C 341 a

K-21

ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

41/72
/1

E4 - 6127

6127

3 41 a

V.G. Kartavenko, R.V. Jolos, F. Dönau,
D. Janssen

PAIR CORRELATIONS AND COLLECTIVE 0^+ STATES OF NUCLEI. II.

1971

E4 - 6127

V.G. Kartavenko, R.V.Jolos, F.Dőnau,
D.Janssen

PAIR CORRELATIONS AND COLLECTIVE 0^+ STATES OF NUCLEI. II.

Submitted to ТМФ

In this paper the investigation of the collective modes associated with nuclear pair correlations which has been performed in ref./1/ is continued. The pair correlations in the system of two kinds of nucleons are considered contrary to ref./1/, where pair correlations in the system of only one kind of nucleons are analysed. This is important, for instance, when we consider nuclei with the number of nucleons $A \gtrsim 40.56$. In these nuclei neutrons and protons fill the same single - particle levels. Hence it is necessary to take into account not only the neutron-neutron and proton-proton pair correlations as in heavy nuclei but also the neutron-proton pair correlations /2/.

There is a large amount of experimental data concerning two-nucleon transfer reactions in nuclei with $A \gtrsim 40.56$. It is seen from these data that the matrix elements for the transitions to some $I^{\pi} = 0^{+}$ states are strongly enhanced compared with the transition into a pure $(j^2)J = 0$ configuration of the same nuclei /3,4/. It means that in these nuclei there exist collective modes associated with the pair correlations.

In the present work the collective Hamiltonian describing such modes is constructed. We use the microscopic model with pairing forces only. It may happen that, in principle, we must take into account other forces, too. For instance, the quadrupole forces which are connected with the symmetry term in the average nuclear potential /3/.

But in order to determine the role of such forces we must first of all perform calculations with the pair forces only. It is evident that the pair forces must be taken into account.

Since in light nuclei the isospin T is a good quantum number we write the Hamiltonian in an isotopically invariant form:

$$H = \sum_{j,m>0} (\varepsilon_j - \lambda) N_{jm} - G \sum_{\substack{j,j' \\ m,m'>0}} \sum_{\tau=0,\pm 1} A^+_{jm\tau} A_{j'm'\tau} , \tag{1}$$

where $$N_{jm} = \sum_{\tau=\pm 1/2} (a^+_{jm\tau} a_{jm\tau} + a^+_{j-m\tau} a_{j-m\tau}) ,$$

$$A^+_{jm\tau} = (-)^{j-m} \sum_{\tau_1,\tau_2} C^{1\tau}_{\frac{1}{2}\tau_1 \frac{1}{2}\tau_2} a^+_{jm\tau_1} a^+_{j-m\tau_2}$$

τ is the isospin projection,

$C^{1\tau}_{\frac{1}{2}\tau_1 \frac{1}{2}\tau_2}$ is the Clebsh-Gordon coefficient.

The other notations are the same as in ref. /1/. Let us introduce the operators $T_{jm\tau}$:

$$T_{jm\tau} = -\frac{1}{\sqrt{2}} \sum_{\tau_1,\tau_2} C^{1\tau}_{\frac{1}{2}\tau_1 \frac{1}{2}\tau_2} (-)^{\frac{1}{2}-\tau_2} (a^+_{jm\tau_1} a_{jm-\tau_2} + a^+_{j-m\tau_1} a_{j-m\tau_2})$$

The operator of the nuclear isospin T_τ is connected with these operators in the following way:

$$T_\tau = \sum_{j,m>0} T_{jm\tau} .$$

The Hamiltonian (1) has been considered in refs /5,6,7/. In these papers the attempt has been made to find the solution in the limit of strong pair correlations where there exists an average pair field. Some qualitative results have been obtained.

But a simple and effective method analogous to the $u-v$ Bogolubov transformation has not been found. The properties of the collective excitations were not considered in these papers.

We shall be interested in the nuclear states which can be obtained by adding or removing the pairs of nucleons from some basic nucleus. As a basic nucleus it is convenient to choose the nucleus with closed subshells. Consequently we shall distinguish between particle (j_+) and hole (j_-) states and use the transformation:

$$A^+_{jm\tau} = \begin{cases} A^+_{j_+ m\tau}, & j\in j_+ \\ (-)^{1-\tau} A_{j_- m,-\tau}, & j\in j_- \end{cases}, \quad N_{jm} = \begin{cases} N_{j_+ m}, & j\in j_+ \\ 4-N_{j_- m}, & j\in j_- \end{cases}, \quad T_{jm\tau} = \begin{cases} T_{j_+ m\tau}, & j\in j_+ \\ T_{j_- m\tau}, & j\in j_- \end{cases}$$

