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NON- ROTATIONAL STATES
OF ODD-Z DEFORMED NUCLEI
IN THE REGION $153 \leq A \leq 177$

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Неротационные состояния деформированных ядер с нечётным числом протонов в области $153 \leq A \leq 177$

В рамках сверхтекучей модели с учётом взаимодействия квазичастиц с фононами рассчитаны энергия и структура неротационных возбужденных состояний в ряде деформированных ядер с нечётным числом протонов в области $153 \leq A \leq 177$. В расчётах использованы волновые функции и одночастичные энергии потенциала Саксона-Вудса. Получено удовлетворительное согласие с экспериментальными данными. Предсказано положение и структура большого числа новых уровней в исследуемых ядрах.

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Non-Rotational States of Odd-Z Deformed Nuclei in the Region $153 \leq A \leq 177$

The energy and structure of the non-rotational excited states for a number of odd-N deformed nuclei in the region $153 \leq A \leq 177$ were calculated in the framework of the superfluid nuclear model. The calculations were performed with the Saxon-Woods single-particle energies and wave functions. A satisfactory agreement of the experimental and theoretical data was obtained. The position and the structure of a large number of new levels in the nuclei in question is predicted.

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The excited non-rotational states of odd-Z nuclei are not pure quasiparticle states, they contain admixtures of many-quasiparticle components. Thus, in studying their structure it is necessary to take into account the connection between the quasiparticle and collective states and, first of all the interaction of quasiparticles with vibrational phonons.

The mathematical formalism for description of the interaction of quasiparticles with phonons in the framework of the superfluid nuclear model is presented in the monograph /1/. In refs. /2-8/ this formalism was used to calculate the non-rotational states of a large number of odd-A deformed nuclei. It was shown that the excited states of odd-A nuclei have a complex structure and only the lowest and a small number of higher excited states were found to be close to the one-quasiparticle states.

The present work is devoted to the study of the structure of nuclei with an odd number of protons in the region $153 \leq A \leq 177$. The main attention was paid to the nuclei the spectra of which were not calculated earlier.

In the present calculations the Saxon-Woods single-particle energies and wave functions calculated by the method suggested in ref.^{/9/} were used. In the expansion of the nuclear radius in the multipoles the quadrupole and hexadecapole deformations have been taken into account. Taking into consideration the fact that the energy and wave functions of the Saxon-Woods potential are A-dependent the deformed nuclei of the region $150 \leq A \leq 190$ were divided into four zones. The nuclei of this region are computed with their parameters which slightly alter in the transition from zone to zone. Table 1 gives the Saxon-Woods parameters for the four zones. In the present paper the nuclei of three zones with $A=155$, 165 and 173 were studied. The calculations of the non-rotational states of odd-A nuclei in zone $A=181$ were carried out in refs.^{/7,10/}.

The atomic nuclei of zone $A=155$ (^{153}Eu , $^{155-161}Tb$, $^{159-161}Ho$) were studied with the quadrupole deformation $\beta_{20}=0.28$ and hexadecapole deformation $\beta_{40}=0.06$, the nuclei of zone $A=165$ (^{163}Ho , $^{165,167,171}Tm$) with $\beta_{20}=0.28$ and $\beta_{40}=0.02$; the nuclei of zone $A=173$ ($^{169,173,175}Lu$, ^{177}Ta) with $\beta_{20}=0.27$ and $\beta_{40}=-0.02$.

The wave function for an odd-Z nuclei is written in the form:

$$\Psi_i(K^\pi) = \frac{1}{\sqrt{2}} N_i(\rho_1 \dots \rho_n) \left\{ \sum_{\rho_n \sigma} C_{\rho_n \sigma}^i a_{\rho_n \sigma}^+ + \sum_{\lambda \mu i \nu \sigma} D_{\nu \sigma}^{\lambda \mu i} (\rho_1 \dots \rho_n) a_{\nu \sigma}^+ Q_i^+(\lambda \mu) \right\} \Psi_0 \quad (1)$$

