ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E4 - 6487

V.G.Soloviev, V.V.Voronov

MAGNETIC MOMENTS OF THE HIGHLY EXCITED STATES OF ATOMIC NUCLEI

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

1972

E4 - 6487

V.G.Soloviev, V.V.Voronov*

MAGNETIC MOMENTS OF THE HIGHLY EXCITED STATES OF ATOMIC NUCLEI

Submitted to "Ядерная физика"

* Moscow State University

In refs.[1,2] the operator form of the wave function of a highly excited state is suggested. In refs.[1-5] the reduced neutron, radiative and alpha widths are expressed in terms of the coefficients of the wave functions of highly excited states. In refs.[4,5] the neutron resonance structure is analysed on the basis of the proposed formalism and some estimates are obtained for the few-quasiparticle components of the wave functions of neutron resonances.

The present paper is devoted to the calculation of the magnetic moments of the highly excited states of spherical and deformed nuclei and the estimation of their magnitude.

The magnetic moment of a spherical nucleus in the state with spin I is connected with the diagonal matrix element of the dipole magnetic transition operator as follows

$$\mu=\left(\frac{4\pi}{3}\right)^{1/2}\langle\Psi_{I^{\pi}M}|\mathcal{M}(\mathcal{M}1)|\Psi_{I^{\pi}M}\rangle\Big|_{M=I} \quad . \tag{I}$$

The operator of the dipole magnetic moment in a quasiparticle representation is of the form[6]

$$\begin{aligned}\mathcal{M}(\mathcal{M}1)=\frac{1}{\sqrt{3}}\sum_{j_1j_2}\langle j_1\|\Gamma(\mathcal{M}1)\|j_2\rangle\{&v^{(+)}_{j_1j_2}B(j_1,j_2;1\mu)+\\+\frac{1}{2}u^{(-)}_{j_1j_2}[A^{+}(j_1,j_2;1\mu)+(-)^{-\mu}A(j_1,j_2;1\mu)]\}&\quad ,\end{aligned} \tag{2}$$

where $u^{(-)}_{j_1j_2}=u_{j_1}v_{j_2}-u_{j_2}v_{j_1}$, $v^{(+)}_{j_1j_2}=u_{j_1}u_{j_2}+v_{j_1}v_{j_2}$ here v_j , u_j

are the Bogolubov transformation coefficients,

$$\mathcal{A}^{+}(j_1,j_2;1\mu)=\sum_{m_1m_2}(j_1m_1j_2m_2|1\mu)\,\alpha^{+}_{j_1m_1}\alpha^{+}_{j_2m_2}\,,$$

$$B(j_1,j_2;1\mu)=\sum_{m_1m_2}(-)^{j_2+m_2}(j_1m_1j_2m_2|1\mu)\,\alpha^{+}_{j_1m_1}\alpha_{j_2-m_2}\,, \qquad (3)$$

α^{+}_{jm} is the quasiparticle creation operator, $\langle j_1|\Gamma(\mathcal{M}1)|j_2\rangle$ are the single-particle matrix elements the explicit form of which is given in ref./6/, $\Psi_{I^{\pi}M}$ is the wave function of a highly excited state.

As an example, we give the expression for the wave function of the highly excited state of an even-even spherical nucleus with spin I, spin projection M and parity π :

$$\Psi_{\lambda}(I^{\pi}M)=\sum_{\substack{j_1j_2\\ m_1m_2}}{}'\sum_{t} b^{\lambda 2t}_{IM}(j_1m_1,j_2m_2)\,\alpha^{+}_{j_1m_1}\alpha^{+}_{j_2m_2}\Psi_0+$$

$$+\sum_{\substack{j_1j_2j_3j_4\\ m_1m_2m_3m_4}}{}'\sum_{tt'} b^{\lambda 2t2t'}_{IM}(j_1m_1,j_2m_2,j_3m_3,j_4m_4)\,\alpha^{+}_{j_1m_1}\alpha^{+}_{j_2m_2}\alpha^{+}_{j_3m_3}\alpha^{+}_{j_4m_4}\Psi_0+\ldots+ \qquad (4)$$

$$\sum_{\substack{j_1j_2\\ m_1m_2}}{}'\sum_{tt'\zeta} b^{\lambda 2t\Omega_\zeta(t')}_{IM}(j_1m_1,j_2m_2)\,\alpha^{+}_{j_1m_1}\alpha^{+}_{j_2m_2}\,\Omega^{+}_{\zeta}(t';j_1m_1,j_2m_2)\Psi_0+\ldots,$$