It was shown earlier /8,9,1/ that in order to construct the collective Hamiltonian it is convenient to use the boson representations of fermion operators. Different methods of the construction of such representations are known. Some of them which have been suggested in refs /8,10,11/ can be combined in one group. These methods create infinite boson expansions and the boson images of the hermitean conjugate operators remain as before hermitean conjugate operators. Since the boson expansions of fermion operators are infinite series there arises a question about the convergence of these series. In the general case it is impossible to answer this question. As a consequence we can not use these methods for the description of nuclei with sufficiently strong pair correlations. But in the case of pair correlations in the system of one type of nucleons it is possible to get a sum of the series. The sum of the series for binar fermion-operators in the physical subspace of the boson space is the polynomial of the third degree [1]. Thus in this case it is possible to construct the finite Hamiltonian in terms of the boson operators. But in the case of two kinds of fermions all the expressions are so complicated that practically it is impossible to use them.

Another method for construction of the boson expansions has been suggested by Dyson in spin-wave theory [12]. This method has been generalized to the case of arbitrary nucleon interaction in ref. [13]. The method gives us finite boson expansions, consequently there are no questions connected with the convergence of the expansions. But this method does not conserve the properties of the hermitean conjugation. If two fermion operators $\hat{A}$ and $\hat{B}$ are connected by the relation $\hat{A}^{+} = \hat{B}$ then the boson images of these operators $\hat{A}$ and $\hat{B}$ are not connected by this relation $\hat{A}^{+} \neq \hat{B}$ generally. As a result we get a nonhermitian Hamiltonian. However, it has been shown in refs [13,14] that the eigenvalues of the transformed Hamiltonian are real. The Dyson's transformation conserves all the commutation relations unchanged.

In the present work we shall follow refs [12,13] using the generalized Dyson's representation in the case of the operator algebra which here is more complicated than in spin-wave theory:

$$A^{+}_{j_+ m \tau} \to b^{+}_{j_+ m \tau} - 9 \sum_{T, M, \tau'} \begin{Bmatrix} 1/2 & 1/2 & 1 \\ 1/2 & 1/2 & 1 \\ 1 & 1 & T \end{Bmatrix} C^{TM}_{1\tau\, 1\tau'} \left[b^{+}_{j_+ m}\, b^{+}_{j_+ m} \right]_{TM} b_{j_+ m \tau'} =$$

$$= b^+_{j_+ m \tau}\Big(1-\sum_K b^+_{j_+ m K}\, b_{j_+ m K}\Big) + \frac{(-)^{\tilde{\tau}}}{2}\sum_K (-)^K b^+_{j_+ m K}\, b^+_{j_+ m -K}\, b_{j_+ m -\tau};$$

$$A_{j_+ m \tau} \to b_{j_+ m \tau};$$

$$A^+_{j_- m \tau} \to b^+_{j_- m \tau};$$

$$A_{j_- m \tau} \to \Big(1-\sum_K b^+_{j_- m K}\, b_{j_- m K}\Big) b_{j_- m \tau} + \frac{(-)^{\tilde{\tau}}}{2}\, b^+_{j_- m -\tau} \sum_K (-)^K b_{j_- m -K}\, b_{j_- m K};$$

$$N_{j_\pm m} \to 2\sum_{\tau} b^+_{j_\pm m \tau}\, b_{j_\pm m \tau};$$

$$T_{j_\pm m \tau} \to \sqrt{2}\sum_{\tau_1 \tau_2} C^{1\tau}_{1\tau_1 1\tau_2} (-)^{1-\tau_2} \big[b^+_{j_\pm m \tau_1}\, b_{j_\pm m \tau_2}\big].$$

Here $\begin{Bmatrix} 1/2 & 1/2 & 1 \\ 1/2 & 1/2 & 1 \\ 1 & 1 & T \end{Bmatrix}$ is the Wigner coefficient, $\big[b^+_{jm} b^+_{jm}\big]_{TM} = \sum_{\tau,K} C^{TM}_{1\tau 1K}\, b^+_{jm\tau}\, b^+_{jmK}$. It is easy to see that this boson representation conserves all the commutation relations unchanged. In such a representation the Hamiltonian will contain only two - and four-boson terms. This is very convenient from the practical point of view.

Let us determine the collective pair modes of excitation. This problem is discussed in detail in ref. /1/. Unlike ref. /1/, after the transformation to the normal form the two-boson part of the Hamiltonian becomes a nonhermitean operator. Thus it is impossible either to diagonalize this part of the Hamiltonian or put it in the same form as in /1/ by means of the linear unitary transformation. But with the help of the linear nonunitary canonical (i.e. conserving all the commutation relations unchanged) transformation it is possible to diagonalize the two-boson part of the Hamiltonian and determine the operators which describe collective modes:

$$b^{+}_{j_\pm m \tau} = \sum_K \left(v_{K\pm, j_\pm m}\, \beta^{+}_{K\pm,\tau} + (-)^{1-\tau}\, v_{K\mp, j_\pm m}\, \beta_{K\mp,-\tau} \right),$$
$$b_{j_\pm m \tau} = \sum_K \left(\tilde{v}_{K\pm, j_\pm m}\, \beta_{K\pm,\tau} + (-)^{1-\tau}\, \tilde{v}_{K\mp, j_\pm m}\, \beta^{+}_{K\mp,-\tau} \right).$$

