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HIGH ENERGY NEUTRONS FROM MUON CAPTURE BY COMPLEX NUCLEI

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1. Introduction

The presently available experimental data on the neutron asymmetry when polarized muons are captured by the complex nuclei [1-3], are rather inconsistent. However it is clear that the asymmetry depends strongly on the energy of the outgoing neutron. There are experimental data only for nuclei, like ^{16}O, ^{28}Si, ^{32}S and ^{40}Ca. The most detailed theoretical investigation of the neutron channel in muon capture by the complex nuclei

$$\mu^- + (A,Z) \to (A-1, Z-1) + n + \nu \qquad (1)$$

is performed [4] for ^4He. This nucleus was chosen for the detailed analysis for the following reason: the structure of the ^4He ground state and the residual nucleus ^3H is known better, than for heavier ones. This enables one to understand reliably the main features of the process (1). The next step is of course, to extend the calculations of the nuclei, for which there is experimental information.

The theoretical analysis of the reaction (1) in ^{4}He shows a strong energy dependence of the neutron asymmetry. At the same time the neutron characteristics appear to be very sensitive to the final state interaction between an outgoing neutron and the residual nuclei. In the present paper we consider the muon capture by ^{16}O and[4] pay attention to the factors which, as was already found in the ^{4}He case, can affect strongly the neutron characteristics.

The following channels are considered:

$$\mu^- + {}^{16}O \longrightarrow \begin{cases} {}^{15}N(1/2^-, g.s.) + n + \nu, & (2a) \\ {}^{15}N(3/2^-, E=6.33\,MeV) + n + \nu, & (2b) \\ {}^{15}N(1/2^+, E=5.3\,MeV) + n + \nu, & (2c) \\ {}^{15}N(5/2^+, E=5.3\,MeV) + n + \nu. & (2d) \end{cases}$$

The capture rates into the positive parity levels of ^{15}N are of a special interest. Their analysis requires the knowledge of some details of nuclear structure, which are unimportant in the transitions to the pure hole states of ^{15}N. The investigation of channels (2c) and (2d) together with analogous ones in the photonuclear reaction can give important information about the ^{16}O and ^{15}N structure.

2. The General Statements

We use the muon-nucleon Hamiltonian given by Primakoff[5]. The generally accepted values are used for the coupling constants. The induced pseudoscalar constant is taken as a function of the four-momentum transfer q; as was shown in the case of ^4He the neutron characteristics in the high energy region are insensitive to the magnitude of this constant:

$$g_P = g_A \frac{2m_\mu M_N}{m_\pi^2 + q^2} \quad .$$

The first order relativistic terms in Hamiltonian are taken into account (the terms proportional to p_N/M, where p_N is the nucleon momentum inside the nucleus and M is the nucleon mass). Calculating the neutron spectrum and asymmetry one needs, as was shown previously, to keep the terms proportional to $|\langle| p_N/M |\rangle|^2$. The neglected terms of the effective Hamiltonian proportional to $(p_N/M)^2$ are unimportant because their matrix elements $\langle|(p_N/M)^2|\rangle$ are smaller than the former ones. The straightforward calculation[6] confirms the correctness of such an approach.

Using the same mathematical technique, as in Ref.[4], we derive the expressions for the energy spectrum and for the asymmetry of the angular distribution relative to muon polarization $\frac{d^2\Lambda}{dE\, d\Omega} = \frac{W(E)}{4\pi}[1+a(E)\cos\theta]$. The spectrum and asymmetry are the functions of the <u>relative energy</u> E of the outgoing neutron and the residual nucleus. However, the asymmetry of the high energy neutrons in muon capture by ^{16}O will differ insignificantly from the calculated value.

The preliminary results of the investigations of the process (2a) and details of the calculation were reported in Ref. /7/. Again, as in the previous papers /4-7/, we shall concentrate our attention on the general features of the process (2).

