q'-\sigma} , \qquad \bar{\mathfrak{B}}(q,q') = \sum_{\sigma} a^{+}_{q-\sigma} a_{q'\sigma} ,$$

$a^{+}_{q\sigma}$ is the quasi-particles creation operator,

$$Y_g = \sum_{q,q'} \frac{(f^{g}_{\sigma}(q,q')U^{(+)}_{qq'})^2 \omega_g (\epsilon(q)+\epsilon(q'))}{[(\epsilon(q)+\epsilon(q'))^2 - \omega_g^2]^2} ,$$

$$Y_n = \sum_{s,s'} \frac{(\sigma^{(1)}_{\sigma}(s,s')U^{(-)}_{s,s'})^2 \omega_n (\epsilon(s)+\epsilon(s'))}{[(\epsilon(s)+\epsilon(s'))^2 - \omega_n^2]^2} .$$

2. Formulation of the Model

The wave function of a nucleus with odd number of neutrons which describes states with a given K^{π} is written as

$$\Psi_i(K^\pi)=C_\rho^i\frac{1}{\sqrt{2}}\sum_\sigma\{a^+_{\rho\sigma}+\sum_{gs}D^{gi}_{\rho s\sigma}a^+_{s\sigma}Q^+_g+\sum_{ns}D^{ni}_{\rho s\sigma}a^+_{s\sigma}Q^+_n+$$

$$+\sum_{g_1g_2}\sum_s F^{g_1g_2i}_{\rho s\sigma}a^+_{s\sigma}Q^+_{g_1}Q^+_{g_2}+\sum_{n_1n_2}\sum_s F^{n_1n_2i}_{\rho s\sigma}a^+_{s\sigma}Q^+_{n_1}Q^+_{n_2}+$$

$$+\sum_{g,n}\sum_s F^{gni}_{\rho s\sigma}a^+_{s\sigma}Q^+_gQ^+_n+\frac{1}{\sqrt{3}}\sum_{g_1g_2g_3}\sum_s R^{g_1g_2g_3i}_{\rho s\sigma}a^+_{s\sigma}Q^+_{g_1}Q^+_{g_2}Q^+_{g_3}+$$

$$+\frac{1}{\sqrt{3}}\sum_{n\,n_2n_3}\sum_s R^{nn_2n_3i}_{\rho s\sigma}a^+_{s\sigma}Q^+_nQ^+_{n_2}Q^+_{n_3}+\sum_{g,g_2n}\sum_s R^{gg_2ni}_{\rho s\sigma}a^+_{s\sigma}Q^+_gQ^+_{g_2}Q^+_n+$$

$$+\sum_{g,n,n_2}\sum_s R^{g,n\,n_2i}_{\rho s\sigma}a^+_{s\sigma}Q^+_gQ^+_nQ^+_{n_2}\}\Psi_0\,,$$

where Ψ_0 is the wave function of the ground state of an even-even nucleus, by $(\rho\sigma)$ we denote the set of quantum numbers of the single-particle state with given K^π and by i the number of the state. The normalization condition is

$$(\Psi_i^*(K^\pi)\Psi_i(K^\pi))=1=(C_\rho^i)^2\{1+\frac{1}{2}\sum_{gs\sigma}(D^{gi}_{\rho s\sigma})^2+\frac{1}{2}\sum_{n,s,\sigma}(D^{ni}_{\rho s\sigma})^2+$$

$$+\sum_{g,g_2,s,\sigma}(F^{gg_2i}_{\rho s\sigma})^2+\sum_{n,n_2s\sigma}(F^{n,n_2i}_{\rho s\sigma})+\frac{1}{2}\sum_{n,g,s,\sigma}(F^{gni}_{\rho s\sigma})^2+\sum_{g,g_2g_3}\sum_{s\sigma}(R^{gg_2g_3i}_{\rho s\sigma})^2+$$

$$+\sum_{g,g_2n}\sum_{s\sigma}(R^{gg_2ni}_{\rho s\sigma})^2+\sum_{g,n,n_2}\sum_{s\sigma}(R^{gnn_2i}_{\rho s\sigma})^2+\sum_{n,n_2,n_3}\sum_{s\sigma}(R^{n,n_2,n_3}_{\rho s\sigma})^2\}=1.\qquad(9)$$

We calculate the average value of $H_M = H_{vq} + H'_\sigma$ over state (8), as a result we get

$$(\Psi_i^* (K^\pi) H_M \Psi_i (K^\pi) = (C_\rho^i)^2 \{ \epsilon(\rho) + \frac{1}{2} \sum_g \sum_{s\sigma} (\epsilon(s) + \omega_g)(D_{\rho s\sigma}^{gi})^2 -$$

$$- \frac{1}{2} \sum_g \sum_{s\sigma} \frac{v_{\rho s}^{(-)}}{\sqrt{Y_g}} f_\sigma^g(\rho s) D_{\rho s\sigma}^{gi} + \frac{1}{2} \sum_n \sum_{s\sigma} (\epsilon(s) + \omega_n)(D_{\rho s\sigma}^{ni})^2 + \frac{1}{2} \sum_n \sum_{s\sigma} \frac{v_{\rho s}^{(+)}}{\sqrt{Y_n}} \sigma_\sigma^{(1)}(\rho s) D_{\rho s\sigma}^{ni} +$$

$$+ \sum_{g;g_2} \sum_{s\sigma} (\epsilon(s) + \omega_g + \omega_{g_2})(F_{\rho s\sigma}^{gg_2 i})^2 + \sum_{n,n_2} \sum_{s\sigma} (\epsilon(s) + \omega_n + \omega_{n_2})(F_{\rho s\sigma}^{nn_2 i})^2 +$$

$$+ \frac{1}{2} \sum_{g,n} \sum_{s\sigma} (\epsilon(s) + \omega_g + \omega_n)(F_{\rho s\sigma}^{gni})^2 + \sum_{g,g_2,g_3} \sum_{s\sigma} (\epsilon(s) + \omega_g + \omega_{g_2} + \omega_{g_3})(R_{\rho s\sigma}^{gg_2 g_3 i})^2 +$$

$$+ \sum_{g,g_2,n} \sum_{s\sigma} (\epsilon(s) + \omega_g + \omega_{g_2} + \omega_n)(R_{\rho s\sigma}^{gg_2 ni})^2 + \sum_{g,n,n_2} \sum_{s\sigma} (\epsilon(s) + \omega_g + \omega_n + \omega_{n_2})(R_{\rho s\sigma}^{gnn_2 i})^2 +$$

$$+ \sum_{n,n_2,n_3} \sum_{s\sigma} (\epsilon(s) + \omega_n + \omega_{n_2} + \omega_{n_3})(R_{\rho s\sigma}^{nn_2 n_3 i})^2 +$$

