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KINEMATICS OF MUON CAPTURE

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PART I. THE KINEMATICS FOR ANY SPINS

1. Introduction

Only the discussion of the angular correlations in various partial transitions in muon capture reaction may lead to a reliable confirmation of the theoretical assumptions concerned with fundamental nature of muon-nuclei weak interaotion as well as with the structure of involved nuclear states. As a consequence of the increasing number of experimental data on muon capture as well as because of the development of nuclear models it seems desirable to calculate the accurate kinematical formulae describing observables in terms of the invariant amplitudes, which are, from a phenomenological point of view, the only quantities that can be obtained from muon capture experiments. Besides them the elements of the density matrix of mesic atom at the moment of capture from K orbit can also be determined.

Our formulas are convenient for aocurate model calculations of concrete partial transitions with realistio wave functions of nuclei and the muon, as well as for the phenomenological analysis of the experimental data. The approximations (e.g. allowed) appropriated in special cases, oan be easily deduced.

The formulas based only on kinematics of the normal muon oapture process are strictly valid for any nuclei both for light and heavier ones.

At present the theory of the muon nuclei interaction (i.e. dynamics) investigates many problems such as:

i. The effects which violate the impulse approximation of Fujii and Primakoff [1] due to the e.g. meson exchange currents in nuclei extensively studied recently by Chemtob and Rho [2].

ii. The shell model structure of light nuclei.

iii. The fundamental nature of the weak muon-nuclei interaction e.g. violation of the G parity [3], etc.

iv. The very important contribution of the higher forbiddennesses [4] as well as the αZ corrections due to the realistic muon wave functions [5].

Besides these, just mentioned "petties" problems we have of course, the main one in the S-matrix approach to weak interactions which is a dynamical theory that goes beyond the lowest order of perturbation theory.

The theoretical study of such problems continually needs new and new calculations of the invariant amplitudes describing the transition under consideration but the general connection presented here between these amplitudes and observables (i.e. a kinematics) is unaltered.

The need of such kinematical tables appears, after discovering of the importance of the contributions of the interference between different forbiddennesses [4]. Such

2. The helicity amplitudes and simple angular correlations

The muon capture process is a slow (weak) binary decay of corresponding mu-mesic atom. The capture rate from K orbit of the atom proceeds from combinations of the two hyperfine states $F_+ = J_i + \frac{1}{2}$ and $F_- = J_i - \frac{1}{2}$, where J_i is the spin of the initial target.

In the rest frame of the mu-mesic atom with spin F, let $\mathbf{q}$ be the momentum of the neutrino. Then, the amplitude describing the decay of spin F, with the z-component M into final nuclei (spin J_f) and a neutrino with helicities λ and h respectively may be written [10]

$$\langle \mathbf{q}[h]; -\mathbf{q}[J_f]\lambda | T | [F] M \rangle = \sqrt{\frac{2F+1}{4\pi}}\, T_\lambda^F D^F_{M\, h-\lambda}(\mathbf{q}) . \qquad (1)$$

We adopt here normalization of the states in Poincaré invariant manner and other notation as in book by Werle /11/. In (1) the constants T_λ^F are the helicity mu-mesic atom decay amplitudes. The helicity h for massless particle is fixed and therefore is not written explicitly in T_λ^F.

In order to calculate the probability and angular distributions for the process $J_i \xrightarrow{\mu^-} J_f \longrightarrow j$ it is necessary to average over the muonic and nuclear polarization states with the density matrix.

Using the amplitude (1) we perform these calculations as described in detail by Bukhvostov and Popov /12/.

Below we denote by p the probability of occupation of the hyperfine (hf) level of the atom with $F = F_+$ (if J_i =0, then $p \equiv 1$) . The existence of the conversion rate R between hf K-shell levels causes the time dependence of the capture rate and of the coefficients determining angular correlations [13] because $p = p(Rt)$. Assuming the statistical initial population of the hf levels (t=0)

$$p = \begin{cases} \frac{J_i+1}{2J_i+1} \exp(-Rt) & \text{if} \quad \mu > 0 \\ 1 - \frac{J_i}{2J_i+1} \exp(-Rt) & \text{if} \quad \mu < 0 \, . \end{cases} \tag{2}$$

Here μ is the dipole magnetic moment of the capturing nuclei. Moreover we introduce a short notation for the helicity amplitudes

$$T_\lambda^F = \begin{cases} T_\lambda^+ & \text{if} \quad F = J_i + \frac{1}{2} \\ T_\lambda^- & \text{if} \quad F = J_i - \frac{1}{2} \end{cases} , \tag{3}$$

1. The transition capture rate.

The transition capture rate is given by:

$$\Lambda^c = q^2 \sum_\lambda \left(p |T_\lambda^+|^2 + (1-p) |T_\lambda^-| \right) \tag{4}$$

$$\equiv \frac{2}{3} q^2 \, \frac{2J_f+1}{2J_i+1} \, \Lambda$$

Here q denotes the neutrino energy

$$\frac{q}{m} = \left(1 - \frac{\Delta + B.E.}{m}\right)\left[1 - \frac{m - \Delta - B.E.}{2(AM + m - B.E.)}\right] ,$$

where the notation is as follows: m and M are muon and nucleon masses respectively,

Δ is an energy difference between the initial and final nuclear states without the recoil energy, and finally B.E. $>$ 0 denote a binding energy of the muon on the K orbit.

The range of the summation over the final nucleus helicities λ in (4) as well as in other subsequent formulas is clear from expression (1).

It is convenient to transform (4) into two forms depending on the sign of the dipole magnetic moment of the capturing nuclei μ using (2)

$$\Lambda = \Lambda^\pm + \left(\Lambda^{stat} - \Lambda^\pm\right) \exp(-Rt) \, .$$

The upper (lower) sign refers to negative (positive) μ , respectively. $\Lambda^\pm$ describe the capture from lower hf level $F_\pm$ respectively. Λ^{stat} and $\Lambda^\pm$ are related by the identity:

$$J_i \Lambda^- + (J_i+1)\Lambda^+ = (2J_i+1)\Lambda^{stat} \, .$$

ii. The gamma-neutrino correlation between the unit vectors of the nuclear gamma deexcitation linear momenta $\mathbf{k}$ and neutrino momenta $\mathbf{q}$ (cf.[5])

$$W = \sum_{S=\text{even}} B_{S\eta}\, a_S P_S(\mathbf{k}\cdot\mathbf{q}) + 2h \sum_{S=\text{odd}} B_{S\eta}\, a_S P_S(\mathbf{k}\cdot\mathbf{q}) \quad . \tag{8}$$

Here the $B_{S\eta}$ term describes the nuclear radiation process and a_S the weak vertex.

Adopting the common normalization $a_0=1$ we may write ($\hat{S} \equiv \sqrt{2S+1}$)

$$\Lambda a_S = \frac{3}{2}\left(\frac{2J_i+1}{2J_f+1}\right) \hat{S} \sum_\lambda C^{J_f\lambda}_{J_f\lambda\, S0} \left(p|T^+_\lambda|^2 + (1-p)|T^-_\lambda|^2\right) \tag{9}$$

which may be presented using (2) in the convenient form

$$= a_S^{\pm} + \left(a_S^{stat} - a_S^{\pm}\right) \exp(-Rt) \tag{10}$$

with notations analogous to formula (6) and a relation like (7)

$$J_i a_S^- + (J_i+1)\, a_S^+ = (2J_i+1)\, a_S^{stat} \quad . \tag{11}$$

The term $B_{S\eta}(J_f \to j)$ looks like

$$B_{S\eta} = \hat{S}\hat{L}\hat{J}_f C^{L\eta}_{L\eta\, S0} W(jLJ_f SJ_f L) + \ldots \quad . \tag{12}$$

Here the parameter of the circular polarization of the nuclear deexcitation gamma-ray is denoted by $\eta = \pm 1$ (for right and left-polarized radiation respectively).

Obviously $B_{S(\eta=-1)} \equiv F_S^1(LLj \xleftarrow{\gamma} J_f)$, where the quantities F_S are given generally not only for a pure 2^L radiation and tabulated in the work by Fraunfelder and Steffen [14] .

$B_{S\eta}$ for S = even is independent of the gamma polarization η.

We mention that our $B_{S\eta}$ are connected with the Z_γ coefficient tabulated for $j = 1/2$ by Baldin et al. [15].

$$\hat{J}_f B_{S1}(J_f \xrightarrow{\gamma} j) = (-)^{J_f - j + 1} Z_\gamma(LJ_f LJ_f jS).$$

For $2J_f \geq S > 2L$, $B_{S\eta} \propto \delta$, where δ is a parameter of a mixing radiation.

The convenient table of B_{S1} quantities are given in Part II.

iii. Angular distributions of recoils is given by [8,5]

$$W = 1 + 2h\alpha\, \boldsymbol{\sigma}\cdot\mathbf{q} \quad , \tag{13}$$

where $\boldsymbol{\sigma}$ denotes the unit pseudovector of muon polarization, and α looks generally as follows

$$\Lambda\alpha = p\lambda_+ \alpha^+ + (1-p)\lambda_- \alpha^- \; .$$

The muon polarization λ is given by

$$\lambda = p\lambda_+ + (1-p)\lambda_- ,$$

where $\lambda_\pm$ characterizes this polarization in hf states $F_\pm$ respectively.

$$\alpha^+ = \left(\frac{2J_i+1}{2J_f+1}\right)\frac{9}{2J_i+3}\sum_\lambda (\lambda-h)\left|T_\lambda^+\right|^2 , \tag{14}$$

$$\alpha^- = -\left(\frac{2J_i+1}{2J_f+1}\right)\frac{9}{2J_i-1}\sum_\lambda (\lambda-h)\left|T_\lambda^-\right|^2 .$$

iv. The circular polarization of the nuclear deexitation gamma quanta with respect to the direction of the muon polarization looks generally like (cf.[5])

$$P_\gamma = B_{11}\,\beta\,\boldsymbol{\sigma}.\mathbf{k}$$

$$\beta = \Lambda^{-1}\left[p\lambda_+\beta^+ + (1-p)\lambda_-\beta^-\right]$$

$$\beta^+ = \frac{3\sqrt{3}}{2}\left(\frac{2J_i+1}{2J_f+1}\right)\left(\frac{2J_i+1}{2J_i+3}\right)^{\frac{1}{2}}\sum_{\lambda,\lambda'} C^{J_f\lambda'}_{J_f\lambda\,1\,\lambda'-\lambda}\, C^{F_+\,\lambda'-h}_{F_+\,\lambda-h\,1\,\lambda'-\lambda}\, T_\lambda^+ \overset{*}{T}{}_{\lambda'}^+$$

$$\beta^- = -\frac{3\sqrt{3}}{2}\left(\frac{2J_i+1}{2J_f+1}\right)\left(\frac{2J_i+1}{2J_i-1}\right)^{\frac{1}{2}}\sum_{\lambda,\lambda'} C^{J_f\lambda'}_{J_f\lambda\,1\,\lambda'-\lambda}\, C^{F_-\,\lambda'-h}_{F_-\,\lambda-h\,1\,\lambda'-\lambda}\, T_\lambda^- \overset{*}{T}{}_{\lambda'}^-$$

an interference is essentially due to large momentum transfer in muon capture reaction. It is obvious that in all effects where such an interference appears the restriction only to the terms of one forbiddenness is not justified. The classification of Morita and Fujii [6] in these cases can not be applied.