The calculation is made in the framework of the direct mechanism of the process by using the distorted wave method. This mechanism is believed to be the basic one for the outgoing neutrons with an energy higher than 10-15 MeV. The final state wave function of the nuclear system $(^{15}N+n)$ is written in the following way

$$\Psi_f = \frac{1}{(2\pi)^3} e^{i\vec{Q}\vec{R}} \Psi_{J_f M_f} \Psi_{1/2\epsilon}(\vec{p};\vec{r}), \tag{3}$$

where $\vec{R}$ is the c.m. coordinate of the nuclear system $(^{15}N+n)$, $\vec{Q}$ its momentum; $\Psi_{J_f M_f}$ is the internal antisymmetrized wave function of ^{15}N and

$$\Psi_{1/2\epsilon}(\vec{p};\vec{r}) = 4\pi \sum i^{\ell} b_{j\ell}(pr) \langle \ell m\, 1/2\mu | jM \rangle \times \langle \ell m\, 1/2\,\epsilon | jM \rangle Y_{\ell m}(\hat{\vec{r}}) Y^{*}_{\ell m}(\hat{\vec{p}})\, \chi_{1/2\mu} \tag{4}$$

is the function of the relative motion of the outgoing neutron and the residual nucleus ^{15}N.

In the expression (4) $b_{j\ell}(pr)$ is the radial function. It was calculated numerically by solving the Shroedinger equation with the optical potential. The potential was chosen in the form:

$$V(r)=Uf(r)+iW\cdot g(r)+U_{s.o.}\frac{1}{r}\frac{df(r)}{dr}\vec{\ell}\cdot\vec{S},$$

$$f(r)=1/\{1+\exp[(r-R_0)/a]\},\ g(r)=\exp\{-[(r-R_0)/b]^2\}, \qquad (5)$$

$$a=0.65\ fm.,\ b=0.98\ fm.,\ R_0=r_0A^{1/3},\ r_0=1.25\ fm$$

with two sets of the parameters

$$\begin{aligned} U &= -(48-0.3E)MeV,\\ W &= -3\sqrt{E}\ MeV, \qquad (6a)\\ U_{s.o.} &= (22-0.3E)MeV\ fm^2, \end{aligned}$$

$$\begin{aligned} U &= -(52.5-0.6E)MeV,\\ W &= -(2.5+0.3E)MeV, \qquad (6b)\\ U_{s.o.} &= (22-0.3E)\ MeV\ fm^2. \end{aligned}$$

The parameters correspond to some average potential in this region of nuclei. The aim of the calculation with the two sets was to check the stability of the results.

The ground state wave function of ^{16}O contains the significant admixture of the two particle-two hole $(2p-2h)$ states to the double magic core $(0p-0h)$. The main $(2p-2h)$ components[8] seem to be $|1p_{1/2}^{-2}\ 01;2s_{1/2}^{2}\ 01\rangle$ — and $|1p_{1/2}^{-2}\ 01;1d_{5/2}^{2}01\rangle$.

Their amplitudes are presently known only roughly. We shall use one of the adopted sets[9]:

$$\Psi(^{16}O,g.s.)=0.82|0p-0h\rangle+0.54|1p_{1/2}^{-2}01,1d_{5/2}^{-2}\ 01\rangle+0.20|1p_{1/2}^{2}\ 01,2s_{1/2}^{2}\ 01\rangle.$$

The ground state and $J^{\pi}=3/2^{-}$, E = 6.33 MeV level of ^{15}N were considered as pure hole ones ($p_{1/2}^{-1}$ and $p_{3/2}^{-1}$, respectively). The positive parity levels are of the $(1p-2h)$-type[10]. The transitions to them are due to the $(2p-2h)$ components in the ground state of ^{16}O. According to Ref.[10], the $5/2^{+}$ level has the structure $|1p^{-2}1d\rangle$ and the $1/2^{+}$ level $-|1p^{-2}2s\rangle$. Therefore the asymmetry coefficient for the transitions to the hole and $(1p-2h)$ states should be independent of the amplitudes of the admixture. This is not valid in the general case of the transitions to the other levels of the residual nucleus.