The transformation coefficients satisfy the relations which follow from the fact that the transformation is canonical:

$$\sum_{j_\pm m} \left(v_{K\pm, j_\pm m}\, \tilde{v}_{K'\pm, j_\pm m} - v_{K'\pm, j_\mp m}\, \tilde{v}_{K\pm, j_\mp m} \right) = \delta_{K\pm, K'\pm},$$
$$\sum_{j_\pm m} \left(v_{K-, j_- m}\, \tilde{v}_{K+, j_- m} - \tilde{v}_{K-, j_+ m}\, v_{K+, j_+ m} \right) = 0,$$
$$\sum_{j_\pm m} \left(\tilde{v}_{K-, j_- m}\, v_{K+, j_- m} - v_{K-, j_+ m}\, \tilde{v}_{K+, j_+ m} \right) = 0,$$
$$\sum_{K\mp} \left(\tilde{v}_{K-, j_- m}\, \tilde{v}_{K-, j_+ m'} - \tilde{v}_{K+, j_- m}\, \tilde{v}_{K+, j_+ m'} \right) = 0,$$
$$\sum_{K\mp} \left(v_{K-, j_- m}\, v_{K-, j_+ m'} - v_{K+, j_- m}\, v_{K+, j_+ m'} \right) = 0,$$
$$\sum_{K\mp} \left(v_{K\pm, j_\pm m}\, \tilde{v}_{K\pm, j'_\pm m'} - \tilde{v}_{K\mp, j'_\pm m'}\, v_{K\mp, j_\pm m} \right) = \delta_{j_\pm j'_\pm}\, \delta_{m m'}.$$

In the paper /1/ we determine the coefficients v_{Kjm} in such a way so that to put the two-boson part of the Hamiltonian in the form:

$$\sum_K \left(\omega_{K+}\, \beta^{+}_{K+} \beta_{K+} + \omega_{K-}\, \beta^{+}_{K-} \beta_{K-} \right) - g\left(\beta^{+}_{1+} \beta^{+}_{1-} + \beta_{1-} \beta_{1+} \right) \qquad (2)$$

We had done this mainly because it was impossible to diagonalize this part of the Hamiltonian if $G > G_{crit}$. But to determine collective modes it is sufficient to put the two-boson part of the Hamiltonian in the form (2). In our case, due to other choice of the boson representation, it is possible to diagonalize the two-boson part of the Hamiltonian and determine coefficients $U_{\kappa jm}, \tilde{U}_{\kappa jm}$ for an arbitrary G. As a result we get the following expressions:

$$\tilde{U}_{\kappa\pm, j_+ m} = \frac{\tilde{X}_{\kappa\pm}(1-2\rho_{j_+ m})}{D_{j_+ m} \mp \omega_{\kappa\pm}}, \quad \tilde{U}_{\kappa\pm, j_- m} = \frac{X_{\kappa\pm}}{D_{j_- m} \pm \omega_{\kappa\pm}},$$

$$U_{\kappa\pm, j_+ m} = \frac{X_{\kappa\pm}}{D_{j_+ m} \mp \omega_{\kappa\pm}}, \quad U_{\kappa\pm, j_- m} = \frac{\tilde{X}_{\kappa\pm}(1-2\rho_{j_- m})}{D_{j_- m} \pm \omega_{\kappa\pm}}.$$

Here $\rho_{j_+ m}$ $(\rho_{j_- m})$ is the average number of particle (hole) pairs in the state without bosons, $\frac{1}{2} D_{j_\pm m}$ is the module of the renormalized single-particle energy reckoned from the Fermi surface. The renormalizations appear when we put the Hamiltonian in the normal form. For $\rho_{j_\pm m}$ and $\frac{1}{2} D_{j_\pm m}$ we get the following expressions:

$$\rho_{j_\pm m} = \frac{3}{2} \sum_{\kappa\mp} U_{\kappa\mp, j_\pm m} \tilde{U}_{\kappa\mp, j_\pm m},$$

$$\frac{1}{2} D_{j_\pm m} = \left| \varepsilon_{j_\pm} \pm \frac{3}{2} \sum_{\kappa\mp} \tilde{X}_{\kappa\mp} U_{\kappa\mp, j_\pm m} - \lambda \right|.$$