Our formulas take into account the full contribution of higher forbiddennesses, i.e. the all multipole amplitudes, and show clearly the existence of the kinematical enhancement of higher multipole in same transitions.

The explicit formulae presented in the tables describe the capture rate, the gamma-neutrino correlations [7] and angular distributions of recoils [8] in lepton capture processes from K-orbit for $J_i \leq 3$, $J_f \leq 5/2$ with change and without change of the parities of nuclear levels. We miss the case $J_i = 2$, because of the absence of such stable targets in nature.

The triple angular correlations, interesting mostly from the point of view of the T-nonconservation are concerned here shortly (cf.[9]).

We refer generally to [5], which was a base for the present work and which contains a rather complete references concerned with angular correlations in normal muon capture.

If we introduce

$$T_J^F = \sqrt{\frac{3}{2}} \sum_J (-)^{F+\lambda-h} \, T_\lambda^F \, C^{Jh}_{J_f \lambda \, F \, h-\lambda} \tag{16}$$

then the latter expressions may be rewritten as

$$\beta^+ = (-)^{2J_i+1} \sqrt{3}\, \hat{J}_i \left(\frac{2J_i+1}{2J_f+1}\right)\left(\frac{2J_i+2}{2J_i+3}\right)^{\frac{1}{2}} \sum_J \hat{J} W(J_f J 1 F_+ F_+ J_f) \left|T_J^+\right|^2$$

$$\beta^- = (-)^{2J_i} \sqrt{3}\, \hat{J}_i \left(\frac{2J_i+1}{2J_f+1}\right)\left(\frac{2J_i}{2J_i-1}\right)^{\frac{1}{2}} \sum_J \hat{J} W(J_f J 1 F_- F_- J_f) \left|T_J^-\right|^2 \tag{17}$$

3. The multipole amplitudes

In terms of the helicity mu-mesic atom decay amplitudes T_λ^F introduced above, the calculation of the angular distributions and polarizations and hence the phenomenological analysis of the experimental data are of course the simplest ones. However these amplitudes calculated e.g. in the current-current model of the weak interactions and in the impulse approximations have no simple structure, and the theoretical interpretation of these constants from the point of view of the weak vertex is very complicated. Therefore it is convenient to introduce another basis in which the theoretical

interpretation of the muon capture amplitudes is the clearest one . From the point of view of the fundamental weak vertex in current current model of interaction the multipole amplitudes are the best ones. In terms of these amplitudes the weak process has an analogy with the usual nuclear radiation. The multipole amplitudes have a great advantage in comparison with all other, because they are almost (up to the αZ corrections, etc) " diagonalized" in vector and axial weak currents.

Following Morita and Fujii [6] we are interested in the calculation of the matrix element of the muon capture reaction between canonical, spin z- component, eigenvectors.

The initial state, i.e. nuclei with the muon on the K-orbit is

$$\left|[J_i]\mu_i ; [\tfrac{1}{2}]\mu'\right\rangle = \sum_F C^{FM}_{J_i \mu_i \frac{1}{2} \mu'} \left|[F]\, M\right\rangle \; . \tag{18}$$

The free neutrino state may be specified by its linear momentum $\mathbf{q}$, the helicity h and the Z-component of the "spin" μ

$$\left|\mathbf{q}\begin{bmatrix} s \\ h \end{bmatrix} \mu\right\rangle = \frac{\hat{s}}{\sqrt{4\pi}} \overset{*}{D}{}^{s}_{\mu h}(\mathbf{q}) \left|\mathbf{q}[h]\right\rangle \; , \tag{19}$$

where s is a "spin" of the free massless particle . Because

$$\sum_{\mu} D^{s}_{\mu h}(\mathbf{q}) \overset{*}{D}{}^{s}_{\mu h}(\mathbf{q}) = 1 ,$$ this "spin" does not contribute to the probability of the process, which is natural as the massless particle is fully described by the helicity alone [x]

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[x] One of us (Z.O.) is deeply indepted to Prof.V.I.Ogievetsky for the explanation of this point .

$$|-\mathbf{q}[J_f]\mu_f\rangle = \sum_{\lambda} \overset{*}{D}{}^{J_f}_{\mu_f\,-\lambda}(\mathbf{q})\,|-\mathbf{q}[J_f]\lambda\rangle . \qquad (20)$$

Using (18)-(20) and (1) the muon capture transition amplitude may be written [11]

$$\langle \mathbf{q}\left[\begin{smallmatrix} s \\ h \end{smallmatrix}\right]\mu\, ; -\mathbf{q}[J_f]\mu_f \,|\, T \,|\, [J_i]\mu_i\, ; \tfrac{1}{2}\mu' \rangle$$

$$= \frac{\hat{S}}{4\pi} \sum_{F,\lambda} \hat{F}\, T^{F}_{\lambda}\, C^{FM}_{J_i \mu_i \frac{1}{2}\mu'}\, D^{s}_{\mu h}(\mathbf{q})\, D^{J_f}_{\mu_f-\lambda}(\mathbf{q})\, \overset{*}{D}{}^{F}_{M,\,h-\lambda}(\mathbf{q})$$

$$= \frac{\hat{S}}{\sqrt{4\pi}} \sum_{F,\lambda,l,\sigma} \frac{\hat{l}}{\hat{F}}\, T^{F}_{\lambda}\, C^{FM}_{J_i \mu_i \frac{1}{2}\mu'}\, C^{F\,h-\lambda}_{l0\,\sigma\,h-\lambda}\, C^{\sigma\,h-\lambda}_{sh\,J_f\,-\lambda} \times \qquad (21)$$

$$\times\, C^{FM}_{l m_l\,\sigma m_\sigma}\, C^{\sigma m_\sigma}_{s\mu\,J_f\mu_f}\, Y^{l}_{m_l}(\mathbf{q}) .$$

Here l is the relative orbital momentum of neutrino and recoil nucleus in the center of mass system, and σ is total spin of the final state.

Now instead of the helicity couplings $\{F, \sigma, \lambda\}$ it is easy to introduce the multipole couplings $\{I, J, \nu\}$ where I denotes multipolarity of the lepton field; J is called usually a "neutrino wave" and finally ν is the muon helicity .

In terms of multipole coupings the muon capture amplitude (21) looks as follows [x] .

$$\langle |T| \rangle = \frac{\hat{S}}{\sqrt{4\pi}} \sum_{I,J,l,\nu} \frac{\hat{l}}{2I+1} (-)^{\frac{1}{2}-\mu_i-\mu'+\mu_f}\, C^{J_f\mu_f}_{J_i\mu_i\,I\,\mu_f-\mu_i} \times \qquad (22)$$

$$\times\, C^{I\,\mu_i-\mu_f}_{J m_J\,\frac{1}{2}-\mu'}\, C^{J m_J}_{l m_l\,s\mu}\, C^{Jh}_{l0\,sh}\, C^{I\,h-\nu}_{Jh\,\frac{1}{2}-\nu}\, a^{\nu}_{I}\, Y^{l}_{m_l}(\mathbf{q}) .$$

The multipole amplitudes a^{ν}_{I} are connected with the helicity amplitudes

$$a^{\nu}_{I} = \frac{\sqrt{2}}{2}\, \frac{2I+1}{2J_f+1} (-)^{I} \sum_{F,\lambda} \hat{F}\, T^{F}_{\lambda}\, C^{J_f\lambda}_{J_i\lambda_i\,I\,h-\nu}\, C^{F\,h-\lambda}_{J_i-\lambda_i\,\frac{1}{2}\nu} . \qquad (2$$

---

[x] This expression is equivalent (10) in [5] .

The inverse relation reads

$$\hat{F}\, T_{\lambda}^{F} = \frac{2}{\sqrt{3}} \sum_{I,\nu} (-)^{I}\, a_{I}^{\nu}\, C^{J_f \lambda}_{J_i \lambda_i\, I\, h-\nu}\, C^{F\, h-\lambda}_{J_i -\lambda_i\, \frac{1}{2} \nu} \,. \tag{24}$$

Manipulation with (16) leads to

$$T_{J}^{F} = \sqrt{2}\, \hat{J}_f \sum_{I,\nu} (-)^{2F+I+\frac{1}{2}-\nu}\, a_{I}^{\nu}\, C^{Jh}_{I\, h-\nu\, \frac{1}{2}\nu} \begin{Bmatrix} F & J_i & \frac{1}{2} \\ I & J & J_f \end{Bmatrix} . \tag{25}$$

Next we consider the restrictions which follow from T conservation in muon capture process. T-conservation in terms of the multipole amplitudes readily reads

$$\begin{aligned} \mathrm{Im}\, a_{I}^{\nu} &= 0 \qquad \text{if} \quad \pi_i \pi_f = +1 \\ \mathrm{Re}\, a_{I}^{\nu} &= 0 \qquad \text{if} \quad \pi_i \pi_f = -1 \,, \end{aligned} \tag{26}$$

where π_i and π_f are the parities of initial and final nuclear state respectively.

Therefore if T conservation holds then

$$\begin{aligned} \mathrm{Im}\, T_{\lambda}^{F} &= 0 \quad \text{if} \quad \pi_i \pi_f = +1 \text{ and } J_f - J_i = \text{even}, \\ &\qquad\quad \text{or} \quad \pi_i \pi_f = -1 \text{ and } J_f - J_i = \text{odd} \\ \mathrm{Re}\, T_{\lambda}^{F} &= 0 \qquad\qquad \text{in the opposite cases.} \end{aligned} \tag{27}$$

For the model calculations it is convenient to introduce instead of the complex a_I^{ν} numbers pure real (if T-invariance holds) amplituedes. We introduce them as follows (cf.[16] and [5]).

$$i^{I} a_{I}^{\nu} \stackrel{df}{=} \begin{cases} \sqrt{\frac{I+1}{I}}\left[(-)^{I} 2h A_I + i M_I\right] & \text{if} \quad \nu = -h \\ \left[(-)^{I} 2h V_I + i P_I\right] & \text{if} \quad \nu = h \,. \end{cases} \qquad (2$$

4. Impulse approximation

Neglecting unitarity corrections due the final state (weak) interaction, we may take the transition operator T in (1) as Hermitian. Next we represent T by the current-current operator and as usually in V-A form. Taking a Foldy-Wouthuysen transformation in the impulse approximation theory (one body weak operators on the mass shell) of Fujii and Primakoff [1] we have up to the term of order 1/M , where M is a nucleon mass [5]

$$\begin{aligned} A_I &= (-)^{I+1} G_A \sqrt{\frac{2I+1}{I+1}} \sqrt{I}\, [1 I I] - \ldots \\ V_I &= (-)^{I}\, G_V \sqrt{2I+1} \sqrt{3}\, [0 I I] + \ldots \\ M_I &= G_A \sqrt{I}\, [1\, I-1\, I] + \ldots \\ P_I &= (G_A - G_P) \sqrt{I}\, [1\, I-1\, I] + \ldots \,. \end{aligned} \qquad ($$

In (29) we neglect important many other contributions of the nuclear matrix elements $[k w I]$ defined by Morita and Fujii [6], as well as αZ corrections. The formulas (29) are given only for the illustration of "names" of new amplitudes defined by (28). The most accurate expressions for these including the second class currents and taking into account rigorously the relativistic corrections to the muon wave functions are given in [5]. Any corrections to (29) are the subject of the extensive theoretical studies.