Calculating the matrix elements we assume that all the shall model functions have their center of mass in the lowest $1S$-state. This holds for all the states of ^{15}N [10] and for the $(0p-0h)$ component. As to the $(2p-2h)$ components, they contain a small fraction of the c.m. excitation therefore in this case our assumption holds only partly.

After integrating over the c.m. coordinate one gets the delta-function of the momentum conservation. The nuclear matrix elements have the following form

$$M_a \sim \Sigma < \ell m' 1/2\, \mu | jM >< \ell \bar{m} 1/2\, \epsilon | jM > Y_{\ell \bar{m}} (\hat{\vec{p}}) \times$$

$$\{ \frac{A}{\sqrt{A}} < \Psi^*_{J_f M_f} (1,2,\ldots, A-1) b^*_{j\ell} (pr) Y^*_{\ell m} (\hat{\vec{r}}) \chi_{1/2\mu} | f^{(a)}_A | \Psi_{J_i M_i} (1,2,\ldots,A) > +$$

$$\frac{A(A-1)}{\sqrt{A}} < \Psi^*_{J_f M_f} (1,2,\ldots,A-1) b^*_{j\ell} (pr) Y^*_{\ell m} (\hat{\vec{r}}) \chi_{1/2\mu} | h^{(a)}_{A-1} | \Psi_{J_i M_i} (1,2,\ldots,A) > \},$$

where $\Psi_{J_i M_i} (1,2,\ldots,A)$ is the internal wave function of the initial nucleus with atomic number A . The operators $f^{(a)}_A$ and $h^{(a)}_{A-1}$ are as follows:

$$f^{(a)}_A = \exp\{ -i \frac{A-1}{A} \vec{p}_\nu \vec{r} \} O^{(a)}_A , \quad h^{(a)}_{A-1} = \exp\{ i \frac{1}{A} \vec{p}_\nu \vec{r} - i \frac{A-1}{A-2} \vec{p}_\nu \vec{r}_c \} O^{(a)}_{A-1};$$

$$O^{(1)}_A = 1, \quad O^{(2)}_A = \vec{\sigma}_A , \quad O^{(3)}_A = -i \vec{\nabla}_r , \quad O^{(4)}_A = -i \vec{\sigma}_A \vec{\nabla}_r ;$$

$$O^{(1)}_{A-1} = 1, \quad O^{(2)}_{A-1} = \vec{\sigma}_{A-1} , \quad O^{(3)}_{A-1} = -i (\vec{\nabla}_{r_c} - \frac{1}{A-1} \vec{\nabla}_r), \quad O^{(4)}_{A-1} = -i \vec{\sigma}_{A-1} (\vec{\nabla}_{r_c} - \frac{1}{A-1} \vec{\nabla}_r);$$

$$\vec{r} = \vec{r}_A - \frac{1}{A-1} \sum_{i=1}^{A-1} \vec{r}_i , \quad \vec{r}_c = \vec{r}_{A-1} - \frac{1}{A-2} \sum_{i=1}^{A-2} \vec{r}_i .$$

The index for $\vec{\sigma}$ indicates on which particle this operator acts. The vectors $\vec{r}$ and $\vec{r}_c$ are the coordinates of the relative notions, $\vec{p}_\nu$ is the neutrino momentum.

The first term in (9) is called the direct term and the second one the exchange term. Usually the latter is neglected. In our calculations we neglect the exchange term, too. At the end of the paper we shall briefly discuss some effects related its inclusion.