λ is the number of the resonance. The index $t=n$ indicates the neutron, and $t=p$ – the proton systems. The summation $\sum'_{j_1j_2m_1m_2}$ implies that the terms $j_1=j_2$ $m_1=m_2$ are absent and that $E(j_1)<E(j_2)$. By Ψ_0 we denote the product of quasiparticle phonon vacua for the neutron and proton systems. The coefficients b^{λ} define the contribution of the appropriate quasiparticle component. They may be written as

$$b_{IM}^{\lambda 2t2t'}(j_1m_1, j_2m_2, j_3m_3, j_4m_4) =$$

$$= \sum_{\substack{J_1 J_2 \\ \mathcal{M}_1 \mathcal{M}_2}} (j_1m_1 j_2m_2 | J_1\mathcal{M}_1)(j_3m_3 j_4m_4 | J_2\mathcal{M}_2)(J_1\mathcal{M}_1 J_2\mathcal{M}_2 | IM)\, b_I^{\lambda 2t2t'}(j_1, j_2, j_3, j_4), \qquad (5)$$

Ω_ζ^+ is the pairing-vibration phonon operator. A detailed discussion of the form of the wave function is made in refs./1-5/ where there are also the expression for the wave functions of odd and odd-odd nuclei.

The calculations have given the following expression for the magnetic moment of the highly excited state of an even-even spherical nucleus:

$$\mu = 4\left[\frac{\pi I}{3(I+1)(2I+1)}\right]^{1/2}\Big\{\sum_t \sum_{j_1j_2j_3} v_{j_1j_2}^{(+)} \langle j_2 \| \Gamma(\mathcal{M}1) \| j_1\rangle (2I+1)(-)^{3I+j_2-j_3} \begin{Bmatrix} I & I & 1 \\ j_1 & j_2 & j_3 \end{Bmatrix} b_I^{*\lambda 2t}(j_1, j_3)\, b_I^{\lambda 2t}(j_2, j_3) +$$

$$+ \sum_{tt_1} (\delta_{tn}\delta_{t_1p} + \delta_{tp}\delta_{t_1n} + 2\delta_{t_1t}) \sum_{j_1^t j_2^t j_3^t j_4^{t_1} j_5^{t_1}} \sum_j v_{j_1^t j_2^t}^{(+)} \langle j_2^t \| \Gamma(\mathcal{M}1) \| j_1^t\rangle (2I+1)(-)^{3I+j_2^t-j} \times$$

$$\times \begin{Bmatrix} I & I & 1 \\ j_1^t & j_2^t & j \end{Bmatrix} b_I^{*\lambda 2t2t_1}(j_1^t, j_3^t, j_4^{t_1}, j_5^{t_1})\, b_I^{\lambda 2t2t_1}(j_2^t, j_3^t, j_4^{t_1}, j_5^{t_1}) + \ldots +$$

$$\sum_{tt_1\zeta} \sum_{j_1j_2j_3} v_{j_1j_2}^{(+)} \langle j_2 \| \Gamma(\mathcal{M}1) \| j_1\rangle (2I+1)(-)^{3I+j_2-j_3} \begin{Bmatrix} I & I & 1 \\ j_1 & j_2 & j_3 \end{Bmatrix} b_I^{*\lambda 2t\Omega_\zeta(t_1)}(j_1, j_3)\; b_I^{\lambda 2t\Omega_\zeta(t_1)}(j_2, j_3) + \ldots +$$

$$\sum_{\zeta\zeta_1t_1} \sum_{j_1j_2j_3} v_{j_1j_2}^{(+)} \langle j_2 \| \Gamma(\mathcal{M}1) \| j_1\rangle (2I+1)(-)^{3I+j_2-j_3} \begin{Bmatrix} I & I & 1 \\ j_1 & j_2 & j_3 \end{Bmatrix} b_I^{*\lambda 2t_1\Omega_{\zeta_1}(t_1)}(j_1, j_3)\; b_I^{\lambda 2t_1\Omega_\zeta(t_1)}(j_2, j_3) \times$$

$$\times\left[\frac{X^{\mathfrak{z}}(j_1)X^{\mathfrak{z}_t}(j_2)+Y^{\mathfrak{z}}(j_2)Y^{\mathfrak{z}_t}(j_1)}{\sqrt{(2j_1+1)(2j_2+1)}}-\frac{X^{\mathfrak{z}}(j_2)X^{\mathfrak{z}_t}(j_2)}{(2j_2+1)}-\frac{Y^{\mathfrak{z}}(j_1)Y^{\mathfrak{z}_t}(j_1)}{(2j_1+1)}\right]+\ldots+$$

$$\sum_{\mathfrak{z}t}\sum_{j_1j_2j_3}u^{(-)}_{j_1j_2}\langle j_2|\Gamma(\mathcal{M}1)|j_1\rangle(2I+1)(-)^{3I+j_2-j_3}\begin{Bmatrix}I&I&1\\j_1&j_2&j_3\end{Bmatrix}\left(\frac{X^{\mathfrak{z}}(j_1)}{\sqrt{2j_1+1}}-\right.$$