The effective coupling constant in (29) G_A, G_V and G_P called axial, vector and pseudoscalar constants were introduced by Primakoff [17]. The physical interpretation of the amplitude (28) is clear from formulas (29). We see that the induced pseudoscalar coupling contributes to P_I only. Moreover in the absence of the vector current $V_I = 0$, and in the absence of the axial current $P_I = 0$ x.

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x These statements are strictly correct up to the αZ corrections.

The contributions of the axial and vector currents cannot be separated even approximatelly in A_I and M_I amplitudes. This is due to the fact that the muon is not on the mass shell at the moment of capture. We return to this point in connection with elementary particle approach to muon capture processes.

5. Balashov-Eramzhyan notation [18] and elementary particle approach

In the frame of the Morita and Fujii formalism [6] Balashov and Eramzhyan use for the calculations of the muon capture rates the "neutrino waves" amplitudes μ_I^J.

The "neutrino waves" amplitudes are defined in terms of the multipole ones

$$\hat{I}\mu_I^J = \sqrt{\frac{2}{3}}\sum_\nu (-)^{\frac{1}{2}-\nu}\, a_I^\nu\, C^{Jh}_{I\,h-\nu\,\frac{1}{2}\nu} . \tag{30}$$

The helicity amplitudes T_J^F (see (16) and (25)) in their terms look

$$T_J^F = \sqrt{3}\,\hat{J}_f \sum_I (-)^{2F+I}\,\hat{I}\mu_I^J \begin{Bmatrix} F & J_i & 1/2 \\ I & J & J_f \end{Bmatrix} . \tag{31}$$

The notations of Balashov and Eramzhyan, using explicitely the T-invariance, are as follows

$$i^I\mu_I^J = \begin{cases} \mu_I(I) & \text{if } J = I - \frac{1}{2},\ \pi_i\pi_f = (-)^I \\ i\mu_I(-I) & \text{if } J = I - \frac{1}{2},\ \pi_i\pi_f = (-)^{I+1} \\ \mu_I(-I-1) & \text{if } J = I + \frac{1}{2},\ \pi_i\pi_f = (-)^I \\ -i\mu_I(I+1) & \text{if } J = I + \frac{1}{2},\ \pi_i\pi_f = (-)^{I+1} \end{cases} . \tag{32}$$

Therefore the multipole amplitudes defined by (28) and (29) in the Balashov and Eramzhyan notation (32) are

$$
\begin{aligned}
A_I &= (-)^I \sqrt{\frac{3I}{2}} \left[\mu_I(I) - \sqrt{\frac{I}{I+1}}\, \mu_I(-I-1) \right] \\
V_I &= (-)^I \sqrt{\frac{3}{2}} \left[\sqrt{I}\, \mu_I(I) + \sqrt{I+1}\, \mu_I(-I-1) \right] \\
M_I &= -\sqrt{\frac{3}{2}} \sqrt{I} \left[\mu_I(-I) + \sqrt{\frac{I}{I+1}}\, \mu_I(I+1) \right] \\
P_I &= -\sqrt{\frac{3}{2}} \left[\sqrt{I}\, \mu_I(-I) - \sqrt{I+1}\, \mu_I(I+1) \right] .
\end{aligned}
\tag{33}
$$

In terms of the "neutrino waves" amplitudes the kinematical formulae for angular distributions are more complicated. These amplitudes have not any advantage in comparison with the helicity and multipole amplitudes (cf.Sec.4) and thus we do not consider them here.

In the elementary particle approach [19] the muon capture process is described as scattering on the threshold of the free particles at rest. This unrealistic simplified assumption enables one to introduce the multipole form factors (constant for muon capture) for any spins of the nuclear states as was performed by Rafael [20]. These formfactors are defined fully by the nuclear currents alone, contrarry to the e.g. helicity amplitudes (1) defined by the full muon capture transition amplitude. Therefore for these formfactors it is easy to find the general restrictions following from CVC, PCAC and G-parity conservation without the help of any nuclear model calculations.

Unrealisticity of such approach follows from the fact that the contributions from the different nuclear currents (V and A) are strictly separated by kinematics alone. Of course, this is not the case, which is shown in Sec.4.

Finally, we give the Galindo and Pascual notations [21]

$$
\begin{aligned}
A_I &= \sqrt{\frac{3}{2\pi}}\, (-)^I\, \hat{I} \sqrt{\frac{I}{I+1}}\; \bar{G}_A^{[I]} \\
V_I &= \sqrt{\frac{3}{2\pi}}\, (-)^I\, \hat{I}\; \bar{G}_V^{[I]} \\
M_I &= \sqrt{\frac{3}{2\pi}}\, \sqrt{I}\; \bar{G}_A^{[I-1]}[I] \\
M_I - P_I &= \sqrt{\frac{3}{2\pi}}\, \sqrt{I}\; \bar{G}_P^{[I-1]}[I] .
\end{aligned}
$$

6. Triple angular correlation including T-noninvariant

In polarized muon capture it is possible, in principle, to measure experimentally triple angular correlations between neutrino and nuclear radiation momenta $\mathbf{q}$ and $\mathbf{k}$ with respect to the direction of muon polarization $\boldsymbol{\sigma}$ [x]

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[x] In our notation $\mathbf{q}$, $\mathbf{k}$ and $\boldsymbol{\sigma}$ are always the unit vectors .

Such a rather complicated angular distribution is the most interesting one mainly from the point of view of proving the T-conservation of the muon nuclei weak interaction. Moreover they may give of course additional independent valuable information on the details of the interaction and nuclear structure. Generally, such a triple angular correlation looks as follows:

$$W(\mathbf{q},\mathbf{k},\boldsymbol{\sigma};\eta) = 1 + \sum_{\substack{S=2\\ S=\mathrm{even}}}^{2J_f} B_{S\eta}\, a_S\, P_S(\mathbf{k}\cdot\mathbf{q}) + 2h \sum_{\substack{S=1\\ S=\mathrm{odd}}}^{2J_f} B_{S\eta}\, a_S\, P_S(\mathbf{k}\cdot\mathbf{q}) + \sum_{S=1}^{2J_f} B_{S\eta}\, \beta_S\, \boldsymbol{\sigma}\cdot\mathbf{k}\, P_S'(\mathbf{k}\cdot\mathbf{q}) + \sum_{f=1}^{2J_f+1} \alpha_f\, \boldsymbol{\sigma}\cdot\mathbf{q}\, P_f'(\mathbf{k}\cdot\mathbf{q}) + \sum_{S=1}^{2J_f} B_{S\eta}\, d_S\, \mathbf{q}\wedge\boldsymbol{\sigma}\cdot\mathbf{k}\, P_S'(\mathbf{k}\cdot\mathbf{q}) \quad . \tag{34}$$

If we introduce the a_{Sf} quantities as follows

$$a_{Sf} = \left[\tfrac{1}{2} - h + (-)^f\left(\tfrac{1}{2}+h\right)\right] \sum_{F,J,J'} \hat{J}'\, C^{Jh}_{J'hf0} \begin{Bmatrix} J_f & J_f & S \\ F & F & 1 \\ J & J' & f \end{Bmatrix} T^F_J \overset{*}{T}{}^F_{J'} \times \left\{ p\lambda_+ \delta_{F,F_+} \sqrt{\frac{J_i+1}{2J_i+1}} - (1-p)\lambda_- \sqrt{\frac{J_i}{2J_i-1}}\, \delta_{F,F_-} \right\} \tag{35}$$

then for the triple angular correlation coefficients we have generally

$$\beta_S = (-)^{S+1}\, 3\sqrt{2}\,(2J_i+1) \frac{\hat{J}_i}{\hat{J}_f} \Lambda^{-1} \times \left[\left(\frac{2S+3}{S+1}\right)^{1/2} a_{S,S+1} + \left(\frac{2S-1}{S}\right)^{1/2} a_{S,S-1} \right]$$

$$\alpha_f = (-)^{f+1}\, \hat{f}\, 3\sqrt{2}\,(2J_i+1) \frac{\hat{J}_i}{\hat{J}_f} \Lambda^{-1} \times \left[f^{-1/2} B_{f-1,\eta}\, a_{f-1,f} + (f+1)^{-1/2} B_{f+1,\eta}\, a_{f+1,f} \right]$$

$$d_S = (-)^S\, 3\sqrt{2}\,(2J_i+1) \frac{\hat{J}_i}{\hat{J}_f} \frac{2S+1}{\sqrt{S(S+1)}} \Lambda^{-1}\, i\, a_{SS} \quad .$$

Here the Λ is defined by (4), and $B_{S\eta}$ by formula (12). The coefficients d_S demonstrate the T-nonconservation in muon capture.

In the particular important case of zero spin targets a_{Sf} (35) looks simply

$$a_{Sf} = \frac{\lambda}{\sqrt{3}} \left[\tfrac{1}{2} - h + (-)^f\left(\tfrac{1}{2}+h\right)\right] \sum_{J,J'} \hat{J}'\, C^{Jh}_{J'hf0} \begin{Bmatrix} I & I & S \\ \tfrac{1}{2} & \tfrac{1}{2} & 1 \\ J & J' & f \end{Bmatrix} T_J \overset{*}{T}_{J'}$$

where

$$T_J = - \sum_\nu (-)^{J+\nu}\, a^\nu_I\, C^{Jh}_{I\,h-\nu\,\frac{1}{2}\,\nu} \quad .$$

PART II

TABLES OF FORMULAE FOR ANGULAR DISTRIBUTIONS FOR LOW VALUES OF NUCLEAR SPINS

USE OF THE TABLES. In the tables we present the formulas for the capture rate Λ gamma-neutrino angular correlation coefficients a_s and the coefficient which describes the angular distributions of recoils defined in Part I by (4) - (6-7), (8-11), and (13-14) respectively.

These formulas are given in terms of the multipole amplitudes (28-29). Any formula for the transition $J_i \xrightarrow{\mu} J_f$ with change of parities of the nuclear levels may be obtained from the formula for the transition $J_i \xrightarrow{\mu} J_f$ without change of parities and inversely by simple substitution [x] [5]

$$M_I \leftrightarrow A_I \qquad (37)$$
$$P_I \leftrightarrow V_I \ .$$

---

[x] The amplitudes A_I and V_I used in this paper differ from the introduced previously [16, 5] .

Therefore in the tables we give these formulas only for one of these two cases.

Next the quantities $\Lambda^{\pm}$ and $a_s^{\pm}$ are related by formulas (7) and (11) respectively, and therefore consequently we calculated only Λ^- and a_s^- which describe the muon capture by nuclei with positive dipole magnetic moments μ .