3. Results of Calculation

The results of the calculation of the energy spectrum and neutron asymmetry are given in Figs. 1-5. Again, as in the case of ^4He, the final state interaction changes the neutron characteristics as compared with the plane wave approximation and gives rise to the nonmonotonic energy dependence. The asymmetry becomes the oscillating function of the energy. Such a property is steady and independent of a special choice of the parameters of the potential. Indeed, the results with the second set (6b) of the parameters (Fig. 3) are almost the same as with the first one (6a). The real interaction is of a more complicated nature, than that used in our calculations. In particular, for the high energy neutrons the volume absorption is found to be important. However, even in this case, the energy dependence of the asymmetry will have the same behaviour /11/. The energy spectrum may be more sensitive to the details of the interaction.

As follows from Fig. 1 and 2, the neutron asymmetry for the transitions to the ground state $(1/2^-)$ has a maximum at about 40 MeV. The neutron asymmetry for the transitions to the $(3/2^-)$ levels at this energy has a minimum. The maximum for the transition to the $(3/2^-)$level is shifted to the lower energies.

The data on the transitions to the two positive parity levels of ^{15}N at 5.3 MeV are given in Fig. 4. The high energy neutron asymmetry for the transition to the

$5/2^+$ level is quite different from the other ones. The capture rate in the channels (2c) and (2d) is not so large, i.e. about 1.5 % of the total one which is adopted to be equal to $1.0.10^5 sec^{-1}$. This value is larger than the predicted one by the giant resonance mechanism in the framework of the particle-hole approximation. However it does not exhaust the measured value. The same situation occurs in photonuclear reactions. For the complete understanding of the excitation mechanism of the positive parity states it is necessary to perform some additional investigations. It seems to be important to take into account the $(2p-2h)$ configurations for the dipole states of the intermediate nucleus ^{16}N in the framework of the resonance model.

Experimentally the final nucleus is not detected in most cases. Therefore we need to sum up the transitions to the all states of the final system. For the reactions (2) the main contribution to the total spectrum for high energy neutrons comes from the channels (2a) and (2b). The contribution of the last two channels (2c) and (2d) is smaller than that of (2a) and (2b). On Fig.5. the energy spectrum and asymmetry summed up over the four levels of ^{15}N are given. The total asymmetry depends only slightly on the neutron energy and its peculiarities manifested in each channel disappear. Thus the investigation of the transitions to the definite state of the final system enables us to reveal the particular features of the process. In a less detailed consideration they are not displayed.

On Fig. 6 we give the ratio R (in percent)

$$R(E) = \int_{E}^{E_{max}} \frac{d\Lambda}{dE}\, dE \Big/ \int_{0}^{E_{max}} \frac{d\Lambda^{tot}}{dE}\, dE$$

of the high energy neutron yield as a function of E in the energy region $\Delta E = E_{max} - E$ to the total neutron yield. The capture rate for the latter case $\int_0^{E_{max}} \frac{d\Lambda^{tot}}{dE} dE$ is adopted to be equal to $1.0.10^5 \sec^{-1}$. The high energy neutron yield is very small. However for these neutrons the asymmetry reaches a large value.

All results given on Figs. 1-6 are obtained disregarding the exchange term. Let us consider its effect taking for example, the transitions to the hole states in ^{15}N. In the ground state wave function of ^{16}O we take into account only the $|0p-0h\rangle$ component its amplitude being 0.82, according to eq. (7). In this case the center of mass of both nuclei ^{16}O and ^{15}N is in the $1S$ state and its motion can be eliminated. The results are given on Fig. 7. The exchange term decreases the neutrons yield and increases the magnitude of asymmetry. The energy dependence differs from that when the exchange term is neglected.

The problem of the exchange term is developed very poorly. There are some open questions, particularly, due to the fact that the initial and final nucleon states are described by the functions of the different phenomenological potential. Therefore the obtained result should necessarily be considered as a pure qualitative one.