$$+ \sum_{n,n_2} \sum_{s,s',\sigma} \frac{v_{ss'}^{(+)}}{\sqrt{Y_{n_2}}} (-\sigma \sigma_{ss'}^{(1)} D_{\rho s'\sigma}^{ni} + \bar{\sigma}_{ss'}^{(1)} D_{\rho s'-\sigma}^{ni}) F_{\rho s\sigma}^{nn_2 i} -$$

$$- \sum_{g,g_2} \sum_{s,s',\sigma} \frac{v_{ss'}^{(-)}}{\sqrt{Y_{g_2}}} (f^{g_2}(ss') D_{\rho s\sigma}^{gi} - \sigma f^{-g_2}(ss') D_{\rho s'-\sigma}^{gi}) F_{\rho s\sigma}^{gg_2 i} +$$

$$
\begin{aligned}
&+ \sum_{g,n} \sum_{s,s',\sigma} \Big[\frac{v^{(+)}_{ss'}}{\sqrt{Y_n}} \left(-\sigma\, \sigma^{(1)}_{ss'} D^{gi}_{\rho s'\sigma} + \bar{\sigma}^{(1)}_{ss'} D^{gi}_{\rho s' -\sigma} \right) F^{gni}_{\rho s \sigma} - \\
&- \frac{v^{(-)}_{ss'}}{\sqrt{Y_g}} \left(f^{g}(ss') D^{ni}_{\rho s' -\sigma} - \bar{f}^{g}(ss') D^{ni}_{\rho s' -\sigma} \right) F^{gni}_{\rho s\sigma} \Big] - \\
&- \sqrt{3} \sum_{g,g_2,g_3} \sum_{s,s'\sigma} \frac{v}{\sqrt{Y_{g_3}}} \left(f^{g_3}(ss') R^{gg_2g_3i}_{\rho s'\sigma} - \sigma f^{g_s}(ss') R^{gg_2g_3i}_{\rho s' -\sigma} \right) F^{gg_2i}_{\rho s\sigma} + \\
&+ \sqrt{3} \sum_{n,n_2,n_3} \sum_{s,s'\sigma} \frac{v^{(+)}_{ss'}}{\sqrt{Y_{n_3}}} \left(-\sigma\, \sigma^{(1)}_{ss'} R^{nn_2n_3i}_{\rho s'\sigma} + \bar{\sigma}^{(1)}_{ss'} R^{nn_2n_3i}_{\rho s'\sigma} \right) F^{nn_2i}_{\rho s\sigma} + \qquad (10) \\
&+ \sum_{g,g_2,n} \sum_{s,s',\sigma} \Big[\frac{v^{(+)}_{ss'}}{\sqrt{Y_n}} \left(-\sigma\, \sigma^{(1)}_{ss'} R^{gg_2ni}_{\rho s'\sigma} + \bar{\sigma}^{(1)}_{ss'} R^{gg_2ni}_{\rho s' -\sigma} \right) F^{gg_2i}_{\rho s\sigma} - \frac{v^{(-)}_{ss'}}{\sqrt{Y_{g_2}}} \Big(f^{g_2}(ss') R^{gg_2ni}_{\rho s'\sigma} - \\
&- \sigma \bar{f}^{g_2}(ss') R^{gg_2ni}_{\rho s' -\sigma} \Big) F^{gni}_{\rho s\sigma} \Big] + \sum_{g,n,n} \sum_{s,s',\sigma} \Big[\frac{v^{(+)}_{ss'}}{\sqrt{Y_{n_2}}} \left(-\sigma\, \sigma^{(1)}_{ss'} R^{gnn_2i}_{\rho s\sigma} + \bar{\sigma}^{(1)}_{ss'} R^{gnn_2i}_{\rho s' -\sigma} \right) F^{gni}_{\rho s\sigma} - \\
&- \frac{v^{(-)}_{ss'}}{\sqrt{Y_g}} \left(f^{g}(ss') R^{gnn_2i}_{\rho s'\sigma} - \sigma \bar{f}^{g}(ss') R^{gnn_2i}_{\rho s' -\sigma} \right) F^{nn_2i}_{\rho s\sigma} \Big] \Big\}.
\end{aligned}
$$

The energies of the nonrotational states η_i and the functions C, D, F and R are determined by means of the variational principle

$$\delta\{(\Psi_i^*(K^\pi)H_M\Psi_i(K^\pi)) - \eta_i[(\Psi_i^*(K^\pi)\Psi_i(K^\pi)) - 1]\} = 0 . \quad (11)$$

We performed a variation and a number of transformations. As a result we have the following system of equations

$$\epsilon(\rho) - \eta_i - \frac{1}{4}\sum_{g,s}\frac{(v^{(-)}_{\rho s})^2}{Y_g}\frac{(f^g_\sigma(\rho s))^2}{\epsilon(s)+\omega_g-\eta_i} - \frac{1}{4}\sum_{n,s}\frac{(v^{(+)}_{\rho s})^2}{Y_n}\frac{(\sigma^{(1)}_\sigma(\rho s))^2}{\epsilon(s)+\omega_n-\eta_i} -$$

$$-\frac{1}{4}\sum\{\frac{1}{2}\sum_{g,g_2}\frac{v^{(-)}_{\rho s}v^{(-)}_{ss'}}{\sqrt{Y_g Y_{g_2}}}[\frac{f^g_\sigma(\rho s)f^{g_2}(ss') + \sigma f^g_{-\sigma}(\rho s)\bar{f}^{g_2}(ss')}{\epsilon(s)+\omega_g-\eta_i} +$$

$$+\frac{f^{g_2}_\sigma(\rho s)f^g(ss') + \sigma f^g_\sigma(\rho s)f^{g_2}(ss')}{\epsilon(s)+\omega_{g_2}-\eta_i}]F^{gg_2 i}_{\rho s'\sigma} - \frac{1}{2}\sum_{n,n_2}\frac{v^{(+)}_{\rho s}v^{(+)}_{ss'}}{\sqrt{Y_n Y_{n_2}}}\times$$

$$\times(\frac{1}{\epsilon(s)+\omega_n-\eta_i} + \frac{1}{\epsilon(s)+\omega_{n_2}-\eta_i})(-\sigma\sigma^{(1)}_\sigma(\rho s)\sigma^{(1)}_{ss'} + \sigma^{(1)}_{-\sigma}(\rho s)\bar{\sigma}^{(1)}_{ss'})F^{nn_2 i}_{\rho s'\sigma} +$$

$$+\sum_{g,n}\frac{1}{\sqrt{Y_g Y_n}}[\frac{v^{(-)}_{\rho s}v^{(+)}_{ss'}}{\epsilon(s)+\omega_g-\eta_i}(-\sigma\sigma^{(1)}_{ss'}f^g_\sigma(\rho s) + \bar{\sigma}^{(1)}_{ss'}f^g_{-\sigma}(\rho s)) +$$