For the convenience we add to each part of the tables the examples of the reactions which are described by the given formulas.

In these examples we restrict ourselves to the captures from g.s. of the light nuclei, however, the corresponding formulae are strictly valid for light and heavy nuclei. The sign of the dipole magnetic moment μ of the capturing nucleus is always shown. The formulas in the Tables are derived using (28) with $h = -1/2$.

To summarize, the following look-up procedure for using the Tables is recommended.

Rule 1. If the change of parities of the nuclear levels in transition under consideration is opposite to that shown in Tables the substitution (37) then should be made.

Rule 2. In the case of the capture by spin nuclei with negative dipole magnetic moment μ in the absence of formulas for Λ^+ and a_s^+ they must be calculated from (7) and (11).

MUON CAPTURE BY ZERO SPIN TARGETS

This most important case includes e.g. the following reactions: $^{12}C \rightarrow B$, $^{16}O \rightarrow N$, $^{18}O \rightarrow N$, $^{20}Ne \rightarrow F$, $^{22}Ne \rightarrow F$, $^{24}Mg \rightarrow Na$, $^{26}Mg \rightarrow Na$, $^{28}Si \rightarrow Al$, etc. Each of such transitions are fully described by two independent amplitudes[x] . They are real numbers if T conservation holds.

---

[x] Exception are the simplest $0 \longrightarrow 0$ transitions each of which is described by one amplitude only.

$\boxed{0 \to 0}$ no

$$\Lambda = V_0^2$$
$$\alpha = -V_0^2$$

$\boxed{0 \to 1}$ no

$$\Lambda = \frac{2}{3} M_1^2 + \frac{1}{3} P_1^2$$
$$a_2 = \frac{\sqrt{2}}{3}\left(M_1^2 - P_1^2\right)$$
$$\alpha = \frac{1}{3}\left(2M_1^2 - P_1^2\right)$$

$\boxed{0 \to 2}$ no

$$\Lambda = \frac{1}{10}\left(3A_2^2 + 2V_2^2\right)$$
$$a_2 = -\frac{1}{2}(105)^{-1/2}\left(3A_2^2 + 4V_2^2\right)$$
$$a_4 = -\frac{3}{5}\left(\frac{2}{7}\right)^{1/2}\left(A_2^2 - 2V_2^2\right)$$
$$\alpha = \frac{1}{10}\left(3A_2^2 - 2V_2^2\right)$$

MUON CAPTURE BY 1/2 SPIN TARGETS

The formulas presented here may describe e.g. the following muon capture reactions.

1. with the positive dipole magnetic moment of capturing nuclei: $p \to n$, $^{13}C \to B$, $^{19}F \to O$, $^{31}P \to Si$ etc.

ii. with the negative dipole magnetic moment of capturing nuclei: $^{3}He \to H$, $^{15}N \to C$, $^{29}Si \to Al$ etc.

$\boxed{\frac{1}{2} \to \frac{1}{2}}$ no

$$\Lambda^{stat} = V_0^2 + \frac{2}{3} M_1^2 + \frac{1}{3} P_1^2$$
$$\Lambda^- = \left(V_0 - \frac{2}{\sqrt{3}} M_1 - \frac{1}{\sqrt{3}} P_1\right)^2$$
$$\Lambda^+ = V_0^2 + \frac{2}{3\sqrt{3}} V_0 (2M_1 + P_1) + \frac{4}{9} M_1 (M_1 - P_1) + \frac{1}{3} P_1^2$$
$$\alpha^+ = -\left(V_0 + \frac{1}{\sqrt{3}} P_1\right)^2 \; ; \; \alpha^- = 0 \, .$$

The gamma-neutrino angular correlation

$$a_1^{stat} = -\frac{2}{\sqrt{3}} V_0 P_1 + \frac{2}{3} M_1^2 \; ; \; a_1^- = \Lambda^-$$

$\boxed{\frac{1}{2} \to \frac{3}{2}}$ no

$$\Lambda^{stat} = \frac{2}{3} M_1^2 + \frac{1}{3} P_1^2 + \frac{3}{10} A_2^2 + \frac{1}{5} V_2^2$$
$$\Lambda^- = \frac{1}{3}\left[M_1 - P_1 - \sqrt{\frac{3}{5}}\left(\frac{3}{2} A_2 + V_2\right)\right]^2$$

$$\Lambda^{+} = \frac{7}{9}M_1^2 + \frac{2}{9}M_1P_1 + \frac{1}{3}P_1^2 + \frac{2}{9}\sqrt{\frac{3}{5}}(M_1 - P_1)\left(\frac{3}{2}A_2 + V_2\right) +$$

$$+ \frac{1}{4}A_2^2 - \frac{1}{5}V_2(A_2 - V_2)$$

$$\alpha^{+} = \left(M_1 + \frac{1}{2}\sqrt{\frac{3}{5}}A_2\right)^2 - \frac{1}{3}\left(P_1 - \sqrt{\frac{3}{5}}V_2\right)^2 ; \quad \alpha^{-} = 0$$

$$a_1^{stat} = \frac{\sqrt{5}}{3}\left[M_1^2 + \frac{3}{5}\left(\sqrt{\frac{3}{5}}M_1A_2 + \frac{2}{\sqrt{15}}P_1V_2 + \frac{9}{20}A_2^2\right)\right]$$

$$3a_2^{stat} = M_1^2 - P_1^2 + 3\sqrt{\frac{3}{5}}M_1A_2 - \frac{9}{20}A_2^2 - \frac{3}{5}V_2^2$$

$$a_3^{stat} = \frac{2}{5}\sqrt{3}\left(M_1A_2 - P_1V_2 - \frac{1}{2}\sqrt{\frac{3}{5}}A_2^2\right)$$

$$a_S^{-} = \sqrt{2S+1}\; C^{3/2\;1/2}_{3/2\;1/2\;S\;0}\; \Lambda^{-}$$

$$3a_2^{+} = (M_1 - P_1)\left(\frac{5}{3}M_1 + P_1 - \frac{2}{\sqrt{15}}V_2\right) + \sqrt{\frac{3}{5}}A_2(3M_1 + P_1) - \frac{3}{5}\left(\frac{1}{2}A_2 - V_2\right)^2$$

$\boxed{\frac{1}{2} \rightarrow \frac{5}{2}}$ yes

$$5\Lambda^{stat} = \frac{3}{2}M_2^2 + P_2^2 + \frac{20}{21}A_3^2 + \frac{5}{7}V_3^2$$

$$5\Lambda^{-} = \left[M_2 - P_2 + \sqrt{\frac{5}{7}}\left(\frac{4}{3}A_3 + V_3\right)\right]^2$$

$$5\alpha^{+} = 2\left(M_2 - \frac{2}{3}\sqrt{\frac{5}{7}}A_3\right)^2 - \left(P_2 + \sqrt{\frac{5}{7}}V_3\right)^2 ; \quad \alpha^{-} = 0$$

$$a_2^{stat} = -\frac{2}{5}\sqrt{\frac{2}{7}}\left(\frac{3}{4}M_2^2 + P_2^2 + \sqrt{\frac{5}{7}}M_2A_3 + \frac{5}{7}A_3^2 + \frac{5}{7}V_3^2\right)$$

$$a_4^{stat} = -\frac{1}{5}\sqrt{\frac{6}{7}}\left(M_2^2 - P_2^2 - \frac{10}{3}\sqrt{\frac{5}{7}}M_2A_3 - \frac{10}{7\cdot 9}A_3^2 - \frac{5}{7}V_3^2\right)$$

$$a_2^{-} = -2\sqrt{\frac{2}{7}}\,\Lambda^{-}$$

$$a_4^{-} = \sqrt{\frac{6}{7}}\,\Lambda^{-}$$

MUON CAPTURE BY SPIN ONE TARGETS

There are the two well known reactions for this case in the region of the very light nuclei both corresponding to positive value of dipole magnetic moment of the target [4]: $^{6}Li \rightarrow He$ and $^{14}N \rightarrow C$.

$\boxed{1 \rightarrow 0}$ no

$$3\Lambda^{stat} = 2M_1^2 + P_1^2$$

$$3\Lambda^{-} = (2M_1 + P_1)^2 = \alpha^{-}$$

$$\alpha^{+} = -\frac{1}{5}(M_1 - P_1)^2$$

$\boxed{1 \rightarrow 1}$ no

$$\Lambda^{stat} = V_0^2 + \frac{1}{3}(2M_1^2 + P_1^2) + \frac{1}{10}(3A_2^2 + 2V_2^2)$$

$$\Lambda^{-} = \left[V_0 - \frac{\sqrt{2}}{3}(2M_1 + P_1)\right]^2 + \left[\frac{1}{3}(M_1 - P_1) - \frac{1}{2\sqrt{5}}(3A_2 + 2V_2)\right]^2$$

$$a_2^{stat} = -\frac{2}{\sqrt{5}}V_0V_2 - \frac{1}{3\sqrt{2}}(M_1^2 - P_1^2) + \frac{3}{\sqrt{10}}M_1A_2 - \frac{1}{20\sqrt{2}}(3A_2^2 + 2V_2^2)$$

$$a_2^{-} = \frac{2}{3}V_0(M_1 - P_1) - \frac{1}{\sqrt{5}}V_0(3A_2 + 2V_2) - \frac{1}{3\sqrt{2}}(M_1 - P_1)(3M_1 + P_1) +$$

$$+ \frac{1}{15\sqrt{10}}\left[3A_2(13M_1 - 11P_1) + 2V_2(M_1 - 13P_1)\right] -$$

$$- \frac{1}{20\sqrt{2}}(3A_2 + 2V_2)^2$$

$$\alpha^{-} = -(V_0 - \sqrt{2}P_1)^2 + (M_1 - P_1)^2 + \frac{1}{\sqrt{5}}(2\sqrt{2}V_0 - 3M_1 - P_1)(3A_2 + 2V_2) +$$

$$+ \frac{1}{20}(3A_2 + 2V_2)^2$$

$$10\alpha^{+} = 2V_0\left[-5V_0 - 5\sqrt{2}P_1 + \sqrt{\frac{2}{5}}(3A_2 + 2V_2)\right] + M_1^2 - 2M_1P_1 - 4P_1^2 +$$

$$+ \frac{1}{\sqrt{5}}\left[3A_2(3M_1 - P_1) - 2V_2(3M_1 - 5P_1)\right] +$$

$$+ \frac{1}{20}(3A_2 + 8V_2)(3A_2 - 4V_2)$$

$$\boxed{1 \to 2} \quad \text{no}$$

$$\Lambda^{stat} = \frac{2}{3} M_1^2 + \frac{1}{3} P_1^2 + \frac{3}{10} A_2^2 + \frac{1}{5} V_2^2 + \frac{4}{3} M_3^2 + P_3^2$$

$$\Lambda^- = \frac{1}{3}(M_1 - P_1)^2 - \frac{1}{5}(M_1 - P_1)(3A_2 + 2V_2) + \frac{7}{20} A_2^2 + \frac{1}{5} V_2 (A_2 + V_2) +$$