4. Discussion

The approach used describes only partly the processes giving rise to the high energy neutron emission. It is known, that at high energy excitations of importance become the clustering phenomena in nuclei. Among them very important are two-nucleon correlations. The capture by two nucleon clusters results mainly in two nucleon emission. Thus, by selecting the transition leading to the formation of the final nucleus $(A-1,Z-1)$ in a definite state we eliminate to a considerable extent such processes. Consequently, the investigation of neutron spectra in coincidence with γ-quanta of the residual $(A-1,Z-1)$ nuclei deexcitation enables us to single out the process described by the single particle direct mechanism. When the final nuclei is not fixed the real neutron yield can differ significantly from the calculated one. Therefore the calculated value for $R(E)$ is to be considered at the lower limit of the total yield.

Some short-range correlation effects on the channel of single-nucleon emission (1) were already considered in the case of the ^4He /12/. Such correlations enrich the high energy components of the nucleon momentum inside the nucleus. As a result, the yield and asymmetry of neutrons in the tail of the spectrum are somewhat changed, but qualitatively the energy dependence of the asymmetry remains the same.

It should be noted, that for the description of the nucleon motion inside the nucleus we choose the wave functions of the harmonic oscillator (H.O.) potential. If we make use of more realistic function, say, of the Woods-Saxon potential it will also result in enhancement of the high momentum components. Thus, one can expect that the substitution of H.O. potential by the Woods-Saxon one will lead to effects which are to some extent similar to the short range correlation effects.

Thus the present analysis of high energy neutron characteristics in muon capture shows that for the understanding of the mechanisms of the process one needs to have more detailed experimental information, which can be extracted from the partial transition characteristic studies.

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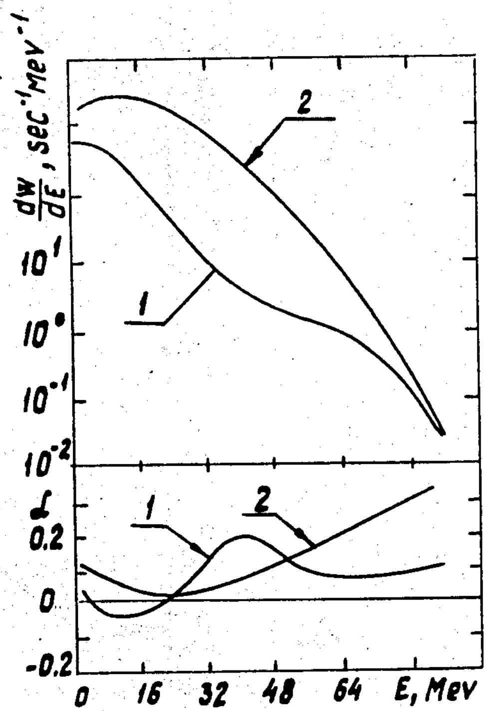
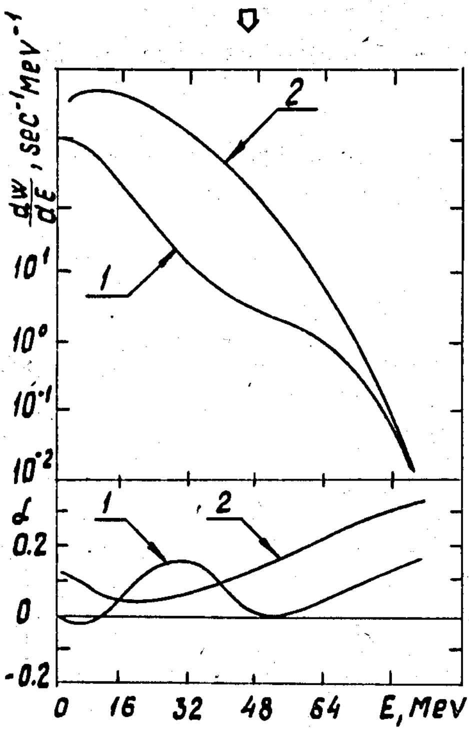


Fig. 1. The energy spectrum and angular distribution asymmetry in the reaction $^{16}O(\mu^-,\nu n)\ ^{15}N\,(1/2^-, g.s.)$: . 1. Distorted wave calculation with the potential (6a). 2. Plane wave approximation.