$$+ \frac{v^{(+)}_{\rho s} \; v^{(-)}_{ss'}}{\epsilon(s)+\omega_n-\eta_i} (\sigma^{(1)}_{\sigma}(\rho s)\, f^{g}(ss') + \sigma\sigma^{(1)}_{-\sigma}(\rho s)\, \bar{f}^{g}(ss'))]\, F^{gni}_{\rho s'\sigma} = 0\,, \quad (12)$$

$$(\epsilon(s)+\omega_g+\omega_{g_2}-\eta_i)\, F^{gg_2 i}_{\rho s\sigma} = \frac{1}{4}\, \frac{1}{\sqrt{Y_g\, Y_{g_2}}} \sum_{s'} \frac{v^{(-)}_{\rho s'}\; v^{(-)}_{ss'}}{\epsilon(s')+\omega_g-\eta_i} \times$$

$$(f^{g_2}(ss')\, f^{g}_{\sigma}(\rho s) - \sigma \bar{f}^{g_2}(ss')\, f^{g}_{-\sigma}(\rho s') + \frac{1}{2} \sum_{g_3, s_2, s_3} \frac{1}{\sqrt{Y_{g_2}\, Y_{g_3}}}\, \frac{v^{(-)}_{\rho s}\; v^{(-)}_{ss'}}{\epsilon(s_2)+\omega_g-\eta_i} \times$$

$$[(f^{g_2}(ss_2)\, f^{g_3}(s_3 s_2) + \bar{f}^{g_2}(ss_2)\, \bar{f}^{g_3}(s_3 s_2))\, F^{gg_3 i}_{\rho s_3 \sigma} +$$

$$+ \sigma (f^{g_2}(ss_2)\, \bar{f}^{g_3}(s_3 s_2) - \bar{f}^{g_2}(ss_2)\, \bar{f}^{g_3}(s_3 s_2))\, F^{gg_3 i}_{\rho s_3 -\sigma}] -$$

$$- \frac{1}{2} \sum_{n, s_2, s_3} \frac{1}{\sqrt{Y_{g_2} Y_n}}\, \frac{v^{(-)}_{ss_2}\; v^{(+)}_{s_3 s_2}}{\epsilon(s_2)+\omega_g-\eta_i} [-\sigma(f^{g_2}(ss_2)\, \sigma^{(1)}_{s_3 s_2} + \bar{f}^{g_2}(ss_2)\, \bar{\sigma}^{(1)}_{s_3 s_2})\, F^{gni}_{\rho s_3 \sigma} +$$

$$+ (f^{g_2}(ss_2)\, \bar{\sigma}^{(1)}_{s_3 s_2} - \bar{f}^{g_2}(ss_2)\, \sigma^{(1)}_{s_3 s_2})\, F^{gni}_{\rho s_3 -\sigma}] + \frac{3}{4} \sum_{g_3, s_2} \frac{1}{Y_{g_3}}\, \frac{1}{\epsilon(s_2)+\omega_g+\omega_{g_2}+\omega_{g_3}-\eta_i} \times$$

$$\times \sum_{s_3} v^{(-)}_{ss_2} v^{(-)}_{s_3 s_2} [(f^{g_3}(ss_2) f^{g_3}(s_3 s_2) + \bar{f}^{g_3}(ss_2) \bar{f}^{g_3}(s_3 s_2)) F^{gg_2 i}_{\rho s_3 \sigma} +$$

$$+ \sigma (f^{g_3}(ss_2) \bar{f}^{g_3}(s_3 s_2) - \bar{f}^{g_3}(ss_2) f^{g_3}(s_3 s_2)) F^{gg_2 i}_{\rho s_3 - \sigma}] -$$

$$- \frac{1}{4} \sum_{n, s_2} \frac{v^{(+)}_{ss_2}}{\sqrt{Y_n}} \frac{1}{\epsilon(s_2) + \omega_g + \omega_{g_2} + \omega_n - \eta_i} \times \tag{13}$$

$$\times \{ \frac{1}{\sqrt{Y_{g_2}}} \sum_{s_3} v^{(-)}_{s_3 s_2} [-\sigma (\sigma^{(1)}_{ss_2} f^{g_2}(s_3 s_2) + \bar{\sigma}^{(1)}_{ss_2} \bar{f}^{g_2}(s_3 s_2)) F^{gni}_{\rho s_3 \sigma} +$$

$$+ (\bar{\sigma}^{(1)}_{ss_2} f^{g_2}(s_1 s_2) - \sigma^{(1)}_{ss_2} \bar{f}^{g_2}(s_3 s_2)) F^{gni}_{\rho s_3 - \sigma}] -$$

$$- \frac{1}{\sqrt{Y_n}} \sum_{s_3} v^{(+)}_{s_3 s_2} [(\sigma^{(1)}_{ss_2} \sigma^{(1)}_{s_3 s_2} + \bar{\sigma}^{(1)}_{ss_2} \bar{\sigma}^{(1)}_{s_3 s_2}) F^{gg_2 i}_{\rho s_3 \sigma} -$$

$$- \sigma (\sigma^{(1)}_{ss_2} \bar{\sigma}^{(1)}_{s_3 s_2} - \bar{\sigma}^{(1)}_{ss_2} \sigma^{(1)}_{s_3 s_2}) F^{gg_2 i}_{\rho s_3 - \sigma}]\},$$

We should also perform a symmetrization in q and q_2 in the r.h.s. of (13),

$$(\epsilon(s) + \omega_n + \omega_{n_2} - \eta_i) F^{nn_2 i}_{\rho s \sigma} =$$

$$= \frac{1}{4} \frac{1}{\sqrt{Y_n Y_{n_2}}} \sum_{s'} v^{(+)}_{\rho s'} v^{(+)}_{ss'} \frac{-\sigma \sigma^{(1)}_{ss'} \sigma^{(1)}_{\sigma}(\rho s') + \bar{\sigma}^{(1)}_{ss'} \sigma^{(1)}_{-\sigma}(\rho s')}{\epsilon(s') + \omega_n - \eta_i} +$$

$$+ \frac{1}{2} \sum_{n_3, s_2, s_3} \frac{1}{\sqrt{Y_{n_2} Y_{n_3}}} \frac{v^{(+)}_{ss_2} v^{(+)}_{s_3 s_2}}{\epsilon(s_2) + \omega_n - \eta_i} [(\sigma^{(1)}_{ss_2} \sigma^{(1)}_{s_3 s_2} + \bar{\sigma}^{(1)}_{ss_2} \bar{\sigma}^{(1)}_{s_3 s_2}) F^{nn_3 i}_{\rho s_3 \sigma} + \sigma(\bar{\sigma}^{(1)}_{ss_2} \sigma^{(1)}_{s_3 s_2} -$$