$$+ \frac{2}{5}\sqrt{\frac{2}{7}}(A_2 - V_2)\left(\frac{4}{3} M_3 + P_3\right) + \frac{1}{7}\left(\frac{4}{3} M_3 + P_3\right)^2$$

$$a_2^{stat} = \frac{1}{3}\sqrt{\frac{7}{10}}\left\{M_1^2 - P_1^2 + 3M_1A_2 - \frac{6}{7}\sqrt{\frac{2}{7}}\left(\frac{4}{3} M_1M_3 + P_1P_3\right) -\right.$$

$$\left. - \frac{3}{7}\left(\frac{3}{4} A_2^2 + V_2^2\right) - \frac{24}{49}\left(\sqrt{\frac{7}{2}} A_2M_3 + M_3^2 + P_3^2\right)\right\}$$

$$a_4^{stat} = -\frac{4}{21}\left\{M_1M_3 - P_1P_3 - \frac{1}{5}\sqrt{\frac{7}{2}}(A_2^2 - V_2^2) - \frac{5}{12} A_2M_3 -\right.$$

$$\left. - \frac{1}{\sqrt{14}}\left(\frac{1}{9} M_3^2 + \frac{1}{2} P_3^2\right)\right\}$$

$$a_2^- = -\frac{1}{3}\sqrt{\frac{7}{10}}\left\{(M_1 - P_1)^2 - \frac{3}{7}(M_1 - P_1)\left[5A_2 + 2V_2 + \sqrt{\frac{2}{7}}\left(\frac{4}{3} M_3 + P_3\right)\right] +\right.$$

$$+ \frac{3}{7}\left[\frac{13}{4} A_2^2 + A_2V_2 + V_2^2 + \frac{1}{3}\sqrt{\frac{2}{7}}(5A_2 - 2V_2)\left(\frac{4}{3} M_3 + P_3\right) +\right.$$

$$\left.\left. + \frac{8}{7}\left(\frac{4}{3} M_3 + P_3\right)^2\right]\right\}$$

$$a_4^- = -\frac{4}{15}\sqrt{\frac{2}{7}}\left\{(M_1 - P_1)\left[A_2 - V_2 + \frac{5}{\sqrt{14}}\left(\frac{4}{3} M_3 + P_3\right)\right] -\right.$$

$$- (A_2 - V_2)\left(A_2 + \frac{1}{2} V_2\right) - \sqrt{\frac{2}{7}}\left(\frac{11}{4} A_2 + V_2\right)\left(\frac{4}{3} M_3 + P_3\right) -$$

$$\left. - \frac{5}{28}\left(\frac{4}{3} M_3 + P_3\right)^2\right\}$$

MUON CAPTURE BY SPIN 3/2 NUCLEI

Examples of reactions with negative μ are $^{9}Be \to Li$, $^{21}Ne \to F$, etc. In the case of positive μ we have $^{11}B \to Be$, $^{23}Na \to Ne$, etc. Muon capture by ^{9}Be and ^{11}B was studied extensively both experimentally and theoretically (see e.g. the recent work of Bukhvostov et al.[4])

$$\boxed{\tfrac{3}{2} \to \tfrac{1}{2}} \quad \text{no}$$

$$\Lambda^{stat} = \frac{1}{3}(2M_1^2 + P_1^2) + \frac{1}{5}\left(\frac{3}{2} A_2^2 + V_2^2\right)$$

$$9\Lambda^- = 11 M_1^2 + 10 M_1P_1 + 3P_1^2 - \sqrt{\frac{3}{5}}(M_1 - P_1)(3A_2 + 2V_2) +$$

$$+ \frac{9}{20}(3A_2 + 2V_2)^2$$

$$3a_1^{stat} = -M_1^2 + \sqrt{\frac{3}{5}}(2P_1V_2 + 3M_1A_2) + \frac{9}{20} A_2^2$$

$$9a_1^- = -7M_1^2 - 2M_1P_1 + P_1^2 + \sqrt{\frac{3}{5}}(5M_1 + 3P_1)(3A_2 + 2V_2) +$$

$$+ \frac{3}{20}(3A_2 + 2V_2)^2$$

$$3\alpha^- = \left[3M_1 + P_1 - \frac{1}{2}\sqrt{\frac{3}{5}}(3A_2 + 2V_2)\right]^2$$

$$5\alpha^+ = -\left[M_1 - P_1 - \frac{1}{2}\sqrt{\frac{3}{5}}(A_2 - 2V_2)\right]^2$$

$$\boxed{\tfrac{3}{2} \to \tfrac{3}{2}} \quad \text{no}$$

$$\Lambda^{stat} = V_0^2 + \frac{1}{3}(2M_1^2 + P_1^2) + \frac{1}{10}(3A_2^2 + 2V_2^2) + \frac{1}{21}(4M_3^2 + 3P_3^2)$$

$$\Lambda^- = V_0\left[V_0 - \frac{2}{3}\sqrt{\frac{5}{3}}(2M_1 + P_1)\right] + \frac{1}{9}\left[4M_1(2M_1 + P_1) + 3P_1^2\right] -$$

$$- \frac{4}{15\sqrt{3}}(M_1 - P_1)(3A_2 + 2V_2) + \frac{1}{5}\left[2A_2(A_2 + V_2) + V_2^2\right] +$$

$$+ \frac{2}{15\sqrt{7}}(A_2 - V_2)(4M_3 + 3P_3) + \frac{1}{63}(4M_3 + 3P_3)^2$$

$$a_2^{stat} = -\frac{2}{\sqrt{5}} V_0 V_2 + \frac{4}{15}(M_1^2 - P_1^2) + \frac{2}{5}\sqrt{3}\, M_1 A_2 -$$

$$- \frac{2}{5\sqrt{21}}(4 M_1 M_3 + 3 P_1 P_3) - \frac{4}{5\sqrt{7}} A_2 M_3 - \frac{4}{35}(M_3^2 + P_3^2)$$

$$a_2^- = \frac{2}{\sqrt{5}} V_0 \left[\frac{4}{\sqrt{3}}(M_1 - P_1) - 6(A_2 + V_2) + \frac{2}{\sqrt{7}}(4M_3 + 3P_3)\right] - \frac{4}{15}(M_1 - P_1)(M_1 - 3P_1) +$$

$$+ \frac{2}{5\sqrt{3}}\left[2M_1(6A_2 + 5V_2) + P_1(3A_2 + 5V_2)\right] - \frac{2}{5\sqrt{21}}(2M_1 + 3P_1)(4M_3 + 3P_3) -$$

$$- A_2\left[\frac{3}{5}A_2 + \frac{4}{5\sqrt{7}}(4M_3 + P_3)\right] - \frac{4}{105}(4M_3 + 3P_3)^2$$

$$\alpha^- = V_0\left[-V_0 + 2\sqrt{\tfrac{5}{3}}\, P_1 + \frac{2}{\sqrt{5}}(3A_2 + 2V_2)\right) + \frac{1}{15}\left(12M_1^2 + 24 M_1 P_1 - 13 P_1^2\right) -$$

$$- \frac{2}{5\sqrt{3}}\left[6M_1(2A_2 + V_2) + P_1(3A_2 + 4V_2)\right] + \frac{1}{5}(3A_2^2 - V_2^2) +$$

$$+ \frac{2}{5\sqrt{7}}(2A_2 + V_2)(4M_3 + 3P_3) + \frac{1}{105}(4M_3 + 3P_3)^2$$

$$\alpha^+ = V_0\left[-V_0 - 2\sqrt{\tfrac{3}{5}}\, P_1 + \frac{2}{5\sqrt{5}}(3A_2 + 2V_2)\right] + \frac{1}{25}\left[4M_1(M_1 - 2P_1) - 11 P_1^2\right] +$$

$$+ \frac{2\sqrt{3}}{25}\left[2M_1(2A_2 - V_2) - P_1(A_2 - 4V_2)\right] + \frac{1}{25}(3A_2^2 - 5V_2^2) +$$

$$+ \frac{2}{25\sqrt{7}}\left[-4M_3(2A_2 - V_2) + 3P_3(2A_2 - 3V_2)\right] +$$

$$+ \frac{1}{525}(16 M_3^2 + 24 M_3 P_3 - 51 P_3^2)$$

$\boxed{\frac{3}{2} \to \frac{5}{2}}$ n0

$$\Lambda^{stat} = \frac{1}{3}(2M_1^2 + P_1^2) + \frac{1}{10}(3A_2^2 + 2V_2^2) + \frac{1}{21}(4M_3^2 + 3P_3^2)$$

$$\Lambda^- = \frac{1}{3}(M_1 - P_1)\left[M_1 - P_1 - \frac{\sqrt{7}}{5}(3A_2 + 2V_2)\right] + \frac{16\sqrt{2}}{315}(A_2 - V_2)(4M_3 + 3P_3) +$$

$$+ \frac{1}{15}(5A_2^2 + A_2 V_2 + 3V_2^2) + \frac{1}{189}(43 M_3^2 + 4 M_3 P_3 + 27 P_3^2)$$

$$a_2^{stat} = \frac{\sqrt{14}}{15}\left[M_1^2 - P_1^2 + \frac{9}{\sqrt{7}} M_1 A_2 - \frac{3}{7}\sqrt{\tfrac{2}{7}}(4M_1M_3 + 3P_1P_3) -\right.$$

$$\left. - \frac{15}{196}(3A_2^2 + 4V_2^2) - \frac{39}{49}\sqrt{2}\, A_2 M_3 - \frac{33}{98}(M_3^2 + P_3^2)\right]$$

$$a_4^{stat} = \frac{1}{7}\sqrt{\frac{2}{21}}\left[3\sqrt{14}(P_1P_3 - M_1M_3) + \frac{24}{5}(A_2^2 - V_2^2) + \frac{5}{\sqrt{2}} A_2 M_3\right.$$

$$\left. - \frac{1}{3}M_3^2 - \frac{3}{2}P_3^2\right]$$

$$a_2^- = -\frac{\sqrt{14}}{15}\left\{(M_1 - P_1)\left[M_1 - P_1 - \frac{1}{7\sqrt{7}}(10V_2 + 39A_2)\right] +\right.$$

$$+ \frac{\sqrt{2}}{21\sqrt{7}}\left[3M_1(47M_3 + 85P_3) + P_1(235M_3 + 27P_3)\right] +$$

$$+ \frac{1}{294}\left[\frac{3}{2}(263A_2^2 + 20V_2(A_2 + 3V_2)) + \frac{\sqrt{2}}{5}\left(A_2(1403M_3 + 107\ldots\right.\right.$$

$$- 18V_2\left(\frac{343}{9}M_3 + 11P_3\right)\Big) + 23\cdot 31 M_3^2 + 33P_3(14M_3 + 9P_3)\Big]$$

$$a_4^- = \frac{\sqrt{2}}{21\sqrt{21}}\left\{-\frac{48}{5}\sqrt{7}(M_1 - P_1)(A_2 - V_2) + \frac{24}{5}(A_2 - V_2)(4A_2 + 3V_2)\right.$$

$$-\sqrt{14}(M_1 - P_1)(7M_3 + 9P_3) + \frac{3}{5\sqrt{2}}\left[A_2(146M_3 + 141P_3) +\right.$$