Here $Q_i^+(\lambda \mu)$ is the phonon operator of multipolarity $(\lambda \mu)$, $a_{\nu \sigma}^+$ is the quasi-particle production operator, $\sigma = \pm 1$, Ψ_0 is the wave function of the ground state of an even-even nucleus. The quantity $N_i^2(\rho_1 \dots \rho_n) (C_{\rho_n}^i)^2$ characterizes the contribution to the state with a given K^π of the one-quasiparticle component ρ_n , the quantity $\frac{1}{2} N_i^2(\rho_1 \dots \rho_n) \sum_{\sigma} (D_{\nu \sigma}^{\lambda \mu i})^2$ is the contribution of the component quasiparticle in the ν -state and the phonon $\lambda \mu i$. The summation over ρ_n means that in the Saxon-Woods single-particle scheme one takes into account simultaneously several states with identical K^π . Using the variational principle, e.g. ref.^{/11/}, one obtains a secular equation the roots of which define the energy of the ground (K_0^π) and excited states of an odd-Z nucleus. A number of excited states has a considerable admixture of the component quasiparticle plus γ -vibrational phonon. Such states are characterized by large E2 transition probabilities. A part of them is given in Table 2. All the reduced $B(E2)$ probabilities (in single-particle units $B_{s.p.}(E2) = 3A^{4/3} e^2 10^{-53} \text{ cm}^4$) are calculated with the effective charge $e_{\text{eff}} = 0.2$. The experimental values of the reduced transition probabilities for even-even nuclei were taken from ref.^{/11/}.

The experimental data on $B(E2)$ values in the odd-Z nuclei question are very poor. The experimental value

$B(E2)$ s.p.u. = 1.5 for the excited $1/2^+$ state of energy 531 keV in ^{159}Tb essentially differs from the calculated value. However, the value $B(E2)$ s.p.u. = 1.5 is surprising if the experimental value $B(E2)$ s.p.u. = 2.7 in ^{158}Gd is true.

The Coriolis forces have been neglected. According to ref. /12/ and other papers these forces are important for levels from spherical subshells with large j and especially for rotational states with large spins. In the region under consideration the Coriolis forces may most strongly affect the $h_{11/2}$ subshell states. Therefore the energies of states 550 \cdot , 541 \cdot , 532 \uparrow , 523 \uparrow , 514 \uparrow and 505 \uparrow may little change due to Coriolis interaction. The latter effect can be estimated for the simple case of mixing of two bands using the matrix elements of the operator i_+ given in the review /13/.

We should bear in mind that the Coriolis forces little affect the energy and the structure of the ground state rotational bands. In the framework of the superfluid nuclear model the Coriolis forces can easily be taken into account in just the same way as it has been done in refs. /14/.

The energy and structure of a state close to the one-quasiparticle state can essentially be affected by the deflection of the equilibrium deformation of a nucleus in the excited state from that for the nucleus in the ground state. A preliminary analysis shown that this effect is especially important for the behaviour of

the levels 404_{\downarrow} , 541_{\downarrow} and 402^{\uparrow} . For example, the investigations /15/ showed that for the 541_{\downarrow} state this deflection $\Delta\beta$ can reach 0.04. For such a change of the quadrupole deformation the single-particle 541_{\downarrow} state energy decreases by 0.5 MeV. Thus, the qualitative analysis of this effect makes it possible to improve the description of the energies of states 404_{\downarrow} , 541_{\downarrow} and 402^{\uparrow} .

The results of calculations of the energies and wave functions for a number of nuclei with an odd number of protons are given in Tables 3-17. The energies and the structure of the ground and excited non-rotational states of nuclei up to 1.5 MeV and some states higher than 1.5 MeV are given there. The fourth column of the tables gives the contribution (in percents) of a few largest components obtained from the wave function normalization condition to the state in question. The second single-particle state with the same K^{π} as that under investigation was included in the table in the case when the value of this component exceeded 1%. For example, in ^{159}Ho the contribution to the $K^{\pi} = 5/2^+$ state comes from: one-quasiparticle 413_{\downarrow} state - 69%, one-quasiparticle 402^{\uparrow} state - 23% and component quasiparticle 532^{\uparrow} plus phonon $Q_1(30)$ - 5%.