It is seen that the renormalization removes the single-particle levels away from the Fermi surface. The equations for $\omega_{\kappa\pm}$ and $\tilde{X}_{\kappa\pm}$ $X_{\kappa\pm}$ are the following:

$$1 = G\left(\sum_{j_\pm, m} \frac{1-2\rho_{j_\pm m}}{D_{j_\pm m} - \omega_{\kappa\pm}} + \sum_{j_\mp, m} \frac{1-2\rho_{j_\mp m}}{D_{j_\mp m} + \omega_{\kappa\pm}} \right) \quad (3)$$

$$1=\tilde{X}_{k\pm}X_{k t}\left(\sum_{j_\pm m}\frac{1-2\rho_{j\pm m}}{(D_{j_\pm m}-\omega_{k t})^2}-\sum_{j_\mp m}\frac{1-2\rho_{j\mp m}}{(D_{j\mp m}+\omega_{k t})^2}\right). \quad (4)$$

These equations will coincide with the equations of quasiboson approximation /15/ if we put $\rho_{j_\pm m}=0$, $\frac{1}{2}D_{j_\pm m}=|\varepsilon_{j_\pm}-\lambda|$
In quasiboson approximation there is the critical value of G (G_{crit}) such that for $G>G_{crit}$ the solutions $\omega_{1\pm}$ corresponding to the collective modes become purely imaginary. The solutions of eq.(3) corresponding to the collective modes are real for arbitrary values of G due to the increase of $\rho_{j_\pm m}$ and $D_{j_\pm m}$ with increasing G. The dependence of $\omega_{1\pm}$ on G is shown on fig.1. Due to this type of the dependence we can diagonalize the two-boson part of the Hamiltonian at arbitrary values of G. The numerical estimates show that we can neglect the contributions of the noncollective modes to $\rho_{j_\pm m}$ and $D_{j_\pm m}$ with a good accuracy. In this case we get a system of equations for $\omega_{1\pm}$, $\tilde{X}_{1\pm}X_{1\pm}$, $\rho_{j_\pm m}$, $D_{j_\pm m}$ which can be solved numerically. In eqs (3) and (4) λ is an arbitrary quantity. We can fixe in such a way so that to get $\omega_{1+}=\omega_{1-}\equiv\omega$. Then we get the following system of equations for

$$1=\tilde{X}_{1\pm 1\pm}\left(\sum_{j_\pm m}\frac{1-2\rho_{j_\pm m}}{(D_{j_\pm m}-\omega)^2}-\sum_{j_\mp m}\frac{1-2\rho_{j_\mp m}}{(D_{j_\mp m}+\omega)^2}\right),$$

$$1=G\sum_{j m}\left(\frac{(1-2\rho_{jm})D_{jm}}{D_{jm}^2-\omega^2}\right), \quad (5)$$

$$\sum_{j_+ m}\frac{1-2\rho_{j_+ m}}{D_{j_+ m}^2-\omega^2}=\sum_{j_- m}\frac{1-2\rho_{j_- m}}{D_{j_- m}^2-\omega^2},$$

$$D_{j_\pm m} = 2|\varepsilon_{j_\pm} - \lambda| + \frac{1}{2}\left(\sqrt{(2|\varepsilon_{j_\pm} - \lambda| + \omega)^2 + 12\tilde{X}_{1\mp}X_{1\mp}} - 2(|\varepsilon_{j_\pm} - \lambda| - \omega)\right)$$

$$1 - 2\rho_{j_\pm m} = \frac{1}{2 - \dfrac{2|\varepsilon_{j_\pm} - \lambda| + \omega}{D_{j_\pm m} + \omega}} .$$

This system of equations can be solved by iterations. After the coefficients $\mathcal{V}_{k\pm, jm}$ and $\tilde{\mathcal{V}}_{1\pm, jm}$ have been found we can determine $\mathcal{V}_{k\pm, jm}$ and $\tilde{\mathcal{V}}_{k\pm, jm}$ for $k \neq 1$. Then we can calculate the constants for the anharmonic terms in H which characterize the interaction of bosons of different types. The results show that the interaction of the collective modes with noncollective ones is much weaker than that between the collective modes themselves. For these reasons we shall consider only the collective part of the Hamiltonian :