$$- \sigma^{(1)}_{ss_2} \bar{\sigma}^{(1)}_{s_3 s_2}) F^{nn_3 i}_{\rho s_3 -\sigma}] - \frac{1}{2} \sum_{g, s_2, s_3} \frac{1}{\sqrt{Y_g Y_{n_2}}} \frac{v^{(+)}_{ss_2} v^{(-)}_{s_3 s_2}}{\epsilon(s_2) + \omega - \eta} [-\sigma(\sigma^{(1)}_{ss_2} f^{g}(s_3 s_2) +$$

$$+ \bar{\sigma}^{(1)}_{s_3 s_2} \bar{f}^{g}(s_3 s_2)) F^{gni}_{\rho s_3 \sigma} + (-\sigma^{(1)}_{ss_2} \bar{f}^{g}(s_3 s_2) + \bar{\sigma}^{(1)}_{ss_2} f^{g}(s_3 s_2)) F^{gni}_{\rho s_3 -\sigma} +$$

$$+ \frac{3}{4} \sum_{n_3 g_2 g_3} \frac{1}{Y_{n_3}} \frac{v^{(+)}_{ss_2} v^{(+)}_{s_2 s_3}}{\epsilon(s_2) + \omega_n + \omega_{n_2} + \omega_{n_3} - \eta_i} \times$$

$$\times [(\sigma^{(1)}_{ss_2} \sigma^{(1)}_{s_3 s_2} + \bar{\sigma}^{(1)}_{ss_2} \bar{\sigma}^{(1)}_{s_3 s_2}) F^{nn_2 i}_{\rho s_3 \sigma} + \sigma(\bar{\sigma}^{(1)}_{ss_2} \sigma^{(1)}_{s_3 s_2} - \sigma^{(1)}_{ss_2} \bar{\sigma}^{(1)}_{s_3 s_2}) F^{nn_2 i}_{\rho s_3 -\sigma}] +$$

$$+\frac{1}{4}\sum_{g,s_2,s_3}\frac{1}{\sqrt{Y_g}}\frac{v^{(-)}_{ss_2}}{\epsilon(s_2)+\omega_g+\omega_n+\omega_{n_2}-\eta_i}\{\frac{v^{(-)}_{s_3s_2}}{\sqrt{Y_g}}[f^g(ss_2)f^g(s_3s_2)+$$

$$+\bar{f}^g(ss_2)\bar{f}^g(s_3s_2))F^{nn_2i}_{\rho s_3\sigma}+\sigma(f^g(ss_2)\bar{f}^g(s_3s_2)-\bar{f}^g(ss_2)f(s_3s_2))F^{nn_2i}_{\rho s_3-\sigma}]-$$

$$-\frac{v^{(+)}_{s_3s_2}}{\sqrt{Y_n}}[-\sigma(f^g(ss_2)\sigma^{(1)}_{s_3s_2}+\bar{f}^g(ss_2)\bar{\sigma}^{(1)}_{s_3s_2})F^{gni}_{\rho s_3\sigma}+ \qquad (14)$$

$$+(f^g(ss_2)\bar{\sigma}^{(1)}_{s_3s_2}-\bar{f}^g(ss_2)\sigma^{(1)}_{s_3s_2})F^{gni}_{\rho s_3-\sigma}]\},$$

and a symmetrization in n and n_2 in the r.h.s. of (14).

$$(\epsilon(s)+\omega_n+\omega_g-\eta_i)F^{gni}_{\rho s\sigma}=$$

$$=\frac{1}{2}\frac{-1}{\sqrt{Y_nY_g}}\sum_{s'}[v^{(+)}_{ss'}v^{(-)}_{\rho s'}\frac{-\sigma\sigma^{(1)}_{ss'}f_\sigma(\rho s')+\bar{\sigma}^{(1)}_{ss'}f^g_\sigma(\rho s')}{\epsilon(s')+\omega_g-\eta_i}+$$

$$+v^{(-)}_{ss'}v^{(+)}_{\rho s'}\frac{f^g(ss')\sigma^{(1)}_\sigma(\rho s')-\sigma f^g(ss')\sigma^{(1)}_{-\sigma}(\rho s')}{\epsilon(s')+\omega_n-\eta_i}]-\sum_{g,s,s}\frac{1}{\sqrt{Y_nY_{g_2}}}\frac{v^{(+)}_{ss_2}v^{(-)}_{s_3s_2}}{\epsilon(s_2)+\omega_g-\eta_i}\times$$

$$\times \Big[-\sigma\big(\sigma^{(1)}_{ss_2}\, f^{g_2}(s_3 s_2) + \bar{\sigma}^{(1)}_{ss_2}\, \bar{f}^{g_2}(s_3 s_2)\big) F^{gg_2 i}_{\rho s_3 \sigma} +$$

$$+ \big(\bar{\sigma}^{(1)}_{ss_2}\, f^{g_2}(s_3 s_2) - \sigma^{(1)}_{ss_2}\, \bar{f}^{g_2}(s_3 s_2)\big) F^{gg_2 i}_{\rho s_3 -\sigma}\Big] +$$

$$+ \sum_{n\, s_2, s_3} \frac{1}{\sqrt{Y_n Y_{n_2}}} \frac{v^{(+)}_{ss_2} v^{(+)}_{s_3 s_2}}{\epsilon(s_2)+\omega_g-\eta_i} \Big[\big(\sigma^{(1)}_{ss_2}\, \sigma^{(1)}_{s_3 s_2} + \bar{\sigma}^{(1)}_{ss_2}\, \bar{\sigma}^{(1)}_{s_3 s_2}\big) F^{gn_2 i}_{\rho s_3 \sigma} + \sigma\big(\bar{\sigma}^{(1)}_{ss_2}\, \sigma^{(1)}_{s_3 s_2} +$$

$$- \sigma^{(1)}_{ss_2}\, \bar{\sigma}^{(1)}_{s_3 s_2}\big) F^{gn_2 i}_{\rho s_3 -\sigma}\Big] + \sum_{g_2, s_2, s_3} \frac{1}{\sqrt{Y_g Y_{g_2}}} \frac{v^{(-)}_{ss_2} v^{(-)}_{s_3 s_2}}{\epsilon(s_2)+\omega - \eta} \Big[\big(f^{g}(ss_2)\, f^{g_2}(s_3 s_2) +$$

$$+ \bar{f}^{g}(ss_2)\, \bar{f}^{g_2}(s_3 s_2)\big) F^{g_2 ni}_{\rho s\, \sigma} + \sigma\big(f^{g}(ss_2)\, \bar{f}^{g_2}(s_3 s_2) - \bar{f}^{g}(ss_2)\, f^{g_2}(s_3 s_2)\big) F^{g_2 ni}_{\rho s_3 -\sigma}\Big] -$$