The experimental data are taken from two reviews /13,16/. They include the data published in literature before May 1, 1970. The experimental data obtained after this data are taken from refs. /17-25/. Some tables give the energies of three-quasiparticle states of the type $(p, 2n)$ with

large spins. The analysis of three-quasiparticle states in odd-A deformed nuclei is performed in ref.^{/26/}.

The calculations of the non-rotational states in ^{155}Eu , ^{165}Ho , ^{169}Tm and ^{171}Lu with the Saxon-Woods wave functions and single-particle energies showed that the improvement of the description of these states is insignificant compared to the description based on the Nilsson potential in ref.^{/6/}. Therefore the present work does not contain the results of calculation for these nuclei. The nuclei at the boundary of the two zones, like ^{161}Ho , ^{169}Lu , ^{171}Tm were calculated with the schemes of both zones. The comparison of the results showed that there are some insignificant differences in the energy and structure of the states under investigation. The present paper gives the results for these nuclei obtained with the schemes which were used for the calculation of the majority of isotopes.

It should be noted that the Saxon-Woods potential calculations are essentially more unambiguous as compared with the Nilsson potential calculations. The Saxon-Woods single-particle energies and wave functions are calculated with the same parameters for all the subshell without additional shifts of some levels and subshells. The calculations given showed that the Saxon-Woods single-particle energies and wave functions makes it possible to describe

satisfactorily a large amount of experimental data on the levels of nuclei with an odd number of protons in the range $153 \leq A \leq 177$. Besides, the position of a large number of levels in the nuclei in question is predicted.

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Table 1

Saxon-Woods Potential Parameters

A	Neutron System				Proton System			
	V_0 Mev	r_0 fm	α fm^2	λ fm^{-1}	V_0 Mev	r_0 fm	α fm^2	λ fm^{-1}
I55	47.2	1.26	0.40	1.67	59.2	1.24	0.36	1.63
I65	44.8	1.26	0.43	1.67	59.2	1.25	0.355	1.63
I73	44.8	1.26	0.42	1.67	59.2	1.25	0.32	1.59
I81	43.4	1.26	0.40	1.67	59.8	1.24	0.33	1.67

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Table 2

Reduced $B(E2, K_0 \rightarrow K_0 \pm 2)$ Transition Probabilities

Nucleus	K^π	Energy (keV)		$B(E2)_{S.P.U.}$	Structure, %
		Experiment	Theory		
^{153}Eu	$1/2^+$	—	670	0.4	$4I3^+ + Q_1(22) - 23$; $4II^+ - 48$
	$9/2^+$	—	I300	I.7	$4I3^+ + Q_1(22) - 100$
	$1/2^+$	—	I380	I.I	$4I3^+ + Q_1(22) - 67$
^{155}Tb	$1/2^+$	—	610	0.5	$4II^+ + Q_1(22) - 20$; $4II^+ - 60$
	$7/2^+$	—	I280	2.I	$4II^+ + Q_1(22) - 98$
	$1/2^+$	—	I310	I.8	$4II^+ + Q_1(22) - 80$; $420^+ - 6$
^{157}Tb	$1/2^+$	598	640	0.6	$4II^+ + Q_1(22) - 26$; $4II^+ - 64$
	$7/2^+$	—	I400	I.8	$4II^+ + Q_1(22) - 97$
	$1/2^+$	—	I425	I.3	$4II^+ + Q_1(22) - 72$; $420^+ - I4$
^{159}Tb	$1/2^+$	581	650	0.4	$4II^+ + Q_1(22) - 27$; $4II^+ - 64$
	$7/2^+$	—	I420	I.2	$4II^+ + Q_1(22) - 97$; $4I3^+ - 2$
	$1/2^+$	—	I650	0.7	$4II^+ + Q_1(22) - 52$; $4II^+ - 30$
^{161}Tb	$1/2^+$	—	590	0.5	$4II^+ + Q_1(22) - 35$; $4II^+ - 54$
	$7/2^+$	—	II80	I.3	$4II^+ + Q_1(22) - 99$
	$1/2^+$	—	I220	0.6	$4II^+ + Q_1(22) - 4I$
^{159}Ho	$3/2^-$	—	II00	2.8	$523^+ + Q_1(22) - 93$; $54I^+ - 5$
	$II/2^-$	—	II30	3.I	$523^+ + Q_1(22) - I00$
^{161}Ho	$3/2^-$	—	II40	2.7	$523^+ + Q_1(22) - 96$; $54I^+ - 3$
	$II/2^-$	—	II60	2.8	$523^+ + Q_1(22) - I00$
^{163}Ho	$3/2^-$	—	I015	2.2	$523^+ + Q_1(22) - 95$; $54I^+ - 4$
	$II/2^-$	—	I040	2.4	$523^+ + Q_1(22) - I00$
^{165}Tm	$5/2^+$	—	990	I.7	$4II^+ + Q_1(22) - 49$; $402^+ - 42$
	$3/2^+$	—	II00	3.2	$4II^+ + Q_1(22) - 94$; $4II^+ - 4$
	$5/2^+$	—	III0	I.6	$4II^+ + Q_1(22) - 46$; $402^+ - 44$