$$\left.-\frac{Y^{\mathfrak{z}}(j_2)}{\sqrt{2j_2+1}}\right)\left[b_I^{*\lambda 2t}(j_1,j_3)b_I^{\lambda 2t\Omega_{\mathfrak{z}}(t)}(j_2,j_3)+b_I^{\lambda 2t}(j_1,j_3)b_I^{*\lambda 2t\Omega_{\mathfrak{z}}(t)}(j_2,j_3)\right]+ \tag{6}$$

$$+\sqrt{\frac{2I+1}{3}}^{-1}\sum_{tt_1}(\delta_{tp}\delta_{t_1n}+3\delta_{tt_1})\sum_{j_1j_2j_3j_4}u^{(-)}_{j_3j_4}\langle j_4|\Gamma(\mathcal{M}1)|j_3\rangle\{b_I^{*\lambda 2t}(j_1,j_2)b_I^{\lambda 2t2t_1}(j_1,j_2,j_3,j_4)+$$

$$+b_I^{\lambda 2t}(j_1,j_2)b_I^{*\lambda 2t2t_1}(j_1,j_2,j_3,j_4)\}+\ldots\}.$$

In the case of the $\mathcal{N}$ odd spherical nucleus the magnetic moment of a highly excited state with spin I and parity π is of the form

$$\mu=\left[\frac{4\pi I}{3(I+1)(2I+1)}\right]^{1/2}\left\{\langle I|\Gamma(\mathcal{M}1)|I\rangle|b_I^{\lambda n}(j^n)|^2\delta_{Ij^n}+\right.$$

$$+\sum_t(\delta_{tp}+3\delta_{tn})\sum_{j_1^nj_2^n}\sum_{j_3j_4j}(2I+1)(-)^{3I+j_1-j}\begin{Bmatrix}I&I&1\\j_3^n&j_2^n&j\end{Bmatrix}v^{(+)}_{j_2^nj_1^n}\langle j_2^n|\Gamma(\mathcal{M}1)|j_1^n\rangle b_I^{*\lambda n2t}(j_1^n,j_3,j_4)b_I^{\lambda n2t}(j_2^n,j_3,j_4)+$$

$$+2\sum_{j_1^nj_2^p}\sum_{j_3^pj_4^pj}(2I+1)(-)^{3I+j_1-j}\begin{Bmatrix}I&I&1\\j_3^p&j_4^p&j\end{Bmatrix}v^{(+)}_{j_3^pj_4^p}\langle j_4^p|\Gamma(\mathcal{M}1)|j_3^p\rangle b_I^{*\lambda n2p}(j_1^n,j_2^p,j_3^p)b_I^{\lambda n2p}(j_1^n,j_2^p,j_4^p)+\ldots+$$

$$+\langle I\|\Gamma(\mu 1)\|I\rangle \delta_{Ij''} \sum_{\tau t} |b_I^{\lambda n \Omega_\tau(t)}(j'')|^2 + \ldots +$$

$$+\sqrt{\frac{2I+1}{3}} \sum_{j_1 j_2 t} (\delta_{tp} + 3\delta_{tn}) \mathcal{U}^{(-)}_{j_1 j_2} \langle j_1\|\Gamma(\mu t)\|j_2\rangle \{ \overset{*}{b}{}_I^{\lambda n}(j'') b_I^{\lambda n 2t}(j_1, j_2, j'') + \quad (7)$$

$$+ b_I^{\lambda n}(j'') \overset{*}{b}{}_I^{\lambda n 2t}(j_1, j_2, j'') \} \delta_{j'' I} + \ldots \} .$$

Here $\left\{ \begin{matrix} I & I & 1 \\ j_1 & j_2 & j_1 \end{matrix} \right\}$ are $6j$ -symbols.

A similar expression is obtained for the magnetic moment of the highly excited state of an odd-odd spherical nucleus.

We calculate the magnetic moments of deformed nuclei. The expression for the dipole magnetic transition operator[6] is to be transformed to the nuclear coordinate system. For highly excited states the projection of the total angular momentum on the symmetry axis K is not a good quantum number and the wave function is a superposition of terms with different K values. The symmetrized wave function for the deformed nucleus is written as[5]

$$\Psi_\lambda(I^\pi M) = \left[\frac{2I+1}{16\pi^2}\right]^{1/2} \sum_K d_I^\lambda(K) \{ \mathcal{D}^I_{MK} \Psi_\lambda(+K^\pi) + (-)^{I+K} \mathcal{D}^I_{M-K} \Psi_\lambda(-K^\pi) \}, \quad (8)$$

where $\mathcal{D}^I_{MK}$ are the well-known functions[7]. The coefficients $d_I^\lambda(K)$ take into account the fact that the components with different K values give different contribution to the wave function. The quantity $\Psi_\lambda(+K^\pi)$ is the inherent wave function with definite values of K and parity π, and $\Psi_\lambda(-K^\pi)$ is obtained from $\Psi_\lambda(+K^\pi)$ by means of the time reversal operation.