$$\begin{aligned}
H_{coll} = {} & \omega \sum_k \left(\beta^+_{1+,k}\beta_{1+,k} + \beta^+_{1-,k}\beta_{1-,k}\right) + \\
& + G(1)\left(\sqrt{5}\left[[\beta^+_{1+}\beta^+_{1+}]_2[\beta^+_{1-}\beta^+_{1-}]_2\right]_0 - \tfrac{1}{2}[\beta^+_{1+}\beta^+_{1+}]_0[\beta^+_{1-}\beta^+_{1-}]_0\right) + \\
& + G(2)\left(\sqrt{5}\left[[\beta_{1-}\beta_{1-}]_2[\beta^+_{1+}\beta^+_{1+}]\right]_0 - \tfrac{1}{2}[\beta_{1-}\beta_{1-}]_0[\beta_{1+}\beta_{1+}]_0\right) + \\
& + G(3):\left(\tfrac{3}{2}[\beta^+_{1+}\beta_{1+}]_0[\beta^+_{1+}\beta_{1+}]_0 - \sqrt{3}\left[[\beta^+_{1+}\beta_{1+}]_1[\beta^+_{1+}\beta_{1+}]_1\right]_0\right): + \\
& + G(4):\left(\tfrac{3}{2}[\beta^+_{1-}\beta_{1-}]_0[\beta^+_{1-}\beta_{1-}]_0 - \sqrt{3}\left[[\beta^+_{1-}\beta_{1-}]_1[\beta^+_{1-}\beta_{1-}]_1\right]_0\right): + \\
& + G(5)\left(\tfrac{3}{2}[\beta^+_{1+}\beta_{1+}]_0[\beta^+_{1-}\beta_{1-}]_0 - \sqrt{3}\left[[\beta^+_{1+}\beta_{1+}]_1[\beta^+_{1-}\beta_{1-}]_1\right]_0\right) + \\
& + G(6)\left(\tfrac{3}{2}[\beta^+_{1+}\beta^+_{1-}]_0[\beta^+_{1+}\beta_{1+}]_0 - \sqrt{3}\left[[\beta^+_{1+}\beta^+_{1-}]_1[\beta^+_{1+}\beta_{1+}]_1\right]_0\right) + \\
& + G(7)\left(\tfrac{3}{2}[\beta^+_{1+}\beta_{1+}]_0[\beta_{1-}\beta_{1+}]_0 - \sqrt{3}\left[[\beta^+_{1+}\beta_{1+}]_1[\beta_{1-}\beta_{1+}]_1\right]_0\right) + \\
& + G(8)\left(\tfrac{3}{2}[\beta^+_{1-}\beta_{1-}]_0[\beta_{1+}\beta_{1-}]_0 - \sqrt{3}\left[[\beta^+_{1-}\beta_{1-}]_1[\beta_{1+}\beta_{1-}]_1\right]_0\right) + \\
& + G(9)\left(\tfrac{3}{2}[\beta^+_{1-}\beta^+_{1+}]_0[\beta^+_{1-}\beta_{1-}]_0 - \sqrt{3}\left[[\beta^+_{1-}\beta^+_{1+}]_1[\beta^+_{1-}\beta_{1-}]_1\right]_0\right),
\end{aligned} \tag{6}$$

where the constants $G(n)$ are expressed through the coefficients $\mathcal{V}_{1\pm,jm}$, $\tilde{\mathcal{V}}_{1\pm,jm}$ and the sign: : means normal product. Moreover $[\beta^+\beta^+]_{IM} = \sum_{\tau K} C^{IM}_{1\tau 1K}\beta^+_\tau\beta^+_K$ and $[\beta^+\beta]_{IM} = \sum_{\tau K} C^{IM}_{1\tau 1K}(-)^{1-K}\beta^+_\tau\beta_{-K}$

If we conserve only the collective bosons then the operators of the particle number and the isotopic spin take the form:

$$N = 2\sum_K (\beta^+_{1+,K}\beta_{1+,K} - \beta^+_{1-,K}\beta_{1-,K}),\quad T_\mu = \sqrt{2}\left([\beta^+_{1+}\beta_{1+}]_{1\mu} + [\beta^+_{1-}\beta_{1-}]_{1\mu}\right).$$

In the case of the weak anharmonicity the bosons $\beta^+_{1\pm,\tau}$ give us a good basis for the construction of the solution. But in the general case the eigenvalues and the eigenfunctions of the Hamiltonian (6) can be found numerically. In order to analyse the situation qualitatively in the case of strong pair correlations it is more convenient to consider the Hamiltonian in some other representation. Since the deviations $[G(1)-G(2)]$ and $[G(3) - G(4)]$ from zero are due to the asymmetry of the single-particle scheme with respect to the Fermi surface, and the asymmetry effect is small in the case of strong pair correlations we can put $G(1) \simeq G(2)$ and $G(3) \simeq G(4)$ in this case.

Let us introduce the operators $q_\tau = \frac{1}{\sqrt{2}}\left(\beta_{1+,\tau} + (-)^{1-\tau}\beta^+_{1-,-\tau}\right)$ and $p_\tau = \frac{i}{\sqrt{2}}\left(\beta^+_{1+,\tau} - (-)^{1-\tau}\beta_{1-,-\tau}\right)$, q^+_τ, p^+_τ which satisfy the following commutation relations:

$$[q_\tau, p_K] = i\delta_{\tau K},\quad [q_\tau, q_K] = [q_\tau, q^+_K] = [p_\tau, p_K] = [q_\tau, p^+_K] = 0.$$