Table 2 (continuation)

Nucleus	K^π	Energy, keV		$B(E2)_{S.P.U.}$	Structure, %
		Experiment	Theory		
^{167}Tm	$3/2^+$	47I	670	0.3	$4II^+ + Q_1(22) - II$; $4II^+ - 82$
	$5/2^+$	—	820	2.4	$4II^+ + Q_1(22) - 84$; $402^+ - I2$
	$3/2^+$	—	990	2.5	$4II^+ + Q_1(22) - 89$; $4II^+ - IO$
^{171}Tm	$5/2^+$	9I3	930	0.3	$4II^+ + Q_1(22) - I8$; $402^+ - 70$
	$5/2^+$	—	I090	I.5	$4II^+ + Q_1(22) - 75$; $402^+ - I6$
	$3/2^+$	—	I100	I.8	$4II^+ + Q_1(22) - 93$
^{169}Lu	$3/2^+$	—	I050	2.0	$404^+ + Q_1(22) - 89$; $402^+ - IO$
	$II/2^+$	—	II30	2.2	$404^+ + Q_1(22) - I00$
^{173}Lu	$3/2^+$	—	I450	0.6	$404^+ + Q_1(22) - I00$
	$II/2^+$	—	I460	0.6	$404^+ + Q_1(22) - I00$
^{177}Ta	$3/2^+$	—	II30	I.0	$404^+ + Q_1(22) - 83$; $402^+ - I5$

Table 3

 ^{153}Eu

$K\pi$	Energy, keV		Structure	%
	Expe- riment	Theo- ry		
$5/2^+$	0	0	413†96%,	411†+Q ₁ (22) 2%
$3/2^+$	103	-20	411†89%,	411†+Q ₁ (22) 5%, 523†+Q ₁ (32) 3%
$5/2^-$	97	20	532†94%,	420†+Q ₁ (32) 1%
$7/2^-$		550	523†86%,	411†+Q ₁ (32) 10%
$1/2^+$		670	411†48%,	411†+Q ₁ (22) 26%, 413†+Q ₁ (22) 23%
$3/2^-$		820	541†86%	550†+Q ₁ (22) 5%, 420†+Q ₁ (31) 2%
$5/2^-$		860		532†+Q ₁ (20) 100%
$5/2^+$		900		413†+Q ₁ (20) 100%
$1/2^+$		920	420†75%,	532†+Q ₁ (32) 8%, 550†+Q ₁ (30) 5%, 422†+Q ₁ (22) 4%
$1/2^-$		1000	550†42%,	532†+Q ₁ (22) 39%, 541†+Q ₁ (22) 8%, 420†+Q ₁ (30) 6%
$3/2^+$		1150		411†+Q ₁ (20) 100%
$9/2^-$		1230	514† 3%,	532†+Q ₁ (22) 97%
$9/2^+$		1300		413†+Q ₁ (22) 100%
$1/2^+$		1380		413†+Q ₁ (22) 67%, 411†+Q ₁ (22) 31%
$1/2^-$		1400	550†19%,	532†+Q ₁ (22) 59%, 541†+Q ₁ (22) 11%, 420†+Q ₁ (30) 9%
$3/2^-$		1400		411†+Q ₁ (30) 100%
$3/2^+$		1510	422†50%,	420†+Q ₁ (22) 26%, 541†+Q ₁ (30) 16%, 532†+Q ₁ (31) 5%
$7/2^+$		1520		411†+Q ₁ (22) 94%, 532†+Q ₁ (31) 5%