$$- \sum_{n_2, s_2, s_3} \frac{1}{\sqrt{Y_g Y_{n_2}}} \frac{v^{(-)}_{ss_2} v^{(+)}_{s_3 s_2}}{\epsilon(s_2)+\omega-\eta} \Big[-\sigma\big(f^{g}(ss_2)\, \sigma^{(1)}_{s_3 s_2} + \bar{f}^{g}(ss_2)\, \bar{\sigma}^{(1)}_{s_3 s_2}\big) F^{nn_2 i}_{\rho s_3 \sigma} +$$

$$+ \big(f^{g}(ss_2)\, \bar{\sigma}^{(1)}_{s_3 s_2} - \bar{f}^{g}(ss_2)\, \sigma^{(1)}_{s_3 s_2}\big) F^{nn_2 i}_{\rho s_3 -\sigma}\Big] +$$

$$+ \frac{1}{2} \sum_{g_2, s_2, s_3} \frac{1}{\sqrt{Y_{g_2}}} \frac{v^{(-)}_{ss_2}}{\epsilon(s_2)+\omega_g+\omega_{g_2}+\omega_n-\eta_i} \Big\{ \frac{v^{(-)}_{s_3 s_2}}{\sqrt{Y_{g_2}}} \times$$

$$\times [(f^{g_2}(ss_2) f^{g_2}(s_3 s_2) + \bar{f}^{g_2}(ss_2) \bar{f}^{g_2}(s_3 s_2)) F^{gni}_{\rho s_3 \sigma} +$$

$$+ \sigma (f^{g_2}(ss_2) \bar{f}^{g_2}(s_3 s_2) - \bar{f}^{g_2}(ss_2) f^{g_2}(s_3 s_2)) F^{gni}_{\rho s_3 - \sigma}] -$$

$$- \frac{v^{(+)}_{s_3 s_2}}{\sqrt{Y_n}} [-\sigma (f^{g_2}(ss_2) \sigma^{(1)}_{s_3 s_2} + \bar{f}^{g_2}(ss_2) \bar{\sigma}^{(1)}_{s_3 s_2}) F^{gg_2 i}_{\rho s_3 \sigma} +$$

$$+ (f^{g_2}(ss_2) \bar{\sigma}^{(1)}_{s_3 s_2} - \bar{f}^{g_2}(ss_2) \sigma^{(1)}_{s_3 s_2}) F^{gg_2 i}_{\rho s_3 - \sigma}]\} - \qquad (15)$$

$$- \frac{1}{2} \sum_{n_2, s_2, s_3} \frac{v^{(+)}_{s_3 s_2}}{\sqrt{Y_{n_2}}} \frac{1}{\epsilon(s_2) + \omega_g + \omega_n + \omega_{n_2} - \eta_i} \{ \frac{v^{(-)}_{s_3 s_2}}{\sqrt{Y_g}} [-\sigma (\sigma^{(1)}_{ss_2} f^{g}(s_3 s_2) +$$

$$+ \bar{\sigma}^{(1)}_{ss_2} \bar{f}^{g}(s_3 s_2)) F^{nn_2 i}_{\rho s_3 \sigma} + (\bar{\sigma}^{(1)}_{ss_2} f^{g}(s_3 s_2) - \sigma^{(1)}_{ss_2} f^{g}(s_3 s_2)) F^{nn_2 i}_{\rho s_3 - \sigma}] -$$

$$- \frac{v^{(+)}_{s_3 s_2}}{\sqrt{Y_{n_2}}} [(\sigma^{(1)}_{ss_2} \sigma^{(1)}_{s_3 s_2} + \bar{\sigma}^{(1)}_{ss_2} \bar{\sigma}^{(1)}_{s_3 s_2}) F^{gni}_{\rho s_3 \sigma} + \sigma (\bar{\sigma}^{(1)}_{ss_2} \sigma^{(1)}_{s_3 s_2} - \sigma^{(1)}_{ss_2} \bar{\sigma}^{(1)}_{s_3 s_2}) F^{gni}_{\rho s_3 - \sigma}]\}.$$

$$D^{gi}_{\rho s\sigma} = \frac{1}{2}\frac{v^{(-)}_{\rho s}}{\sqrt{Y_g}}\frac{f^{g}_{\sigma}(\rho s)}{\epsilon(s)+\omega_g-\eta_i} + \frac{1}{\epsilon(s)+\omega_g-\eta_i}\sum_{g_2,s_2}\frac{v^{(-)}_{ss_2}}{\sqrt{Y_{g_2}}}(f^{g_2}(ss_2)F^{gg_2 i}_{\rho s_2\sigma} - \tag{16}$$

$$-\sigma\bar{f}^{g_2}(ss_2)F^{gg_2 i}_{\rho s_2 -\sigma}) - \frac{1}{\epsilon(s)+\omega_g-\eta_i}\sum_{n,s_2}\frac{v^{(+)}_{ss_2}}{\sqrt{Y_n}}(-\sigma\sigma^{(1)}_{ss_2}F^{gni}_{\rho s_2\sigma} + \bar{\sigma}^{(1)}_{ss_2}F^{gni}_{\rho s_2 -\sigma}),$$

$$D^{ni}_{\rho s\sigma} = -\frac{1}{2}\frac{v^{(+)}_{\rho s}}{\sqrt{Y_n}}\frac{\sigma^{(1)}_{\sigma}(\rho s)}{\epsilon(s)+\omega_n-\eta_i} + \frac{1}{\epsilon(s)+\omega_n-\eta_i}\sum_{g,s_2}\frac{v^{(-)}_{ss_2}}{\sqrt{Y_g}}(f^{g}(ss_2)F^{gni}_{\rho s_2\sigma} - \tag{17}$$

$$-\sigma\bar{f}^{g}(ss_2)F^{gni}_{\rho s_2 -\sigma}) - \frac{1}{\epsilon(s)+\omega_n-\eta_i}\sum_{n_2,s_2}\frac{v^{(+)}_{ss_3}}{\sqrt{Y_{n_2}}}(-\sigma\sigma^{(1)}_{ss_2}F^{gn_2 i}_{\rho s_2\sigma} + \bar{\sigma}^{(1)}_{ss_2}F^{gn_2 i}_{\rho s_2 -\sigma}).$$

$$R^{gg_2g_3 i}_{\rho s\sigma} = \frac{\sqrt{3}}{2}\frac{1}{\sqrt{Y_{g_3}}}\frac{\sum_{s'} v^{(-)}_{ss'}(f^{g_3}(ss')F^{gg_2 i}_{\rho s'\sigma} - \sigma\bar{f}^{g_3}(ss')F^{gg_2 i}_{\rho s' -\sigma})}{\epsilon(s)+\omega_g+\omega_{g_2}+\omega_{g_3}-\eta_i}, \tag{18}$$

$$R^{gg_2 ni}_{\rho s\sigma} = \frac{1}{2}\frac{1}{\epsilon(s)+\omega_g+\omega_{g_2}+\omega_n-\eta_i}\sum\Big\{\frac{v^{(-)}_{ss'}}{\sqrt{Y_{g_2}}}(f^{g_2}(ss')F^{gni}_{\rho s'\sigma} - \tag{19}$$