In this representation the operators of the particle number and isotopic spin take the form:

$$N = 2i \sum_K (q_K^+ p_K^+ - q_K p_K),$$

$$T_\mu = i\sqrt{2} \sum_{\tau, K} C^{1\tau}_{1K\,1\mu} (q_\tau^+ p_K^+ - q_K p_\tau).$$

For the Hamiltonian we get the following expression:

$$H_{coll} = M_0 [p^+p]_0 - C[q^+q]_0 + A\left([q^+q]_0[q^+q]_0 - \frac{1}{2}[q^+q^+]_0[qq]_0\right) + \\ + B\left([q^+q]_0[p^+p]_0 + h.c.\right) - \frac{B}{2}\Big(2[q^+p]_0[p^+q]_0 + 2[p^+q]_0[q^+p]_0 \\ - [qq]_0[pp]_0 - [pp]_0[qq]_0 + N[q^+q]_0[p^+p]_0 - N[p^+p]_0[q^+q]_0 - \frac{N^2}{2}\Big). \qquad (7)$$

Here $[q^+q]_0 = \sum_K q_K^+ q_K$, $[q^+q^+]_0 = \sum_K (-)^K q_K^+ q_{-K}^+$; $[p^+p]_0 = \sum_K p_K^+ p_K$, $[q^+p]_0 = \sum_K (-)^K q_K^+ p_{-K}$. The coefficients A,B,C,Mo are expressed through ω and the boson interaction constants. The coefficient $C > 0$ in the case of strong pair correlations and $C < 0$ in the case of weak pair correlations.

Let us use the differential representation of the operators q_μ, p_μ:

$$q_\mu = z_\mu, \quad p_\mu = -i\frac{\partial}{\partial z_\mu},$$

where z_μ is a complex variable since q_μ is a nonhermitean operator. Let us introduce also the new variables $\Delta, \vartheta, \varphi, \psi_1, \psi_2, \psi_3$

$$z_\mu = \sum_{K=0,\pm 1} D^{1\,*}_{\mu K}(\psi_1, \psi_2, \psi_3)\, a_K, \quad a_1 = a_{-1} = \frac{\Delta}{\sqrt{2}} e^{i\varphi} \sin\vartheta, \quad a_0 = \Delta e^{i\varphi} \cos\vartheta. \qquad (8)$$

Here $D^{1\,*}_{\mu K}(\psi_1, \psi_2, \psi_3)$ is the Wigner function. The three Euler angles describe the rotation in isotopic space. Since ψ_1, ψ_2, ψ_3 are the dynamical variables, the transformation (8) is equivalent to the transition to the coordinate system rotating in isospace. The variables a_K are the inner coordinates with respect to the new coordinate system. The nuclear isotopic spin operator is connected with the operators of the isospin projections on the new coordinate axes ξ, η, ζ in the following way:

$$T_\mu = \sum_K D^{1*}_{\mu K}(\psi_1, \psi_2, \psi_3)\, \tilde{T}_K ,$$

$$\tilde{T}_0 = T_\zeta , \qquad \tilde{T}_{\pm 1} = \mp \frac{T_\xi \pm i T_\eta}{\sqrt{2}} ,$$

where

$$T_\xi = i\left\{ \frac{\cos\psi_3}{\sin\psi_2}\left(\frac{\partial}{\partial\psi_1} - \cos\psi_2\frac{\partial}{\partial\psi_3}\right) - \sin\psi_3\frac{\partial}{\partial\psi_2} \right\},$$

$$T_\eta = i\left\{ -\frac{\sin\psi_3}{\sin\psi_2}\left(\frac{\partial}{\partial\psi_1} - \cos\psi_2\frac{\partial}{\partial\psi_3}\right) - \cos\psi_3\frac{\partial}{\partial\psi_2} \right\},$$

$$T_\zeta = -i\frac{\partial}{\partial\psi_3}$$

The variable φ , canonically conjugated to the particle number operator had been introduced in ref. /1/. The particle number operator in the new variables takes the form $N = 2i\frac{\partial}{\partial\varphi}$. The quantities Δ and ϑ describe the pairing vibrations. The Hamiltonian can be written in terms of new variables as follows:

$$H_{coll} = T_{vib} + T_{rot} + V$$

$$T_{vib} = \frac{M_0}{4}\left(-\frac{1}{\Delta^5}\frac{\partial}{\partial\Delta}\Delta^5\frac{\partial}{\partial\Delta} - \frac{1}{\Delta^2\sin 4\vartheta}\frac{\partial}{\partial\vartheta}\sin 4\vartheta\frac{\partial}{\partial\vartheta} \right),$$

$$T_{rot} = \frac{M_0}{4\Delta^2}\left(\frac{\left(\frac{N}{2} + \sin 2\vartheta\, T_\xi\right)^2}{\cos^2 2\vartheta} + T_\xi^2 + \frac{T_\eta^2}{\cos^2\vartheta} + \frac{T_\zeta^2}{\sin^2\vartheta} \right), \qquad (9)$$