Table 4

 ^{155}Tb

$K\pi$	Energy, keV		Structure	%
	Expe- riment	Theo- ry		
$3/2^+$	0	0	411†92%,	411†+Q ₁ (22) 4%
$7/2^-$	545	341	523†94%,	411†+Q ₁ (32) 2%
$5/2^+$	271	390	413†96%,	411†+Q ₁ (22) 2%
$5/2^-$	227	530	532†93%,	550†+Q ₁ (22) 2%
$1/2^+$		610	411†60%,	411†+Q ₁ (22) 20%, 413†+Q ₁ (22) 8%
$3/2^+$		1000		411†+Q ₁ (20) 100%
$5/2^+$		1100		413†+Q ₁ (20) 100%
$7/2^+$		1280	413†1%,	411†+Q ₁ (22) 98%
$1/2^-$		1300	550†18%,	532†+Q ₁ (22) 76%, 541†+Q ₁ (22) 3%
$1/2^+$		1310	420† 6%	411†+Q ₁ (22) 80%
$1/2^+$		1330		413†+Q ₁ (22) 100%
$9/2^+$		1330		413†+Q ₁ (22) 100%
$3/2^-$		1350	541†52%,	411†+Q ₁ (30) 35%, 550†+Q ₁ (22) 3%
$5/2^+$		1420	402†73%,	523†+Q ₁ (31) 9%, 660†+Q ₁ (22) 6%, 532†+Q ₁ (30) 5%
$9/2^-$		1460	514† 7%,	532†+Q ₁ (22) 92%
$3/2^-$		1650	541† 6%,	523†+Q ₁ (22) 56%, 411†+Q ₁ (30) 34%
$7/2^+$		1680	404†68%,	523†+Q ₁ (30) 28%, 651†+Q ₁ (22) 3%

Table 5

 ^{157}Tb

K π	Energy, keV		Structure %	
	Expe-	Theo-		
	ri-	ry		
3/2 ⁺	0	0	4II ⁺ 93%,	4II ⁺ + q_1 (22) 4%
7/2 ⁻	572	360	523 ⁺ 95%,	4II ⁺ + q_1 (32) 2%
5/2 ⁺	328	380	4I3 ⁺ 96%,	4II ⁺ + q_1 (22) 2%
5/2 ⁻	326	530	532 ⁺ 94%,	550 ⁺ + q_1 (22) 2%
1/2 ⁺	598	640	4II ⁺ 64%,	4II ⁺ + q_1 (22) 26%, 4I3 ⁺ + q_1 (22) 7%
3/2 ⁺	993	I300		4II ⁺ + q_1 (20)100%
3/2 ⁻		I370	54I ⁺ 48%,	4II ⁺ + q_1 (30) 39%
5/2 ⁺		I390	402 ⁺ 49%,	4I3 ⁺ + q_1 (20) 32%, 523 ⁺ + q_1 (3I)10%, 660 ⁺ + q_1 (22)4%
7/2 ⁺		I400		4II ⁺ + q_1 (22) 97%
1/2 ⁻		I410	550 ⁺ 21%,	532 ⁺ + q_1 (22) 72%, 54I ⁺ + q_1 (22) 3%
1/2 ⁺		I425	420 ⁺ 14%,	4II ⁺ + q_1 (22) 72%, 532 ⁺ + q_1 (32) 2%
5/2 ⁺		I450	402 ⁺ 21%,	4I3 ⁺ + q_1 (20) 68%, 523 ⁺ + q_1 (3I) 5%
9/2 ⁺		I460		4I3 ⁺ + q_1 (22)100%