As an example, we give the expression for $\Psi_\lambda(+K^\pi)$ in the case of the odd $\mathcal{N}$ deformed nucleus:

$$\Psi_\lambda(+K^\pi)=\sum_s b^{\lambda n}_{+K^\pi}(s)\alpha^+_{s+}\Psi_0+$$

$$+\sum_{\substack{s q_2 q_3\\ \sigma\sigma_2\sigma_3}}{}' \sum_t b^{\lambda n 2t}_{+K^\pi}(s\sigma;q_2\sigma_2;q_3\sigma_3)\alpha^+_{s\sigma}\alpha^+_{q_2\sigma_2}\alpha^+_{q_3\sigma_3}\Psi_0+$$

$$+\sum_{\substack{s q_2 q_3 q_4 q_5\\ \sigma\sigma_2\sigma_3\sigma_4\sigma_5}}{}' \sum_{tt'} b^{\lambda n 2t 2t'}_{+K^\pi}(s\sigma;q_2\sigma_2;q_3\sigma_3;q_4\sigma_4;q_5\sigma_5)\alpha^+_{s\sigma}\alpha^+_{q_2\sigma_2}\alpha^+_{q_3\sigma_3}\alpha^+_{q_4\sigma_4}\alpha^+_{q_5\sigma_5}\Psi_0+\ldots+ \qquad (9)$$

$$+\sum_{st\zeta} b^{\lambda n \Omega_\zeta(t)}_{+K^\pi}(s)\alpha^+_{s+}\Omega^+_\zeta(t;s)\Psi_0+$$

$$+\sum_{s q_2 q_3}{}' \sum_{\sigma\sigma_2\sigma_3}\sum_{tt'\zeta} b^{\lambda n 2t \Omega_\zeta(t')}_{+K^\pi}(s\sigma;q_2\sigma_2;q_3\sigma_3)\alpha^+_{s\sigma}\alpha^+_{q_2\sigma_2}\alpha^+_{q_3\sigma_3}\Omega^+_\zeta(t';s,q_2,q_3)\Psi_0+\ldots .$$

The set of quantum numbers defining the single-particle state is denoted by $(s\sigma)$ for the neutron, $(r\sigma)$ the proton and $(q\sigma)$ for both the systems. The summation in (9) over σ_1, $\sigma_2\ldots$ is performed in such a manner that the conditions

$$\sigma_1 K_1+\sigma_2 K_2+\sigma_3 K_3=K,$$

$$\sigma_1 K_1+\sigma_2 K_2+\sigma_3 K_3+\sigma_4 K_4+\sigma_5 K_5=K, \qquad (10)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

are valid. For the functions $\Psi_\lambda(-K^\pi)$, the conditions

$$\sigma_1 K_1+\sigma_2 K_2+\sigma_3 K_3=-K,$$

$$\sigma_1 K_1+\sigma_2 K_2+\sigma_3 K_3+\sigma_4 K_4+\sigma_5 K_5=-K, \qquad (10')$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

should be fulfilled, instead of (10).

The operators of pairing vibration phonons $\Omega^+_\zeta(t;s,q_2,q_3)$ are determined in refs./1,5,6/. The wave function (9) is normalized as follows:

$$\left(\Psi^{*}_{\lambda}(\pm K^{\pi})\Psi_{\lambda}(\pm K^{\pi})\right)=1=\sum_{s}|b^{\lambda n}_{\pm K^{\pi}}(s)|^{2}+$$

$$+\sum_{\substack{s q_2 q_3\\ \sigma\sigma_2\sigma_3}}{}'\sum_{t}|b^{\lambda n 2t}_{\pm K^{\pi}}(s\sigma;q_2\sigma_2;q_3\sigma_3)|^{2}+\ldots+\sum_{s\sigma}\sum_{t\zeta}|b^{\lambda n\Omega_{\zeta}(t)}_{\pm K^{\pi}}(s)|^{2}+\ldots\,. \tag{11}$$

From the normalization condition of the wave function (8), the orthogonality of $\mathcal{D}$ functions and eq.(II) we get

$$\left(\Psi^{*}_{\lambda}(I^{\pi}M)\Psi_{\lambda}(I^{\pi}M)\right)=1=\sum_{K}|d^{\lambda}_{I}(K)|^{2}. \tag{12}$$