$$V = -C\Delta^2 + \frac{A\Delta^4}{4}\left\{3 - \cos 4\vartheta\right\}$$

When we passed from (7) to (9) we conserved in T_{vib} and T_{rot} only the terms which are important for qualitative consideration. The first term in (9) is the kinetic energy of Δ and ϑ vibrations. The term proportional to N^2 in T_{rot} is known from the consideration of the systems of one type of nucleons /1/. The collective motion connected with them is the pairing rotation /3/. The other part of T_{rot} for the exception of the term proportional to $N \cdot T_\xi$ has the same form as the Hamiltonian of the asymmetric rotator in the three-dimensional space. We shall call this part of the Hamiltonian the energy of the rotation in isospace. The fact that in H_{coll} there is a term proportional to $N \cdot T_\xi$ means that the rotations in isospace and in gauge space are not separated from one another. The third term in H_{coll} is the potential energy of Δ and ϑ vibrations. Due to the dependence of the momenta of inertia on Δ and ϑ the rotations are connected with the vibrations.

As the inner axes η and ζ are not distinguishable from the physical point of view (the axis is singled out by the coupling of T_ξ with N) we can choose these axes in two physically equivalent ways. As a result H_{coll} is invariant under the transformation:

$$\vartheta \to \pi + \vartheta, \quad \vartheta \to \frac{\pi}{2} - \vartheta, \quad \vartheta \to -\frac{\pi}{2} - \vartheta.$$

Consequently the $\Delta - \vartheta$ plane is divided into four physically equivalent sectors:

$$-\frac{\pi}{4} \leq \vartheta \leq \frac{\pi}{4}, \quad \frac{\pi}{4} \leq \vartheta \leq \frac{3\pi}{4}, \quad \frac{3\pi}{4} \leq \vartheta \leq \frac{5\pi}{4}, \quad -\frac{3\pi}{4} \leq \vartheta \leq -\frac{\pi}{4}.$$

We single out the sector $-\frac{\pi}{4} \leq \vartheta \leq \frac{\pi}{4}$ and consider that ϑ changes in these limits.

The potential energy V has the minimum at $\vartheta = 0$ and $\Delta = \Delta_c = \sqrt{\frac{G}{A}}$. The quantity Δ_c is the analog of the energy gap in the superfluid nuclear model. As is seen from (9) the stiffness with respect to Δ and ϑ vibrations increases with increasing Δ_c. In the limit of strong pair correlations ($\Delta_c \gg G$) the potential energy can be replaced by the first terms of expansion of V power series in $(\Delta - \Delta_o)$ and ϑ

$$V \simeq -\frac{1}{2} A \Delta_c^4 + 2 A \Delta_c^2 (\Delta - \Delta_c)^2 + 2 A \Delta_c^4 \vartheta^2 .$$

In this limit we can put in T_{rot} $\Delta = \Delta_c$, $\vartheta = 0$ and in T_{vib} $\Delta = \Delta_c$, $\sin 4\vartheta \simeq 4\vartheta$. As a result, we get for H_{coll}:

$$H_{coll} \simeq -\frac{1}{2} A \Delta_c^4 + \frac{M_c}{4\Delta_c^2} \left[\frac{N^2}{4} + T(T+1) - T_\zeta^2 \right] + \frac{M_c}{4} \left[-\frac{\partial^2}{\partial (\Delta - \Delta_c)^2} + \frac{8 A \Delta_c^2}{M_c} (\Delta - \Delta_c)^2 \right] + \frac{M_c}{4\Delta_c^2} \left[-\frac{1}{\vartheta} \frac{\partial}{\partial \vartheta} \vartheta \frac{\partial}{\partial \vartheta} + \frac{T_\zeta^2}{\vartheta^2} + \frac{8 A \Delta_c^6}{M_c} \vartheta^2 \right].$$

The eigenvalues of this Hamiltonian are:

$$E(N, T, T_\zeta, n_\Delta, n_\vartheta) = -\frac{A\Delta_c^4}{2} + \frac{M_o}{4\Delta_c^2} \left[\frac{N^2}{4} + T(T+1) - T_\zeta^2 \right] + \sqrt{2 A M_c \Delta_c^2} \left(n_\Delta + 2 n_\vartheta + |T_\zeta| + \frac{3}{2} \right),$$

where $n_\Delta = 0,1,2,\ldots$ and $n_\vartheta = 0,1,2\ldots$

The eigenfunctions of the Hamiltonian are proportional to:

$$e^{-\frac{iN}{2}\gamma}\left[D^{T}_{M T_\zeta} + (-)^{T+T_\zeta} D^{T}_{M -T_\zeta}\right]\vartheta^{|T_\zeta|} H_{n_\Delta}\left[\frac{1}{2}\left(\frac{M_c}{A\Delta_c^2}\right)^{1/2}(\Delta-\Delta_0)\right]\times$$

$$\times F\left(-n_\vartheta, 1+|T_\zeta|, \vartheta^2\Delta_0^2\sqrt{\frac{8A\Delta_c^2}{M_c}}\right),$$

Where H_{n_Δ} is the Hermite polynomial and $F(-n, a, x)$ is a degenerate hypergeometric function.