$$-\sigma\bar{f}^{g_2}(ss')F^{gni}_{\rho s' -\sigma}) - \frac{v^{(+)}_{ss'}}{\sqrt{Y_n}}(-\sigma\sigma^{(1)}_{ss'}F^{gg_2 i}_{\rho s'\sigma} + \bar{\sigma}^{(1)}_{ss'}F^{gg_2 i}_{\rho s' -\sigma})\Big\},$$

$$R^{gn n_2 i}_{\rho s \sigma} = \frac{1}{2}\frac{1}{\epsilon(s)+\omega_g+\omega_n+\omega_{n_2}-\eta_i}\sum\left\{\frac{v^{(-)}_{ss'}}{\sqrt{Y_g}}\left(f^{g}(ss')\,F^{nn_2 i}_{\rho s'-\sigma} - \sigma\,\bar{f}^{g}(ss')\,F^{nn_2 i}_{\rho s'-\sigma}\right) - \frac{v^{(+)}_{ss'}}{\sqrt{Y_{n_2}}}\left(-\sigma\sigma^{(1)}_{ss'}F^{gni}_{\rho s'\sigma} + \bar{\sigma}^{(1)}_{ss'}F^{gni}_{\rho s'-\sigma}\right)\right\}, \tag{20}$$

$$R^{nn_2n_3 i}_{\rho s \sigma} = -\frac{\sqrt{3}}{2}\frac{1}{\sqrt{Y_{n_3}}}\frac{\sum_{s'} v^{(+)}_{ss'}\left(-\sigma\sigma^{(1)}_{ss'}F^{nn_2 i}_{\rho s'\sigma} + \bar{\sigma}^{(1)}_{ss'}F^{nn_2 i}_{\rho s'-\sigma}\right)}{\epsilon(s)+\omega_n+\omega_{n_2}+\omega_{n_3}-\eta_i}. \tag{21}$$

The r.h.s. of expressions (18-21) should be symmetrized in g, g_2, g_3; g, g_2; n, n_2 and n, n_2, n_3, respectively.

If we find the functions F from eqs. (13), (14) and (15) and insert them in (12) we get a secular equation for determining the energies of nonrotational states η_i. Knowing η_i we obtain the functions D and R from eqs.(16-21) and the function C from the normalization condition (9). If we reject the spin phonons Q_n we are led to the system of equations which was derived in ref.[5]

3. An Approximate Solution

Eqs. (13), (14) and (15) contain coherent terms with squared matrix elements, i.e. $(f^{g}_{\sigma}(s,s'))^2$ and $(\sigma^{(1)}_{\sigma}(ss'))^2$ and noncoherent terms with the products

$f^{g}_{\sigma}(ss'')\,f^{g'}_{\sigma}(s's'')$, $\sigma^{(1)}_{\sigma}(ss'')\,\sigma^{(1)}_{\sigma}(s's'')$ $\quad f^{g}_{\sigma}(ss'')\,\sigma^{(1)}_{\sigma}(s's'')$.

We keep the noncoherent free terms and reject the noncoherent terms containing F. As a result we get expressions for F in the explicit form

$$F^{gg_2 i}_{\rho s \sigma} = \frac{1}{8\sqrt{Y_g Y_{g_2}}} \frac{1}{\epsilon(s) + \omega_g + \omega_{g_2} - \eta_i - S^i_{gg_2}(s) - T^{i(-)}_{gg_2}(s) - T^{i(+)}_{gg_2}(s)} \times$$

$$\times \sum_{s'} v^{(-)}_{\rho s'} v^{(-)}_{ss'} \Big[\frac{f^{g_2}(ss') f^{g}_{\sigma}(\rho s') - \sigma \bar{f}^{g_2}(ss') f^{g}_{\sigma}(\rho s')}{\epsilon(s') + \omega_g - \eta_i} + \qquad (22)$$

$$+ \frac{f^{g_2}(ss') f^{g_2}_{\sigma}(\rho s') - \sigma \bar{f}^{g}(ss') f^{g_2}_{-\sigma}(\rho s')}{\epsilon(s') + \omega_{g_2} - \eta_i} \Big],$$

$$F^{nn_2 i}_{\rho s \sigma} = \frac{1}{8\sqrt{Y_n Y_{n_2}}} \frac{1}{\epsilon(s) + \omega_n + \omega_{n_2} - \eta_i - S^i_{nn_2}(s) - T^{i(+)}_{nn_2}(s) - T^{i(-)}_{nn_2}(s)} \times$$

$$(23)$$

$$\times \sum_{s'} v^{(+)}_{\rho s'} v^{(+)}_{ss'} \Big[\frac{1}{\epsilon(s') + \omega_n - \eta_i} + \frac{1}{\epsilon(s') + \omega_{n_2} - \eta_i} \Big] (-\sigma \sigma^{(1)}_{ss'} \sigma^{(1)}_{\sigma}(\rho s') + \bar{\sigma}^{(1)}_{ss'} \sigma^{(1)}_{\sigma}(\rho s')),$$

$$F^{gni}_{\rho s \sigma} = \frac{-1}{2\sqrt{Y_g Y_n}} \frac{1}{\epsilon(s) + \omega_g + \omega_n - \eta_i - S^i_{gn}(s) - T^{i(-)}_{gn}(s) - T^{i(+)}_{gn}(s)} \times$$

$$\times \sum_{s'} \{ v^{(+)}_{ss'} v^{(-)}_{\rho s'} \frac{-\sigma\sigma^{(1)}_{ss'} f^{g}_{\sigma}(\rho s') + \bar{\sigma}^{(1)}_{ss'} f^{g}_{-\sigma}(\rho s')}{\epsilon(s') + \omega_g - \eta_i} + v^{(+)}_{ss'} v^{(+)}_{\rho s'} \times$$

$$\times \frac{f^{g}(ss')\sigma^{(1)}_{\sigma}(\rho s') - \sigma \bar{f}^{g}(ss')\sigma^{(1)}_{-\sigma}(\rho s')}{\epsilon(s') + \omega_n - \eta_i} \}. \qquad (24)$$