Using (8) and (9) for the magnetic moment of the highly excited state of an odd $\mathcal{N}$ deformed nucleus we obtain the following expression:

$$\mu=\sqrt{\frac{4\pi}{3}}\sum_{K}\frac{K}{I+1}|d^{\lambda}_{I}(K)|^{2}\Big\{\sum_{ss'}v^{(+)}_{ss'}\langle s+|\Gamma(\mathcal{M}1)|s'+\rangle b^{*\lambda n}_{+K^{\pi}}(s)\,b^{\lambda n}_{+K^{\pi}}(s')+$$

$$+\sum_{t}(\delta_{tp}+3\delta_{tn})\sum_{ss'}\sum_{\substack{q_2q_3\\ \sigma_2\sigma_3}}{}'(\delta_{\sigma+}-\delta_{\sigma-})v^{(+)}_{ss'}\langle s+|\Gamma(\mathcal{M}1)|s'+\rangle b^{*\lambda n2t}_{+K^{\pi}}(s\sigma;q_2\sigma_2;q_3\sigma_3)\,b^{\lambda n2t}_{+K^{\pi}}(s'\sigma;q_2\sigma_2;q_3\sigma_3)+$$

$$+2\sum_{rr'}\sum_{\substack{s_1r_2\\ \sigma_1\sigma_2}}(\delta_{\sigma+}-\delta_{\sigma-})v^{(+)}_{rr'}\langle r+|\Gamma(\mathcal{M}1)|r'+\rangle b^{*\lambda n2p}_{+K^{\pi}}(s_1\sigma_1;r\sigma;r_2\sigma_2)\,b^{\lambda n2p}_{+K^{\pi}}(s_1\sigma_1;r'\sigma;r_2\sigma_2)+\ldots+$$

$$+\sum_{t\zeta}\sum_{ss'}v^{(+)}_{ss'}\langle s+|\Gamma(\mathcal{M}1)|s'+\rangle b^{*\lambda n\Omega_{\zeta}(t)}_{+K^{\pi}}(s)\,b^{\lambda n\Omega_{\zeta}(t)}_{+K^{\pi}}(s')+$$

$$+\frac{1}{2}\sum_{t\zeta\zeta_1}\sum_{ss'}v^{(+)}_{ss'}\langle s+|\Gamma(\mathcal{M}1)|s'+\rangle\left[X^{\zeta_1}_{(s)}X^{\zeta}_{(s')}-X^{\zeta_1}_{(s')}X^{\zeta}_{(s')}+Y^{\zeta_1}_{(s')}Y^{\zeta}_{(s)}-Y^{\zeta_1}_{(s)}Y^{\zeta}_{(s)}\right]b^{*\lambda n\Omega_{\zeta}(t)}_{+K^{\pi}}(s)\,b^{\lambda n\Omega_{\zeta_1}(t)}_{+K^{\pi}}(s')+$$

$$+\ldots-\sum_{t}(\delta_{tp}+3\delta_{tn})\sum_{sqq'\sigma}u^{(-)}_{qq'}\langle q+|\Gamma(\mathcal{M}1)|q'-\rangle\left[b^{*\lambda n}_{+K^{\pi}}(s)\,b^{\lambda n2t}_{+K^{\pi}}(s+;q\sigma;q'-\sigma)+b^{*\lambda n2t}_{+K^{\pi}}(s+;q\sigma;q'-\sigma)\,b^{\lambda n}_{+K^{\pi}}(s)\right]-$$

$$-\frac{1}{\sqrt{2}}\sum_{\zeta ss'}u^{(-)}_{ss'}\left[X^{\zeta}_{(s')}-Y^{\zeta}_{(s)}\right]\langle s+|\Gamma(\mathcal{M}1)|s'+\rangle\left[b^{*\lambda n}_{+K^{\pi}}(s')\,b^{\lambda n\Omega_{\zeta}(n)}_{+K^{\pi}}(s)+b^{*\lambda n\Omega_{\zeta}(n)}_{+K^{\pi}}(s)\,b^{\lambda n}_{+K^{\pi}}(s')\right]+\ldots+$$

$$+(-)^{I+K}\delta_{K,1/2}\Big(\sum_{ss'} v^{(+)}_{ss'}\langle s+|\Gamma(\mathcal{M}1)|s'-\rangle b^{*\lambda n}_{-K\pi}(s)\, b^{\lambda n}_{+K\pi}(s')+$$

$$+\sum_{t}(\delta_{tp}+3\delta_{tn})\sum_{ss'}\sum_{\substack{q_2q_3\\ \sigma_2\sigma_3}}{}' v^{(+)}_{ss'}\langle s+|\Gamma(\mathcal{M}1)|s'-\rangle b^{*\lambda n2t}_{-K\pi}(s-;q_2\sigma_2;q_3\sigma_3)\, b^{\lambda n2t}_{+K\pi}(s'+;q_2\sigma_2;q_3\sigma_3)+$$

$$+2\sum_{\tau\tau'}\sum_{\substack{s_1\tau_2\\ \sigma_1\sigma_2}} v^{(+)}_{\tau\tau'}\langle \tau+|\Gamma(\mathcal{M}1)|\tau'-\rangle b^{*\lambda n2p}_{-K\pi}(s_1\sigma_1;\tau-;\tau_2\sigma_2)\, b^{\lambda n2p}_{+K\pi}(s_1\sigma_1;\tau'+;\tau_2\sigma_2)+\ldots+$$