These functions can be used as the basis ones in the limit of strong pair correlations.

In the expression (10) the term $M_c \frac{N^2}{16\Delta_c^2}$ corresponds to the energy of the pairing rotations, the term $\frac{M_c}{4\Delta_c^2}\left[T(T+1) - T_\zeta^2\right]$ is the energy of rotations in isospace and the term $\sqrt{2AM_c\Delta_c^2}\left(n_\Delta + 2n_\vartheta + |T_\zeta| + \frac{1}{2}\right)$ is the energy of Δ and ϑ vibrations.

The experimental data /16/ concerning the energies and cross sections of the two-nucleon transfer reactions are in some oases in agreement with this strong coupling scheme. In other cases the model of approximately independent collective bosons is in better agreement with the experimental data than the strong coupling scheme. It means that the intermediate coupling model corresponds mainly to the experimental situation and we must diagonalized H_{coll} exactly. This problem will be solved elsewhere.

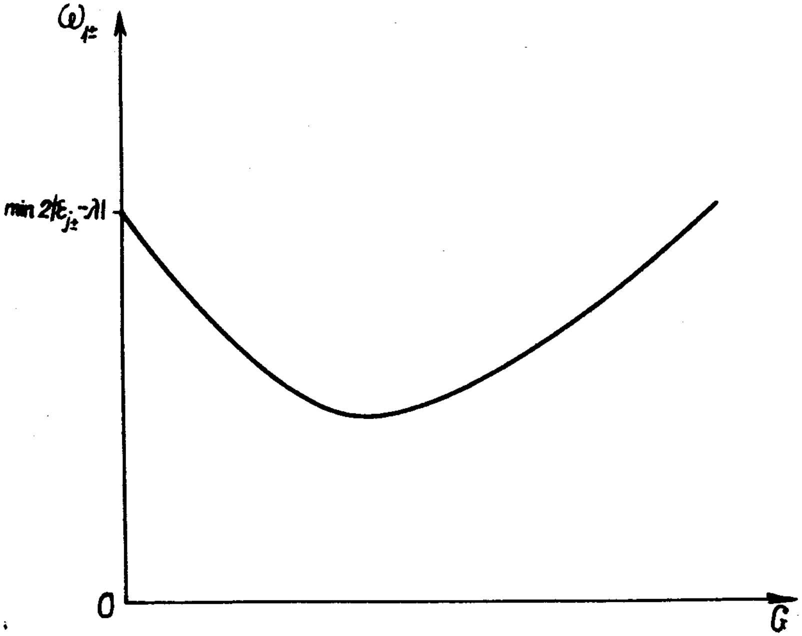

Fig.1 The dependence of the energy of the collective bosons ($\omega_{1\pm}$) on pairing interaction constant (G).

References

1. Р.В.Джолос. Теоретическая и математическая физика, 6, 403, 1971.
2. В.Г.Соловьёв. Влияние парных корреляций сверхпроводящего типа на свойства атомных ядер, Атомиздат, 1963.
3. A.Bohr. Nuclear Structure Dubna Symposium 1968, International Atomic Energy Agency, Vienna, 1968.
4. O.Nathan. Nuclear Structure Dubna Symposium 1968, International Atomic Energy Agency, Vienna, 1968.
5. B.H.Flowers, S.Szpikowski.Proc.Phys.Soc.,84, 193, 1964.
6. P.Camiz, A.Covello, M.Jean. Nuovo Cim., 42B, 199, 1966.
7. Г.Н.Афанасьев. Препринт Р4-3431, ОИЯИ, 1967.
8. С.Т.Беляев, В.Г.Зелевинский. ЖЭТФ, 42, 1590, 1962.
9. B.Sørensen. Prog.Theor.Phys., 39, 1468, 1968.
10. T.Holstein, H.Primakoff.Phys.Rev., 58, 1098, 1940.
11. T.Marumori, M.Yamamura and A.Tokunaga. Prog.Theor.Phys. 31, 1009, 1964.
12. F.Dyson. Phys.Rev., 102, 1217, 1956.
13. D.Janssen, F.Dönau, S.Frauendorf, R.V.Jolos. Preprint ZFK 1970.
14. T.Oguchi. Prog.Theor.Phys., 25, 721 (1961).
15. B.Sørensen. Nucl.Phys., A97, 1, 1967.
16. B.F.Bayman, D.R.Bes, R.A.Broglia. Phys.Rev.Lett., 23, 1299, 1969.

Received by Publishing Department
on November 18, 1971.