$$+ 2V_2(41M_3 + 89P_3)\Big] - \frac{1}{18}\left[111M_3^2 - 9P_3(14M_3 + 9P\ldots\right.$$

$$\alpha^- = \frac{1}{5}(M_1 - P_1)\left[P_1 - M_1 - \sqrt{7}(A_2 - 2V_2) - \frac{8}{3}\sqrt{\tfrac{2}{7}}(4M_3 + 3P_3)\right]\ldots$$

$$+ \frac{1}{140}\left[79A_2^2 - 4V_2(15A_2 + 17V_2)\right] + \frac{4\sqrt{2}}{105}\left[A_2(11M_3 + 9\ldots\right.$$

$$+ 6V_2(M_3 + 2P_3)\Big] + \frac{1}{315}(113M_3^2 - 18M_3P_3 - 63P_3^2)$$

$$5\alpha^+ = \frac{1}{15}(97M_1^2 + 6M_1P_1 - 23P_1^2) + \frac{\sqrt{7}}{25}\left[M_1(11A_2 + 2V_2) +\right.$$

$$+ P_1(A_2 + 6V_2)\Big] - \frac{8}{15}\sqrt{\tfrac{2}{7}}(M_1 - P_1)(4M_3 + 3P_3) +$$

$$+ \frac{1}{140}\left[3A_2(73A_2 - 20V_2) - 110V_2^2\right] +$$

$$- \frac{4\sqrt{2}}{35}\left[2M_3(7A_2 - V_2) - 3P_3(A_2 - 4V_2)\right] +$$

$$+ \frac{1}{105}(71M_3^2 - 6M_3P_3 - 81P_3^2)$$

MUON CAPTURE BY SPIN 5/2 NUCLEI

The formulas presented here described the muon-capture processes by targets with negative μ : ${}^{17}O \rightarrow N$, ${}^{25}Mg \rightarrow Na$, etc. and by targets with positive μ as e.g. ${}^{27}Al \rightarrow Mg$.

$\boxed{\frac{5}{2} \rightarrow \frac{1}{2}}$ no

$$\Lambda^{stat} = \frac{3}{10} A_2^2 + \frac{1}{5} V_2^2 + \frac{1}{21}(4M_3^2 + 3P_3^2)$$

$$\Lambda^- = \frac{1}{25}(11A_2^2 + 14A_2V_2 + 5V_2^2) + \frac{1}{15\sqrt{35}}(A_2 - V_2)(4M_3 + 3P_3) + \\ + \frac{1}{63}(4M_3 + 3P_3)^2$$

$$a_1^{stat} = -\frac{1}{10}A_2^2 - \frac{2}{3\sqrt{35}}(3V_2P_3 + 4A_2M_3) + \frac{4}{63}M_3^2$$

$$15a_1^- = -\frac{3}{5}(5A_2^2 + 2A_2V_2 - V_2^2) - \frac{2}{\sqrt{35}}\left[A_2(28M_3 + 21P_3) + \right. \\ \left. + V_2(20M_3 + 9P_3)\right] + \frac{1}{21}(4M_3 + 3P_3)^2$$

$$\alpha^- = 3\left[\frac{1}{5}(2A_2 + V_2) + \frac{1}{3\sqrt{35}}(4M_3 + 3P_3)\right]^2$$

$$7\alpha^+ = -\left[\sqrt{\frac{3}{5}}(A_2 - V_2) + \frac{1}{\sqrt{21}}(2M_3 - 3P_3)\right]^2$$

$\boxed{\frac{5}{2} \rightarrow \frac{3}{2}}$ no

$$\Lambda^{stat} = \frac{1}{3}(2M_1^2 + P_1^2) + \frac{1}{10}(3A_2^2 + 2V_2^2) + \frac{1}{21}(4M_3^2 + 3P_3^2) + \ldots$$

$$15\Lambda^- = 17M_1^2 + 14M_1P_1 + 5P_1^2 - \frac{3}{5}\sqrt{7}(M_1 - P_1)(3A_2 + 2V_2) + \\ + \frac{3}{5}\left(\frac{41}{4}A_2^2 + 11A_2V_2 + 5V_2^2\right) + \frac{16}{35}\sqrt{2}(A_2 - V_2)(4M_3 + 3P_3) + \\ + \frac{11}{3}M_3^2 + \frac{34}{7}M_3P_3 + \frac{15}{7}P_3^2 + \ldots$$

$$a_2^{stat} = \frac{1}{15}(M_1^2 - P_1^2) - \frac{\sqrt{7}}{5}M_1A_2 - \frac{4}{15}\sqrt{\frac{2}{7}}(4M_1M_3 + 3P_1P_3) + \\ + \frac{3}{84}(3A_2^2 + 2V_2^2) - \frac{4\sqrt{2}}{35}A_2M_3 + \frac{1}{35}(M_3^2 + P_3^2) + \ldots$$

$$5a_2^- = \frac{1}{15}(19M_1^2 - 14M_1P_1 - 5P_1^2) + \frac{1}{25\sqrt{7}}\left[M_1(27A_2 + 34V_2) + \right. \\ \left. + P_1(69A_2 - 62V_2)\right] + \frac{4}{3}\sqrt{\frac{2}{7}}\left[M_1(5M_3 + 3P_3) + 3P_1(M_3 + P_3)\right] \\ + \frac{1}{35}\left(\frac{11}{2}A_2 + 5V_2\right)^2 - \frac{2\sqrt{2}}{105}(11A_2 + 10V_2)(M_3 + 3P_3) - \\ - \frac{1}{7}\left(\frac{11}{9}M_3^2 + \frac{10}{3}M_3P_3 - P_3^2\right) + \ldots$$

$$25\alpha^- = 61M_1^2 + 28M_1P_1 + P_1^2 - 3\sqrt{7}\left[M_1(7A_2 + 4V_2) + 2P_1(A_2 + V_2 \right. \\ - 4\sqrt{\frac{2}{7}}(M_1 - P_1)(4M_3 + 3P_3) + \frac{3}{7}(51A_2^2 + 10A_2V_2 - 5V_2^2) + \\ + \frac{2}{7}\sqrt{2}\left[A_2(58M_3 + 39P_3) + 2V_2(13M_3 + 12P_3) + \frac{17}{3}M_3^2 + \right. \\ + \frac{1}{7}P_3(22M_3 - 3P_3) + \ldots$$

$$5\alpha^+ = -(M_1 - P_1)^2 + \frac{3}{\sqrt{7}}(M_1 - P_1)(A_2 - 2V_2) - \frac{2}{7}\sqrt{\frac{2}{7}}(M_1 - P_1)(4M_3 + 3P_3 \\ - \frac{3}{49}\left(\frac{33}{2}A_2^2 - 5A_2V_2 + 13V_2^2\right) - \frac{\sqrt{2}}{49}\left[A_2(38M_3 - 39P_3) + \right. \\ \left. + 2V_2(24P_3 - 13M_3)\right] + \frac{1}{49}\left(\frac{22}{3}M_3^2 + 11M_3P_3 - 24P_3^2\right) + \ldots$$

$\boxed{\frac{5}{2} \rightarrow \frac{5}{2}}$ no

$$\Lambda^{stat} = V_0^2 + \frac{1}{3}(2M_1^2 + P_1^2) + \frac{1}{10}(3A_2^2 + 2V_2^2) + \frac{1}{21}(4M_3^2 + 3P_3^2) + \ldots$$

$$a_2^{stat} = -\frac{\sqrt{2}}{15\sqrt{7}}\left\{3\sqrt{\frac{14}{5}}V_0V_2 + 8(M_1^2 - P_1^2) - 9\sqrt{2}M_1A_2 - \frac{15}{28}(3A_2^2 + 2V_2^2 \right. \\ \left. + \frac{27}{7}\sqrt{3}A_2M_3 - \frac{1}{7}(M_3^2 + P_3^2)\right\} + \ldots$$

$$a_4^{stat} = \frac{1}{7\sqrt{42}}\left\{16\sqrt{\frac{2}{3}}(M_1M_3 - P_1P_3) - \frac{27}{5}(A_2^2 - V_2^2) - 10\sqrt{3}A_2M_3 - \right. \\ \left. - \frac{2}{3}M_3^2 - 3P_3^2\right\} + \ldots$$

$$a_2^- = \frac{\sqrt{2}}{5\sqrt{7}}\Big\{\frac{16}{3}\sqrt{\frac{7}{5}}V_0(M_1-P_1)+\sqrt{\frac{14}{5}}V_0(3A_2-5V_2)+3\sqrt{\frac{6}{35}}V_0(4M_3+3P_3)-$$
$$-\frac{8}{15}(9M_1^2-4M_1P_1-5P_1^2)+\frac{\sqrt{2}}{105}\big[2M_1(159A_2+205V_2)+$$
$$+P_1(123A_2+325V_2)\big]+\frac{6\sqrt{6}}{35}\big[M_1(2M_3-3P_3)-6P_1M_3\big]-$$
$$-\frac{1}{35}(42A_2^2+60A_2V_2-25V_2^2)-\frac{6\sqrt{3}}{35}\big[A_2(4M_3+7P_3)+5V_2P_3\big]+$$
$$+\frac{1}{105}(18M_3^2+196M_3P_3+5P_3^2)\Big\}+\ldots$$

$$a_4^- = \frac{3}{35}\sqrt{\frac{3}{14}}\Big\{-\frac{8}{9}\sqrt{\frac{70}{3}}V_0(2M_3-P_3)+\frac{16}{5}\sqrt{2}(M_1-P_1)(A_2-V_2)+$$
$$+\frac{16}{9}\sqrt{\frac{2}{3}}\big[M_1(11M_3-2P_3)-P_1(4M_3+5P_3)\big]-$$
$$-\frac{3}{5}(A_2-V_2)(11A_2+5V_2)-\frac{2}{45\sqrt{3}}\big[3A_2(4\cdot 47M_3+91P_3)+$$
$$+V_2(4\cdot 49M_3+47P_3)\big]-8M_3^2-8M_3P_3-5P_3^2\Big\}+\ldots$$

$$\alpha^- = -V_0^2+2\sqrt{\frac{7}{5}}V_0P_1+\frac{2}{5}\sqrt{\frac{14}{5}}V_0(3A_2+2V_2)+\frac{1}{25}(16M_1^2-32M_1P_1-$$
$$-19P_1^2)-\frac{2\sqrt{2}}{25}\big[6M_1(3A_2+V_2)+P_1(3A_2+8V_2)\big]+$$
$$+\frac{3}{35}\Big(\frac{61}{10}A_2^2-A_2V_2-3V_2^2\Big)+\frac{18\sqrt{3}}{175}\big[3A_2(2M_3+P_3)+V_2(2M_3+3P_3)\big]+$$
$$+\frac{1}{735}(164M_3^2-4M_3P_3-79P_3^2)+\ldots$$

$$\alpha^+ = -V_0^2-2\sqrt{\frac{5}{7}}V_0P_1+\sqrt{\frac{2}{35}}V_0(3A_2+2V_2)+\frac{1}{35}(8M_1^2-16M_1P_1-$$
$$-17P_1^2)+\frac{\sqrt{2}}{35}\big[6M_1(3A_2-V_2)-P_1(3A_2-16V_2)\big]+$$
$$+\frac{3}{490}\Big(\frac{61}{2}A_2^2-5A_2V_2-96V_2^2\Big)-$$
$$-\frac{9\sqrt{3}}{245}\big[3A_2(2M_3-P_3)-2V_2(M_3-3P_3)\big]+$$
$$+\frac{1}{735}(82M_3^2-2M_3P_3-105P_3^2)+\ldots$$