$$+\frac{1}{2}\sum_{t\,\zeta\zeta_1}\sum_{ss'} v^{(+)}_{ss'}\langle s+|\Gamma(\mathcal{M}1)|s'-\rangle[X^{\zeta_1}(s)X^{\zeta}(s')-X^{\zeta_1}(s')X^{\zeta}(s')+Y^{\zeta_1}(s')Y^{\zeta}(s)-Y^{\zeta_1}(s)Y^{\zeta}(s)]\, b^{*\lambda n\Omega_\zeta(t)}_{-K\pi}(s)\, b^{\lambda n\Omega_{\zeta_1}(t)}_{+K\pi}(s')+$$

$$+\sum_{t}(\delta_{tp}+3\delta_{tn})\sum_{sqq'\sigma}\sigma\, u^{(-)}_{qq'}\langle q+|\Gamma(\mathcal{M}1)|q'-\rangle[b^{*\lambda n}_{-K\pi}(s)\, b^{\lambda n2t}_{+K\pi}(s-;q\sigma;q'\sigma)+$$

$$+b^{*\lambda n2t}_{-K\pi}(s+;q-\sigma;q'-\sigma)\, b^{\lambda n}_{+K\pi}(s)]- \tag{13}$$

$$-\frac{1}{\sqrt{2}}\sum_{\zeta ss'} u^{(-)}_{ss'}\langle s+|\Gamma(\mathcal{M}1)|s'-\rangle[X^{\zeta}(s')-Y^{\zeta}(s)][b^{*\lambda n}_{-K\pi}(s')\, b^{\lambda n\Omega_\zeta(n)}_{+K\pi}(s)+b^{*\lambda n\Omega_\zeta(n)}_{-K\pi}(s')\, b^{\lambda n}_{+K\pi}(s)]+\ldots\Big)\Big\}.$$

In those terms in which there is no summation over σ the quantity $\sigma = \pm 1$ is determined from the condition (10). The terms containing $b^{*}_{-K\pi}b_{+K\pi}\langle q+|\Gamma(\mathcal{M}1)|q'-\rangle v^{(+)}_{qq'}$, differ from zero, provided that $K_q+K_{q'}=1$ and $\sigma_2 K_{q_2}+\sigma_3 K_{q_3}=0$.

The terms containing $b^{*}_{-K\pi}b_{+K\pi}\langle q+|\Gamma(\mathcal{M}1)|q'-\rangle u^{(-)}_{qq'}$, differ from zero, provided that $K_q+K_{q'}=1$ and $K_s=1/2$.

For the magnetic moment of the highly excited state of an even-even deformed nucleus we derive the following expression:

$$\mu=-2\sqrt{\frac{4\pi}{3}}\sum_{K}\frac{K}{I+1}\left|d_I^{\lambda}(K)\right|^2\Big\{\sum_{q_1\sigma_1}\sum_{qq't}(\delta_{\sigma+}-\delta_{\sigma-})v^{(+)}_{qq'}\langle q+\|\Gamma(M1)\|q'+\rangle b^{*\lambda 2t}_{+K\pi}(q_1\sigma_1;q\sigma)\,b^{\lambda 2t}_{+K\pi}(q_1\sigma_1;q'\sigma)+$$

$$+\sum_{tt'}(\delta_{tn}\delta_{t'p}+2\delta_{tt'})\sum_{qq'}\sum_{\substack{q_1q_2q_3\\ \sigma_1\sigma_2\sigma_3}}{}^{'}(\delta_{\sigma+}-\delta_{\sigma-})v^{(+)}_{qq'}\langle q+\|\Gamma(M1)\|q'+\rangle b^{*\lambda 2t2t'}_{+K\pi}(q_1\sigma_1;q_2\sigma_2;q_3\sigma_3;q\sigma)\times$$

$$\times b^{\lambda 2t2t'}_{+K\pi}(q_1\sigma_1;q_2\sigma_2;q_3\sigma_3;q'\sigma)+\ldots+\sum_{tt'\zeta}\sum_{qq'q_1\sigma_1}(\delta_{\sigma+}-\delta_{\sigma-})v^{(+)}_{qq'}\langle q+\|\Gamma(M1)\|q'+\rangle b^{*\lambda 2t\Omega_\zeta(t')}_{+K\pi}(q_1\sigma_1;q\sigma)\,b^{\lambda 2t\Omega_\zeta(t')}_{+K\pi}(q_1\sigma_1;q'\sigma)$$

$$+\ldots-\sum_{tt'}(\delta_{tp}\delta_{t'n}+3\delta_{tt'})\sum_{qq'}\sum_{\substack{q_1q_2\\ \sigma_1\sigma_2}}{}^{'}u^{(-)}_{qq'}\langle q+\|\Gamma(M1)\|q'+\rangle\Big[b^{*\lambda 2t}_{+K\pi}(q_1\sigma_1;q_2\sigma_2)\,b^{\lambda 2t2t'}_{+K\pi}(q_1\sigma_1;q_2\sigma_2;q\sigma;q'-\sigma)+$$