Fig. 2. The energy spectrum and angular distribution asymmetry in the reaction $^{16}O(\mu^-,\nu n)^{15}N(3/2^-, E=6.33\ MeV)$:
1. Distorted wave calculation with the potential (6a).
2. Plane wave approximation.



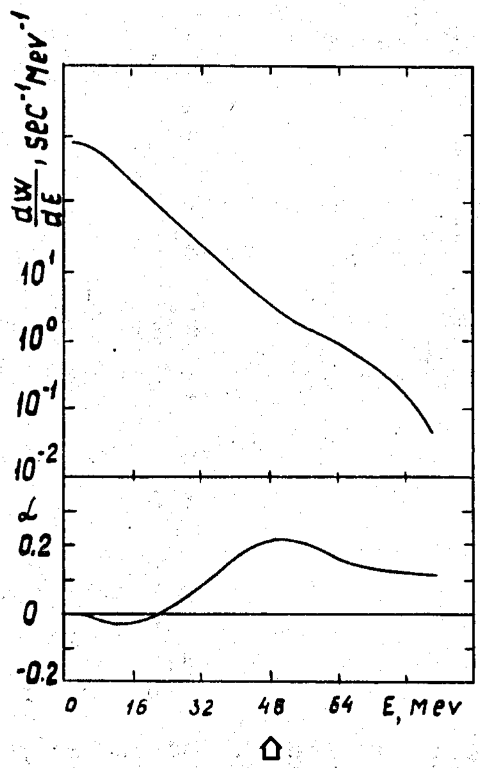
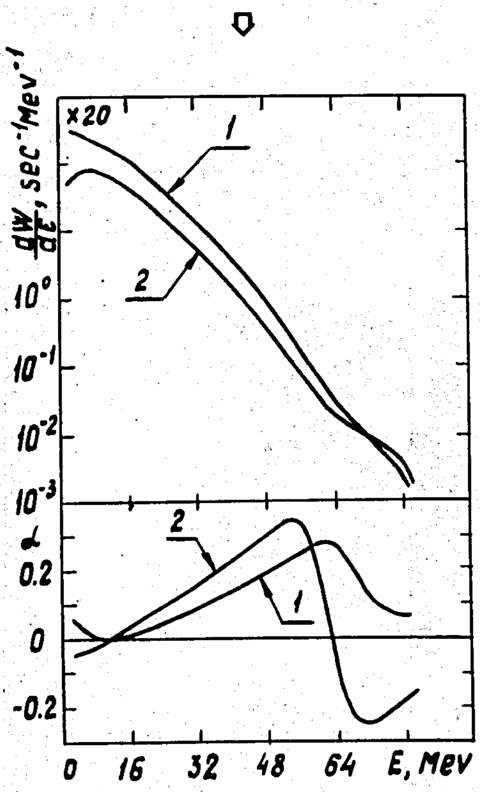


Fig. 3. The energy spectrum and asymmetry in the reaction $^{16}O(\mu^-,\nu n)\ ^{15}N(1/2^-, g.s.)$, calculated with the second set (6b) of optical potential parameters.

Fig. 4. The energy spectrum and asymmetry, calculated with optical potential (6a): 1. the reaction $^{16}O(\mu^-,\nu n)^{15}N(1/2^+, E = 5.3\ MeV)$, 2. the reaction $^{16}O(\mu^-,\nu n)^{15}N(5/2^+, E = 5.3\ MeV)$.