THE MUON CAPTURE REACTIONS $^{10}B(3^+) \longrightarrow Be$

This target has the positive magnetic moment.*

$\boxed{3 \to 0}$ no

$$\Lambda^{stat} = \frac{1}{21}(4M_3^2+3P_3^2)$$
$$7\Lambda^- = \Big(\frac{4}{3}M_3+P_3\Big)^2$$
$$\alpha^- = -\frac{3}{5}\Lambda^-, \quad \alpha^+ = -\frac{1}{21}(M_3-P_3)^2$$

$\boxed{3 \to 1}$ no

$$\Lambda^{stat} = \frac{1}{10}(3A_2^2+2V_2^2)+\frac{1}{21}(4M_3^2+3P_3^2)+\ldots$$
$$9\Lambda^- = \frac{3}{10}(13A_2^2+16A_2V_2+4A_2^2)+\frac{4}{7\sqrt{5}}(A_2-V_2)(4M_3+3P_3)+$$
$$+\frac{1}{7}\Big(\frac{47}{3}M_3^2+22M_3P_3+9P_3^2\Big)+\ldots$$
$$a_2^{stat} = \frac{1}{7\sqrt{2}}\Big\{-\frac{1}{10}(3A_2^2+4V_2^2)+\frac{4}{\sqrt{5}}A_2M_3+M_3^2+P_3^2\Big\}+\ldots$$
$$a_2^- = \frac{1}{63\sqrt{2}}\Big\{-\frac{3}{10}(5A_2^2+32A_2V_2+12V_2^2)+\frac{4}{\sqrt{5}}\big[A_2(16M_3+3P_3)$$
$$+V_2(5M_3-3P_3)\big]+\frac{35}{3}M_3^2+22M_3P_3+9P_3^2\Big\}+\ldots$$
$$105\alpha^- = \frac{3}{5}\Big(\frac{173}{2}A_2^2+72A_2V_2+13V_2^2\Big)+\frac{4}{\sqrt{5}}\big[A_2(38M_3+27P$$
$$+V_2(18M_3+15P_3)\big]+\frac{61}{3}M_3^2+18M_3P_3+3P_3^2+\ldots$$
$$21\alpha^+ = -\frac{9}{5}(A_2-V_2)^2-\frac{3}{\sqrt{5}}(A_2-V_2)(2M_3-3P_3)-$$
$$-\frac{1}{4}(M_3^2-6M_3P_3+6P_3^2)+\ldots$$

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* These formulas were derived in collaboration with Drs. N.P. Popov and A.P. Bukhvostov.

$$\boxed{3 \to 2} \quad \text{no}$$

$$\Lambda^{stat} = \frac{1}{3}(2M_1^2 + P_1^2) + \frac{1}{10}(3A_2^2 + 2V_2^2) + \frac{1}{21}(4M_3^2 + 3P_3^2) + \dots$$

$$3\Lambda^- = \frac{10}{3}M_1^2 + \frac{8}{3}M_1P_1 + P_1^2 - \frac{4}{15}\sqrt{6}(M_1 - P_1)\left(\frac{3}{2}A_2 + V_2\right) + \frac{6}{5}A_2^2 +$$
$$+ \frac{6}{5}A_2V_2 + \frac{3}{5}V_2^2 + \frac{6}{35}\sqrt{6}(A_2 - V_2)\left(\frac{4}{3}M_3 + P_3\right) + \frac{5}{7}M_3^2 + \frac{6}{7}M_3P_3 +$$
$$+ \frac{3}{7}P_3^2 - \frac{10}{21\sqrt{3}}(M_3 - P_3)\left(\frac{5}{4}A_4 + V_4\right) + \frac{1}{9}\left(\frac{73}{16}A_4^2 + \frac{13}{2}A_4V_4 + 3V_4^2\right) +$$
$$+ \frac{2}{3}\sqrt{\frac{2}{33}}(A_4 - V_4)\left(\frac{6}{5}M_5 + P_5\right) + \frac{3}{11}\left(\frac{6}{5}M_5 + P_5\right)^2$$

$$a_2^{stat} = + \frac{\sqrt{2}}{3\sqrt{35}}\Big\{M_1^2 - P_1^2 - 3\sqrt{6}M_1A_2 - \frac{12}{7}(4M_1M_3 + 3P_1P_3) +$$
$$+ \frac{3}{7}(3A_2^2 + 4V_2^2) - \frac{3}{7}\sqrt{6}A_2M_3 + \frac{11}{14}(M_3^2 + P_3^2)\Big\} + \dots$$

$$a_4^{stat} = \frac{2}{21}\sqrt{\frac{2}{7}}\Big\{-M_1M_3 + P_1P_3 + \frac{9}{20}(A_2^2 - V_2^2) + \frac{5}{2}\sqrt{\frac{3}{2}}A_2M_3 + \frac{1}{3}M_3^2 +$$
$$+ \frac{3}{2}P_3^2\Big\} + \dots$$

$$a_2^- = \frac{\sqrt{2}}{3\sqrt{35}}\Big\{\frac{1}{3}(11M_1^2 - 8M_1P_1 - 3P_1^2) - \frac{\sqrt{2}}{7\sqrt{3}}\big[16V_2(M_1 - P_1) +$$
$$+ 3A_2(39M_1 + 10P_1)\big] + \frac{3}{7}\left[\frac{11}{2}A_2^2 + 4V_2(2A_2 + V_2)\right] -$$
$$- \frac{4}{7}\big[9P_1(M_3 + P_3) + M_1(19M_3 + 12P_3)\big] + \frac{\sqrt{6}}{7}\big[M_3(3A_2 - V_2) -$$
$$- P_3(5A_2 + 7V_2)\big] + \frac{1}{2}M_3^2 + \frac{11}{7}P_3\left(M_3 + \frac{1}{2}P_3\right)\Big\} + \dots$$

$$a_4^- = \frac{2}{21}\sqrt{\frac{2}{7}}\Big\{-\frac{\sqrt{6}}{5}(M_1 - P_1)(A_2 - V_2) + \frac{1}{3}M_1(-10M_3 + 4P_3) + P_1(M_3 + P_3) +$$
$$+ \frac{9}{20}(3A_2^2 - 2A_2V_2 - V_2^2) + \frac{\sqrt{3}}{5\sqrt{2}}\Big[M_3\left(\frac{23}{3}V_2 + 19A_2\right) +$$
$$+ 2P_3\left(4A_2 - \frac{7}{3}V_2\right)\Big] + 3M_3P_3 + \frac{3}{2}P_3^2\Big\} + \dots$$

HELICITY AMPLITUDES

Here we give the explicit formulas for the helicity amplitudes in terms of the multipole amplitudes calculated from (24) and (28) for low values of the nuclear spins $J_i \to J_f$ for the case of left handed neutrino emission $(h = -1/2)$.

To save the space we introduced x_I and y_I defined as follows.

I	$\pi_i\pi_f$	x_I	y_I
even	no	$-i^{-I}A_I$	$-i^{-I}V_I$
	yes	$i^{1-I}M_I$	$i^{1-I}P_I$
odd	no	$i^{1-I}M_I$	$i^{1-I}P_I$
	yes	$i^{-I}A_I$	$i^{-I}V_I$

$$\boxed{0 \to I}$$

$$T_0 = \sqrt{\frac{2}{3}}\,y_I\ , \quad T_{-1} = \sqrt{\frac{2}{3}}\sqrt{\frac{I+1}{I}}\,x_I\ , \quad T_{-1}(I=0) \equiv 0$$

$$\boxed{I \to 0}$$

$$T_0^+ = (-)^{I+1}\frac{1}{2I+1}\sqrt{\frac{2}{3}}(x_I - y_I)$$

$$T_0^- = \frac{(-)^I}{2I+1}\sqrt{\frac{2}{3}}\left(\frac{I+1}{I}x_I + y_I\right)\ ; \quad T_0^-(I=0) \equiv 0$$

$$\boxed{\frac{1}{2} \to J}$$

$$T_{-1/2}^+ = \frac{1}{3}\sqrt{\frac{2J+1}{2J}}\left\{x_{J-\frac{1}{2}} + y_{J-\frac{1}{2}} - \sqrt{\frac{J}{J+1}}\left(\frac{2J+3}{2J+1}x_{J+1/2} - y_{J+1/2}\right)\right\}$$

$$T^{+}_{+1/2}=\frac{1}{3}\sqrt{\frac{2J-1}{J}}\left(y_{J-1/2}-\sqrt{\frac{J}{J+1}}\,y_{J+1/2}\right)$$

$$T^{+}_{-3/2}=\frac{1}{3}\left[\frac{(2J+1)(2J+3)}{J(2J-1)}\right]^{1/2}\left(x_{J-1/2}+\frac{2J-1}{2J+1}\sqrt{\frac{J}{J+1}}\,x_{J+1/2}\right);\ T^{+}_{-3/2}(J=\tfrac{1}{2})\equiv 0$$

$$T^{-}_{-1/2}=-\left(\frac{2J+1}{6J}\right)^{1/2}\left\{x_{J-1/2}-y_{J-1/2}-\sqrt{\frac{J}{J+1}}\left(\frac{2J+3}{2J+1}\,x_{J+1/2}+y_{J+1/2}\right)\right\}$$

$\boxed{J\rightarrow 1/2}$

$$T^{+}_{-1/2}=\frac{(-)^{J+1/2}}{\sqrt{6J(J+1)}}\left\{x_{J-1/2}-y_{J-1/2}+\sqrt{\frac{J}{J+1}}\left(\frac{2J+3}{2J+1}\,x_{J+1/2}-y_{J+1/2}\right)\right\}$$

$$T^{+}_{1/2}=\frac{(-)^{J+1/2}}{\sqrt{6J(J+1)}}\left(\frac{2J+3}{2J+1}\right)^{1/2}\left\{x_{J-1/2}-y_{J-1/2}-\sqrt{\frac{J}{J+1}}\left(\frac{2J-1}{2J+1}\,x_{J+1/2}-y_{J+1/2}\right)\right\}$$

$$T^{-}_{-1/2}=\frac{(-)^{J-1/2}}{J\sqrt{6}}\left\{x_{J-1/2}+y_{J-1/2}+\sqrt{\frac{J}{J+1}}\left(\frac{2J+3}{2J+1}\,x_{J+1/2}+y_{J+1/2}\right)\right\}$$

$$T^{-}_{+1/2}=\frac{(-)^{J-1/2}}{J\sqrt{6}}\left(\frac{2J-1}{2J+1}\right)^{1/2}\left\{\frac{2J+3}{2J-1}\,x_{J-1/2}+y_{J-1/2}-\sqrt{\frac{J}{J+1}}\left(\frac{2J+3}{2J+1}\,x_{J+1/2}+y_{J+1/2}\right)\right\}$$