The following notation is used

$$S^{i}_{gg_2}(s) = \frac{1}{4\sqrt{Y_g Y_{g_2}}} \sum (v^{(1)}_{ss'})^2 [\frac{(f^{g_2}_{\sigma}(ss'))^2}{\epsilon(s')+\omega_g - \eta_i} + \frac{(f^{g}_{\sigma}(ss'))^2}{\epsilon(s')+\omega_{g_2} - \eta_i}], \qquad (25)$$

$$S^{i}_{nn_2}(s) = \frac{1}{4\sqrt{Y_n Y_{n_2}}} \sum_{s'} (v^{(+)}_{ss'})^2 [\frac{1}{\epsilon(s') + \omega_n - \eta_i} + \frac{1}{\epsilon(s) - \omega_{n_2} - \eta_i}](\sigma^{(1)}_{\sigma}(ss'))^2, \qquad (25')$$

$$S^{i}_{gn}(s) = \frac{1}{Y_n} \sum_{s'} (v^{(+)}_{ss'})^2 \frac{(\sigma^{(1)}_{\sigma}(ss'))^2}{\epsilon(s')+\omega_g - \eta_i} + \frac{1}{Y_g} \sum_{s'} (v^{(-)}_{ss'})^2 \frac{(f^{g}_{\sigma}(ss'))^2}{\epsilon(s) + \omega_n - \eta_i}, \qquad (25'')$$

$$T^{i(-)}_{gg_2}(s) = \frac{3}{4} \sum_{g_3} \sum_{s'} \frac{(v^{(-)}_{ss'})^2}{Y_{g_3}} \frac{(f^{g_3}_{\sigma}(ss'))^2}{\epsilon(s') + \omega_g + \omega_{g_2} + \omega_{g_3} - \eta_i}, \qquad (26)$$

$$T^{i(+)}_{gg_2}(s) = \frac{1}{4} \sum_{n} \sum_{s'} \frac{(v^{(+)}_{ss'})^2}{Y_n} \frac{(\sigma^{(1)}_{\sigma}(ss'))^2}{\epsilon(s')+\omega_g + \omega_{g_2} - \omega_n - \eta_i}, \qquad (26')$$

$$T^{i(+)}_{nn_2}(s) = \frac{3}{4}\sum_{n_3}\sum_{s'} \frac{(v^{(+)}_{ss'})^2}{Y_{n_3}} \frac{(\sigma^{(1)}_{\sigma}(ss'))^2}{\epsilon(s') + \omega_n + \omega_{n_2} + \omega_{n_3} - \eta_i}, \qquad (27)$$

$$T^{i(-)}_{nn_2}(s) = \frac{1}{4}\sum_{g}\sum_{s'} \frac{(v^{(-)}_{ss'})^2}{Y_g} \frac{(f^{g}_{\sigma}(ss'))^2}{\epsilon(s') + \omega_n + \omega_{n_2} + \omega_g - \eta_i}, \qquad (27')$$

$$T^{i(-)}_{gn_2}(s) = \frac{1}{2}\sum_{g_2}\sum_{s'} \frac{(v^{(-)}_{ss'})^2}{Y_{g_2}} \frac{(f^{g_2}_{\sigma}(ss'))^2}{\epsilon(s') + \omega_g + \omega_{g_2} + \omega_n - \eta_i}, \qquad (28)$$

$$T^{i(+)}_{gn}(s) = \frac{1}{2}\sum_{n_2}\sum_{s'} \frac{(v^{(+)}_{ss'})^2}{Y_{n_2}} \frac{(\sigma^{(1)}_{\sigma}(ss'))^2}{\epsilon(s') + \omega_g + \omega_n + \omega_{n_2} - \eta_i}, \qquad (28')$$

The functions F defined by (22), (23) and (24) are inserted in (12). Then we obtain the explicit form of the secular equation for determining the energies η_i of nonrotational states. Now it is not difficult to calculate the functions C, D, F and R and thereby derive the expressions for the wave function (8).

An approximate solution of the secular equation must not strongly differ from the exact one since the role of the rejected noncoherent terms is not great. However, this leads to the appearance of extraneous roots. To exclude these roots it is necessary to take into account a part of noncoherent terms in eqs.(13), (14) and (15).

As a first step toward studying this model we should investigate a particular case when in the wave function (8) we put $R = 0$. Then the equations read

$$\epsilon(\rho) - \eta_i - \frac{1}{4} \sum_g \sum_{s\sigma} \frac{v^{(-)}_{\rho s}}{\sqrt{Y_g}} f^g_\sigma(\rho s) D^{gi}_{\rho s \sigma} + \tag{29}$$

$$+ \frac{1}{4} \sum_n \sum_{s\sigma} \frac{v^{(+)}_{\rho s}}{\sqrt{Y_n}} \sigma^{(1)}_\sigma(\rho s) D^{ni}_{\rho s \sigma} = 0,$$

$$(\epsilon(s) + \omega_g - \eta_i) D^{gi}_{\rho s \sigma} - \frac{1}{2} \frac{v^{(-)}_{\rho s}}{\sqrt{Y_g}} f^g_\sigma(\rho s) - \frac{1}{2} \sum_{s_2 s_3} \{ \sum_{g_2} \frac{1}{Y_{g_2}} v^{(-)}_{ss_2} v^{(-)}_{s_3 s_2} \times$$

$$\times \frac{(f^{g_2}(ss_2) f^{g_2}(s_3 s_2) + \bar{f}^{g_2}(ss_2) \bar{f}^{g_2}(s_3 s_2)) D^{gi}_{\rho s_3 \sigma}}{\epsilon(s_2) + \omega_g + \omega_{g_2} - \eta_i} +$$

$$+ \frac{\sigma (f^{g_2}(ss_2) \bar{f}^{g_2}(s_3 s_2) - \bar{f}^{g_2}(ss_2) f^{g_2}(s_3 s_2)) D^{gi}_{\rho s_2 \sigma}}{\epsilon(s_2) + \omega_g + \omega_{g_2} - \eta_i} +$$

$$+ \sum_n \frac{1}{Y_n} \frac{v^{(+)}_{ss_2} v^{(+)}_{s_3 s_2}}{\epsilon(s_2) + \omega_n + \omega_g - \eta_i} [(\sigma^{(1)}_{ss_2} \sigma^{(1)}_{s_3 s_2} + \bar{\sigma}^{(1)}_{ss_2} \bar{\sigma}^{(1)}_{s_3 s_2}) D^{gi}_{\rho s_3 - \sigma} +$$

$$+ \sigma (\bar{\sigma}^{(1)}_{ss_2} \sigma^{(1)}_{s_3 s_2} - \sigma^{(1)}_{ss_2} \bar{\sigma}^{(1)}_{s_3 s_2}) D^{gi}_{\rho s_3 - \sigma}] +$$

$$+\sum_{n}\frac{1}{\sqrt{Y_g Y_n}}\frac{v^{(+)}_{ss_2}\,v^{(-)}_{s_3 s_2}}{\epsilon(s_2)+\omega_n+\omega_g-\eta_i}[\sigma(\sigma^{(1)}_{ss_2} f^{g}(s_3 s_2)+\bar{\sigma}^{(1)}_{ss_2}\bar{f}^{g}(s_3 s_2))D^{ni}_{\rho s_3\sigma}+$$

$$+(\sigma^{(1)}_{ss_2}\bar{f}^{g}(s_3 s_2)-\bar{\sigma}^{(1)}_{ss_2} f^{g}(s_3 s_2))D^{ni}_{\rho s_3 -\sigma}]\}=0, \qquad (30)$$