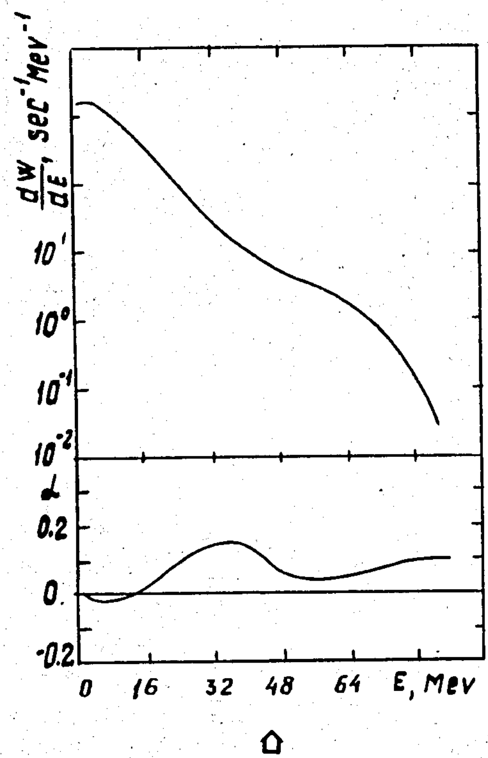
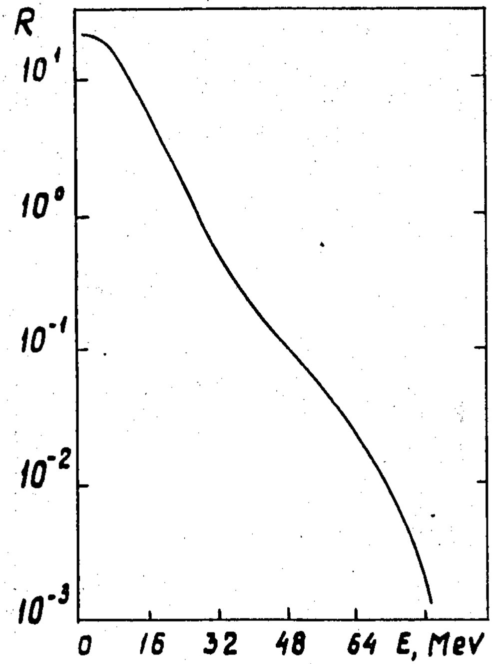


Fig. 5. The energy spectrum and asymmetry summed up over the four final states (g.s., $3/2^-$, $1/2^+$ and $5/2^+$) of ^{15}N .

Fig. 6. The total yield (in percent) of the high energy neutrons as the function of the cut off energy E .



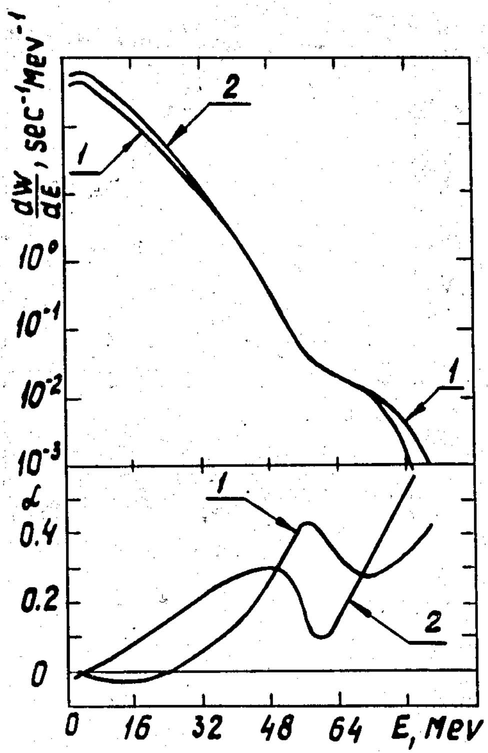


Fig. 7. The energy spectrum and asymmetry calculated with optical potential (6a). The exchange term is included: 1. the reaction $^{16}O(\mu^-,\nu n)^{15}N(1/2^-,g.s.)$, 2. the reaction $^{16}O(\mu^-,\nu n)^{15}N(3/2^-, E=6.33\ MeV)$.