Note: $T^{-}_{1/2}(J=\tfrac{1}{2})\equiv 0$

$\boxed{1\rightarrow 1}$

$$T^{+}_{0}=\frac{1}{3}\left(\sqrt{2}\,y_0-x_1-\frac{2}{\sqrt{5}}\,y_2+\frac{3}{2\sqrt{5}}\,x_2\right)$$

$$T^{+}_{-1}=\frac{1}{3\sqrt{2}}\left(\sqrt{2}\,y_0+y_1-2x_1-\frac{3}{\sqrt{5}}\,x_2+\frac{1}{\sqrt{5}}\,y_2\right)$$

$$T^{+}_{1}=\sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}\,y_0-\frac{1}{2}\,y_1+\frac{1}{2\sqrt{5}}\,y_2\right)$$

$$T^{-}_{0}=\frac{2}{3}\left(\frac{1}{\sqrt{2}}\,y_0+x_1-\frac{1}{\sqrt{5}}\,y_2-\frac{3}{2\sqrt{5}}\,x_2\right)$$

$$T^{-}_{-1}=\frac{\sqrt{2}}{3}\left(\sqrt{2}\,y_0+x_1+y_1+\frac{1}{\sqrt{5}}\,y_2+\frac{3}{2\sqrt{5}}\,x_2\right)$$

$\boxed{1\rightarrow 2}$

$$T^{+}_{-2}=\sqrt{\frac{2}{3}}\left(x_1+\frac{1}{2}x_2+\frac{1}{3}\sqrt{\frac{2}{7}}\,x_3\right)$$

$$T^{+}_{-1}=\frac{1}{3\sqrt{2}}\left(y_1+2x_1+y_2-x_2+\sqrt{\frac{2}{7}}\,y_3-\frac{8}{3}\sqrt{\frac{2}{7}}\,x_3\right)$$

$$T^{+}_{0}=\frac{1}{3\sqrt{3}}\left(2y_1+x_1-\frac{3}{2}x_2-3\sqrt{\frac{2}{7}}\,y_3+2\sqrt{\frac{2}{7}}\,x_3\right)$$

$$T^{+}_{1}=\frac{1}{\sqrt{6}}\left(y_1-y_2+\sqrt{\frac{2}{7}}\,y_3\right)$$

$$T^{-}_{-1}=\frac{\sqrt{2}}{3}\left(y_1-x_1+y_2+\frac{1}{2}x_2+\sqrt{\frac{2}{7}}\,y_3+\frac{4}{3}\sqrt{\frac{2}{7}}\,x_3\right)$$

$$T^{-}_{0}=\frac{2}{3\sqrt{3}}\left(y_1-x_1+\frac{3}{2}x_2-\frac{3}{2}\sqrt{\frac{2}{7}}\,y_3-2\sqrt{\frac{2}{7}}\,x_3\right)$$

$\boxed{\frac{3}{2}\rightarrow\frac{3}{2}}$

$$T^{+}_{-3/2}=\frac{1}{5\sqrt{3}}\left\{\sqrt{5}\,y_0+\sqrt{3}\,(y_1-2x_1)+y_2-3x_2+\frac{1}{\sqrt{7}}(y_3-4x_3)\right\}$$

$$T^{+}_{-1/2}=\frac{\sqrt{2}}{5\sqrt{3}}\left\{\sqrt{5}\,y_0+\frac{1}{\sqrt{3}}(y_1-4x_1)-y_2-\frac{1}{\sqrt{7}}(3y_3-4x_3)\right\}$$

$$T^{+}_{1/2}=\frac{1}{5}\left\{\sqrt{5}\,y_0-\frac{1}{\sqrt{3}}(y_1+2x_1)-y_2+x_2+\frac{1}{\sqrt{7}}\left(3y_3-\frac{4}{3}x_3\right)\right\}$$

$$T^{+}_{3/2}=\frac{2}{5\sqrt{3}}\left\{\sqrt{5}\,y_0-\sqrt{3}\,y_1+y_2-\frac{1}{\sqrt{7}}\,y_3\right\}$$

$$T^{-}_{-3/2}=\frac{1}{\sqrt{15}}\left\{\sqrt{5}\,y_0+\sqrt{3}\left(y_1+\frac{2}{3}x_1\right)+y_2+x_2+\frac{1}{\sqrt{7}}\left(y_3+\frac{4}{3}x_3\right)\right\}$$

$$T^{-}_{-1/2}=\frac{\sqrt{2}}{3\sqrt{5}}\left\{\sqrt{5}\,y_0+\frac{1}{\sqrt{3}}(y_1+4x_1)-y_2-\frac{1}{\sqrt{7}}(3y_3+4x_3)\right\}$$

$$T^{-}_{1/2}=\frac{1}{3\sqrt{5}}\left\{\sqrt{5}\,y_0-\frac{1}{\sqrt{3}}(y_1-6x_1)-y_2-3x_2+\frac{1}{\sqrt{7}}(3y_3+4x_3)\right\}$$

$\boxed{\frac{3}{2}\rightarrow\frac{5}{2}}$

$$T^{+}_{-5/2}=\sqrt{\frac{2}{15}}\left(2x_1-\frac{3}{\sqrt{7}}\,x_2+\sqrt{\frac{2}{7}}\,x_3+\frac{1}{2}\sqrt{\frac{5}{42}}\,x_4\right)$$

$$T^{+}_{-3/2}=\frac{\sqrt{6}}{5}\left\{\frac{1}{3}y_1+x_1+\frac{1}{\sqrt{7}}\left(y_2-\frac{1}{2}x_2\right)+\frac{1}{\sqrt{14}}\left(y_3-\frac{7}{3}x_3\right)+\sqrt{\frac{5}{42}}\left(\frac{1}{3}y_4-\frac{5}{4}x\right.\right.$$

$$T^{+}_{-1/2}=\frac{\sqrt{2}}{5}\left\{x_1+y_1+\frac{1}{\sqrt{7}}\left(y_2-\frac{5}{2}x_2\right)-\sqrt{\frac{2}{7}}\left(y_3+\frac{1}{3}x_3\right)-\sqrt{\frac{10}{21}}\left(y_4-\frac{5}{4}x_4\right)\right.$$

$$T^{+}_{1/2}=2\sqrt{3}\left\{y_1+\frac{1}{3}x_1-\frac{1}{\sqrt{7}}\left(y_2+\frac{3}{2}x_2\right)-\sqrt{\frac{2}{7}}(y_3-x_3)+\sqrt{\frac{10}{21}}\left(y_4-\frac{5}{12}x_4\right)\right\}$$

$$T^{+}_{3/2}=\frac{2}{5}\sqrt{\frac{2}{3}}\left\{y_1-\frac{3}{\sqrt{7}}\,y_2+\frac{3}{\sqrt{14}}\,y_3-\sqrt{\frac{5}{42}}\,y_4\right\}$$

$$T^{-}_{-3/2}=\sqrt{\frac{2}{15}}\left\{y_1-x_1+\frac{1}{\sqrt{7}}\left(3y_2+\frac{1}{2}x_2\right)+\frac{1}{\sqrt{14}}\left(3y_3+\frac{7}{3}x_3\right)+\sqrt{\frac{5}{42}}\left(y_4+\frac{5}{4}x_4\right.\right.$$

$$T^{-}_{-1/2}=\sqrt{\frac{2}{15}}\left\{y_1-x_1+\frac{1}{\sqrt{7}}\left(y_2+\frac{5}{2}x_2\right)-\sqrt{\frac{2}{7}}\left(y_3-\frac{1}{3}x_3\right)-\sqrt{\frac{10}{21}}\left(y_4+\frac{5}{4}x_4\right)\right\}$$

$$T^{-}_{1/2}=2\sqrt{\frac{5}{3}}\left\{y_1-x_1-\frac{1}{\sqrt{7}}\left(y_2-\frac{9}{2}x_2\right)-\sqrt{\frac{2}{7}}(y_3+3x_3)+\sqrt{\frac{10}{21}}\left(y_4+\frac{5}{4}x_4\right)\right.$$

THE TABLE OF THE $B_{S1}(J_f \xrightarrow{\gamma} j)$

[defined by formulas (8) and (12)]

$\xrightarrow{\tilde{\gamma}} J_f \xrightarrow{\gamma} j$	$S=1$	$S=2$	$S=3$	$S=4$
$\to 1 \to 0$	$\sqrt{\frac{3}{2}}$	$\frac{1}{\sqrt{2}}$		
$\to 1 \to 1$	$\frac{1}{2}\sqrt{\frac{3}{2}}+\dots$	$-\frac{1}{2\sqrt{2}}+\dots$		
$\to 1 \to 2$	$-\frac{1}{2}\sqrt{\frac{3}{2}}+\dots$	$\frac{1}{10\sqrt{2}}+\dots$		
$\to \frac{3}{2} \to \frac{1}{2}$	$\frac{\sqrt{5}}{2}+\dots$	$\frac{1}{2}+\dots$	$\propto \delta$	
$\to \frac{3}{2} \to \frac{3}{2}$	$\frac{1}{\sqrt{5}}+\dots$	$-\frac{2}{5}+\dots$	$\propto \delta$	
$\to \frac{3}{2} \to \frac{5}{2}$	$-\frac{3}{2\sqrt{5}}+\dots$	$\frac{1}{10}+\dots$	$\propto \delta$	
$\to 2 \to 0$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{70}}$	$-\sqrt{2}$	$-2\sqrt{\frac{2}{7}}$
$\to 2 \to 1$	$\frac{3}{2\sqrt{2}}+\dots$	$\frac{1}{2}\sqrt{\frac{7}{10}}+\dots$	$\propto \delta$	$\propto \delta$
$\to 2 \to 2$	$\frac{1}{2\sqrt{2}}+\dots$	$-\frac{1}{2}\sqrt{\frac{7}{10}}+\dots$	$\propto \delta$	$\propto \delta$
$\to 2 \to 3$	$-\frac{1}{\sqrt{2}}+\dots$	$\frac{1}{\sqrt{70}}+\dots$	$\propto \delta$	$\propto \delta$
$\to \frac{5}{2} \to \frac{1}{2}$	$\sqrt{\frac{7}{15}}+\dots$	$-\sqrt{\frac{2}{7}}+\dots$	$-\sqrt{\frac{6}{5}}+\dots$	$-2\sqrt{\frac{2}{21}}+\dots$
$\to \frac{5}{2} \to \frac{3}{2}$	$\frac{1}{2}\sqrt{\frac{21}{5}}+\dots$	$\frac{1}{5}\sqrt{\frac{7}{2}}+\dots$	$\propto \delta$	$\propto \delta$
$\to \frac{5}{2} \to \frac{5}{2}$	$\sqrt{\frac{3}{35}}+\dots$	$-\frac{4}{5}\sqrt{\frac{2}{7}}+\dots$	$\propto \delta$	$\propto \delta$

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