$$(\epsilon(s)+\omega_n-\eta_i)D^{ni}_{\rho s\sigma}+\frac{1}{2}\frac{v^{(+)}_{\rho s}}{\sqrt{Y_n}}\sigma^{(1)}_{\sigma}(\rho s)-$$

$$-\frac{1}{2}\sum_{s_2 s_3}\{\sum_{n}\frac{1}{Y_{n_2}}\frac{v^{(+)}_{ss_2}\,v^{(+)}_{s_3 s_2}}{\epsilon(s_2)+\omega_n+\omega_{n_2}-\eta_i}[(\sigma^{(1)}_{ss_2}\sigma^{(1)}_{s_3 s_2}+\bar{\sigma}^{(1)}_{ss_2}\bar{\sigma}^{(1)}_{s_3 s_2})D^{ni}_{\rho s_3\sigma}+$$

$$+\sigma(\bar{\sigma}^{(1)}_{ss_2}\sigma^{(1)}_{s_3 s_2}-\sigma^{(1)}_{ss_2}\bar{\sigma}^{(1)}_{s_3 s_2})D^{ni}_{\rho s_3 -\sigma}]+$$

$$+\sum_{g}\frac{1}{Y_g}\frac{v^{(-)}_{ss_2}\,v^{(-)}_{s_3 s_2}}{\epsilon(s_2)+\omega_g+\omega_n-\eta_i}[(f^{g}(ss_2)f^{g}(s_3 s_2)+\bar{f}^{g}(ss_2)\bar{f}^{g}(s_3 s_2))D^{ni}_{\rho s_3\sigma}+$$

$$+\sigma(f^{g}(ss_2)\bar{f}^{g}(s_3 s_2)-\bar{f}^{g}(ss_2)f^{g}(s_3 s_2))D^{ni}_{\rho s_3 -\sigma}]+$$

$$+\sum_n \frac{1}{\sqrt{Y_n Y_g}} \frac{v^{(-)}_{ss_2} v^{(-)}_{s_3 s_2}}{\epsilon(s_2)+\omega_g+\omega_n-\eta_i} [\sigma(f^g(ss_2)\sigma^{(1)}_{s_3 s_2} + \bar{f}^g(ss_2)\bar{\sigma}^{(1)}_{s_3 s_2}) D^{gi}_{\rho s \sigma} +$$

$$+(\bar{f}^g(ss_2)\sigma^{(1)}_{s_3 s_2} - f^g(ss_2)\bar{\sigma}^{(1)}_{s_3 s_2}) D^{gi}_{\rho s_3 -\sigma}]\} = 0, \quad (31)$$

$$F^{gg_2 i}_{\rho s \sigma} = \frac{1}{2\sqrt{Y_{g_2}}} \cdot \frac{\sum_{s'} v^{(-)}_{ss'} [f^g(ss_2) D^{gi}_{\rho s' \sigma} - \sigma \bar{f}^g(ss') D^{gi}_{\rho s' -\sigma}]}{\epsilon(s)+\omega_g+\omega_{g_2}-\eta_i}, \quad (32)$$

$$F^{nn_2 i}_{\rho s \sigma} = \frac{1}{2\sqrt{Y_{n_2}}} \frac{\sum_{s'} v^{(+)}_{ss'} [\sigma \sigma^{(1)}_{ss'_2} D^{ni}_{\rho s' \sigma} - \bar{\sigma}^{(1)}_{ss'} D^{ni}_{\rho s' -\sigma}}{\epsilon(s)+\omega \quad +\omega \quad -\eta_i}, \quad (33)$$

$$F^{gni}_{\rho s \sigma} = \frac{1}{2} \frac{1}{\epsilon(s)+\omega_n+\omega_g-\eta_i} \sum_{s'} \{ \frac{v^{(+)}_{ss'}}{\sqrt{Y_n}} (\sigma \sigma^{(1)}_{ss'} D^{gi}_{\rho s' \sigma} - \bar{\sigma}^{(1)}_{ss'} D^{gi}_{\rho s' -\sigma}) +$$

$$+ \frac{v^{(-)}_{ss'}}{\sqrt{Y_g}} (f^g(ss') D^{ni}_{\rho s' \sigma} - \sigma \bar{f}^g(ss') D^{ni}_{\rho s' -\sigma})\}. \quad (34)$$

If in eqs. (30) and (31) we reject the noncoherent terms, obtain the functions D and insert them in eq. (29) the secular equation takes on the following form:

$$\epsilon(\rho) - \eta_i - \frac{1}{4}\sum_g \sum_s \frac{(v^{(-)}_{\rho s})^2}{Y_g} \frac{(f^g_\sigma(\rho s))^2}{\epsilon(s) + \omega_g - \eta_i - \mathfrak{L}^{i(-)}_g(s) - \mathfrak{L}^{i(+)}_g(s)} -$$

$$- \frac{1}{4}\sum_n \sum_s \frac{v^{(+)}_{\rho s}}{Y_n} \frac{(\sigma^{(1)}_\sigma(\rho s))^2}{\epsilon(s) + \omega_n - \eta_i - \mathfrak{L}^{i(+)}_n(s) - \mathfrak{L}^{i(-)}_n(s)} = 0 \qquad (35)$$

where

$$\mathfrak{L}^{i(-)}_g(s) = \frac{1}{4}\sum_{g_2}\sum_{s'} \frac{(v^{(-)}_{ss'})^2}{Y_g} \frac{(f^{g_2}_\sigma(ss'))^2(1+\delta_{gg_2})}{\epsilon(s') + \omega_g + \omega_{g_2} - \eta_i}, \qquad (36)$$

$$\mathfrak{L}^{i(+)}_g(s) = \frac{1}{2}\sum_n\sum_{s'} \frac{(v^{(+)}_{ss'})^2}{Y_n} \frac{(\sigma^{(1)}_\sigma(ss'))^2}{\epsilon(s') + \omega_g + \omega_n - \eta_i}, \qquad (36')$$

$$\mathfrak{L}^{i(+)}_n(s) = \frac{1}{4}\sum_{n_2}\sum_{s'} \frac{(v^{(+)}_{ss'})^2}{Y_{n_2}} \frac{(\sigma^{(1)}_\sigma(ss'))^2(1+\delta_{nn_2})}{\epsilon(s') + \omega_n + \omega_{n_2} - \eta_i}, \qquad (37)$$

$$\mathfrak{L}^{i(-)}_n(s) = \frac{1}{2}\sum_g\sum_s \frac{(v^{(-)}_{ss'})^2}{Y_g} \frac{(f^g(ss'))^2}{\epsilon(s') + \omega_g + \omega_n - \eta_i}. \qquad (37')$$

This model may serve as a basis for studying the structure of highly excited nuclear states.

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