$$+b^{\lambda 2t}_{+K\pi}(q_1\sigma_1;q_2\sigma_2)\,b^{*\lambda 2t2t'}_{+K\pi}(q_1\sigma_1;q_2\sigma_2;q\sigma;q'-\sigma)\Big]- \qquad (14)$$

$$-\sqrt{2}\sum_{t\zeta}\sum_{qq'q_1\sigma_1}(\delta_{\sigma+}-\delta_{\sigma-})u^{(-)}_{qq'}\left[X^{\zeta}(q')-Y^{\zeta}(q)\right]\Big[b^{*\lambda 2t}_{+K\pi}(q_1\sigma_1;q'\sigma)\,b^{\lambda 2t\Omega_\zeta(t)}_{+K\pi}(q_1\sigma_1;q\sigma)+$$

$$+b^{\lambda 2t}_{+K\pi}(q_1\sigma_1;q'\sigma)\,b^{*\lambda 2t\Omega_\zeta(t')}_{+K\pi}(q_1\sigma_1;q\sigma)\Big]\langle q+\|\Gamma(M1)\|q'+\rangle+\ldots\Big\}.$$

The contribution to the formulas for the magnetic moments is seen to come from all the components of the wave functions of highly excited states. Let us estimate, for example, the magnitude of the magnetic moment of the highly excited state of an odd spherical nucleus. To this end, in eq.(7) assuming the single-particle matrix elements to be identical we take them out of the sign of summation. We put $j_1=j_2$, $v^{(+)}_{j_1j_1}=1$ and reject the terms containing $u^{(-)}_{jj'}$. Taking into account the fact that $(2I+1)(-)^{3I+j_1-j}\begin{Bmatrix} I & I & 1\\ j_1 & j_1 & j\end{Bmatrix}\approx 1$ we obtain an approximate value for the magnetic moment in the form:

$$\mu\simeq\langle I\|\Gamma(M1)\|I\rangle\left[\frac{4\pi I}{3(I+1)(2I+1)}\right]^{1/2}\Big\{\left|b_I^{\lambda n}(j^n)\right|^2\delta_{Ij^n}+\sum_{\zeta t}\left|b_I^{\lambda n\Omega_\zeta(t)}(j^n)\right|^2\delta_{Ij^n}+$$

$$+\sum_t (\delta_{tp}+\delta_{tn}) \sum_{j_1^n j_3 j_4} |b_I^{\lambda n 2t}(j_1^n, j_3, j_4)|^2 + \ldots + 2\sum_{j_1 j_2 j_3} |b_I^{\lambda 3n}(j_1, j_2, j_3)|^2 +$$

$$+ 2\,\frac{\langle j^p | \Gamma(\mathcal{M}(1)) | j^p \rangle}{\langle I | \Gamma(\mathcal{M}(1)) | I \rangle} \sum_{j_1^n j_2^p j_3^p} |b_I^{\lambda n 2p}(j_1^n, j_2^p, j_3^p)|^2 + \ldots \Big\}. \qquad (15)$$

Using the normalization condition for the wave function of the highly excited state of an odd $\mathcal{N}$ spherical nucleus, we can write the expression in braces in the form $1 + \delta$, where δ is a small addition containing the ratio of the proton and neutron single-particle matrix elements multiplied by the squared coefficients of the corresponding wave function components which are estimated to be small. The term standing in front of the braces gives the well-known Schmidt lines.

Similar estimates can easily be obtained for the magnetic moments of even-even and odd-odd nuclei. In the case of the even-even nucleus the magnetic moment of a highly excited state is approximately equal to the magnetic moment of a two-quasiparticle state. Thus, the magnetic moments of highly excited states must be equal, in the order of magnitude, to the magnetic moments of low-lying states.

In ref./8/ it is reported on the measurement of the magnetic moments of two neutron resonances in ^{168}Er. The magnetic moment of the resonance with $\mathcal{E} - B_n$ = 0.460 eV and $I^{\pi} = 4^+$ is $\mu = (-0.45 \pm 0.74)\mu_0$ and the resonance with $\mathcal{E} - B_n$ = 0.584 eV and $I^{\pi} = 3^+$ is $(5.9 \pm 1.2)\mu_0$, where μ_0 is the nuclear magneton. The measurement of the magnetic moments of these resonances is also performed in ref.[9] where it is obtained for the resonance with $\mathcal{E}-B_n$=0.460 eV μ =(0.9 ± 0.4)μ_0 for the resonance $\mathcal{E}-B_n$ =0.584 eV μ =(I.8 ± 0.9)μ_0 . These experimental data are in disagreement with each other, however, the values of the magnetic moments in both measurements coincide, in the order of magnitude, with the magnetic moments of two-quasiparticle states. The magnetic moment of the ground state of ^{167}Er with $7/2^+$ 7/2 633↑ is $\mu = -0.564\mu_0$. These experimental data confirm our conclusion on the magnitude of the magnetic moments of highly excited states.

The situation with the magnetic moments of highly excited states differs in principle from the situation with the radiative widths. The matrix elements for the $E\lambda$ and $\mathcal{M}\lambda$ transitions from highly excited states to low-lying states of spherical and deformed nuclei are given in refs.[1,3] and analysed in refs.[4,5]. It follows from the experimental data on the reduced partial widths for $E1$ transitions from neutron resonances that these transitions are hindered by a factor of 10^5 - 10^7 compared with the one-particle values. In ref.[5], on the basis of the experimental data on $E1$ transitions to the ground and low-lying states, one has obtained the following estimates for the single- and two-quasiparticle components $|\bar{b}|^2$ of the wave functions of neutron resonances:

a. for the region of the isotopes $Cr-Ni-Zn$ $|\bar{b}|^2 \sim 10^{-5}$,

b. for nuclei in the region $Mo-Sn$ $|\bar{b}|^2 \sim 10^{-6}-10^{-7}$,

c. for deformed nuclei $|\bar{b}|^2 \sim 10^{-8}$,

d. for nuclei in the region $A = 200$ $|\bar{b}|^2 \sim 10^{-7}-10^{-8}$.

These estimates are rather rough, but in the order of magnitude, they coincide with the $|\bar{b}|^2$ estimates obtained from the average neutron widths.

Thus, the $E\lambda$ and $M\lambda$ transitions from highly excited to low-lying states involve one-quasiparticle and three-quasiparticle (in the case of odd nuclei) or two-quasiparticle and four-quasiparticle (in the case of even nuclei) components of the wave functions of highly excited states. Because the values of these components are small, the reduced probabilities of $E\lambda$ and $M\lambda$ transitions from highly excited states are smaller by several orders of magnitude compared with the one-particle values. The expressions for the magnetic moments contain all the components of the appropriate wave functions. Therefore, the magnetic moments of neutron resonances and of states of intermediate excitation energy must coincide, in the order of magnitude, with the magnetic moments of the ground and low-lying states of atomic nuclei.

References

1. В.Г.Соловьев. ЯФ,13, 48 (1971).
2. В.Г.Соловьев. Известия АН СССР,сер. физ.,35, 666 (1971).
3. В.В.Воронов, В.Г.Соловьев. Сообщение ОИЯИ Р4-5562 (1971).
4. В.Г.Соловьев. ЯФ,15, 733 (1972).
5. В.Г.Соловьев. ЭЧАЯ,3, № 4 (1972).
6. В.Г.Соловьев. Теория сложных ядер, Наука, 1971.
7. А.Эдмонс. Деформация атомных ядер, ИЛ, Москва, 1958.
8. K.U.Berckurts, G.Brunhart. Phys.Rev. C1, 726 (1970).
9. В.П.Алфименоков, Г.П.Жуков, Г.Н.Зимин и др. Contributions Conference on Nuclear Structure Study with Neutrons, Budapest, 1972.

Received by Publishing Department
on June 2, 1972.

Соловьев В.Г., Воронов В.В. **E4 - 6487**

Магнитные моменты высоковозбужденных состояний атомных ядер

На основе полумикросокпического подхода получены формулы для магнитных моментов высоковозбужденных состояний. Показано, что магнитные моменты выражены через все компоненты волновых функций высоковозбужденных состояний. Согласно грубой оценке величины магнитных моментов состояний промежуточной энергии возбуждения и высоковозбужденных состояний, включая и нейтронные резонансы, по порядку величины должны быть равны одночастичным значениям. Теоретические результаты согласуются с имеющимися экспериментальными данными по магнитным моментам нейтронных резонансов.

Препринт Объединенного института ядерных исследований.

Дубна, 1972

Soloviev V.G., Voronov V.V. **E4 - 6487**

Magnetic Moments of the Highly Excited States of Atomic Nuclei

Formulas for the magnetic moments of the highly excited states of atomic nuclei are derived on the basis of the semimicroscopic approach. It is shown that the magnetic moments are expressed in terms of all the components of the wave functions of highly excited states. According to a rough estimation, the magnetic moments of intermediate excitation states and highly excited states, including neutron resonances, must be equal, in the order of magnitude, to the single-particle values. The situation with the magnetic moments differs essentially from that with the probabilities of the E1 and M1 transitions from highly excited to low-lying states which are smaller than the single particle values by a factor of 10^5-10^7. The theoretical results are in agreement with the available experimental data on the magnetic moments of neutron resonances.

Preprint. Joint Institute for Nuclear Research.

Dubna, 1972