Table 6

 ^{159}Tb

K π	Energy, keV		Structure %	
	Experi-	Theo-		
	ment	ry		
3/2 ⁺	0	0	4II ⁺ 93%,	4II ⁺ + q_1 (22) 4%
5/2 ⁺	348	370	4I3 ⁺ 96%,	
7/2 ⁻		380	523 ⁺ 96%,	4II ⁺ + q_1 (32) 2%
5/2 ⁻	364	570	532 ⁺ 93%,	4I3 ⁺ + q_1 (30) 2%, 550 ⁺ + q_1 (22) 2%
1/2 ⁺	581	650	4II ⁺ 64%,	4II ⁺ + q_1 (22)27%, 4I3 ⁺ + q_1 (22) 7%
3/2 ⁻		I170	54I ⁺ 17%,	4II ⁺ + q_1 (30)80%
1/2 ⁺		I350	420 ⁺ 12%,	4I3 ⁺ + q_1 (22)63%, 4II ⁺ + q_1 (22)19%
5/2 ⁺		I360	402 ⁺ 29%,	532 ⁺ + q_1 (30)64%, 660 ⁺ + q_1 (22) 2%
1/2 ⁻		I390	550 ⁺ 26%,	532 ⁺ + q_1 (22)65%, 54I ⁺ + q_1 (22) 4%
7/2 ⁺		I420		4II ⁺ + q_1 (22)97%
9/2 ⁺		I470		4I3 ⁺ + q_1 (22)100%
3/2 ⁺		I480		4II ⁺ + q_1 (20)100%
3/2 ⁻		I580	54I ⁺ 35%,	4I3 ⁺ + q_1 (3I)44%, 4II ⁺ + q_1 (30)12%
1/2 ⁺		I650	4II ⁺ 30%,	4II ⁺ + q_1 (22)52%, 4I3 ⁺ + q_1 (22)14%
5/2 ⁺		I860		p523 ⁺ _n 52I ⁺ 642 ⁺

Table 7

 ^{161}Tb

K	Energy, keV		Structure %		
	Experi- ment	Theo- ry			
$3/2^+$	0	0	411†93%,	411†+Q ₁ (22) 5%	
$5/2^+$	315	360	413†97%,	411†+Q ₁ (22) 2%	
$7/2^-$	417	390	523†97%,	411†+Q ₁ (32) 1%	
$1/2^+$	590		411†54%,	411†+Q ₁ (22) 35%,	413†+Q ₁ (22)10%
$5/2^-$	480	600	532†96%,	550†+Q ₁ (22) 2%	
$7/2^+$	1180		411†+Q ₁ (22) 99%		
$1/2^+$	1220		413†+Q ₁ (22) 58%,	411†+Q ₁ (22)41%	
$9/2^+$	1230		413†+Q ₁ (22)100%		
$1/2^-$	1250	550†12%,	532†+Q ₁ (22) 85%,	541†+Q ₁ (22) 2%	
$9/2^-$	1370	514† 4%,	532†+Q ₁ (22) 96%		
$3/2^-$	1420	541†16%,	411†+Q ₁ (30) 78%,	523†+Q ₁ (22) 4%	
$5/2^+$	1530	402†65%,	413†+Q ₁ (20) 12%,	532†+Q ₁ (30)11%	
$3/2^+$	1550		411†+Q ₁ (20) 100%		
$5/2^+$	1560		413†+Q ₁ (20) 100%		
$15/2^-$	1680		p412†n642†n523†		
$17/2^+$	1730		p523†n642†n523†		

Table 8

 ^{159}Ho

K	Energy, keV		Structure %		
	Experi- ment	Theo- ry			
$7/2^-$	0	0	523†97%		
$3/2^+$	250		411†94%		
$1/2^+$	206	380	411†88%,	411†+Q ₁ (22) 9%	
$5/2^+$	700		402†56%,	413†28%,	660†+Q ₁ (22) 8%, 411†+Q ₁ (22)3%
$5/2^+$	650	730	413†69%,	402†23%,	532†+Q ₁ (30) 5%
$5/2^-$	624	900	532†90%,	550†+Q ₁ (22) 3%, 413†+Q ₁ (30)3%	
$7/2^+$	1000		404†90%,	651†+Q ₁ (22) 5%, 523†+Q ₁ (30) 3%	
$7/2^+$	1050		413† 2%,	411†+Q ₁ (22)97%	
$3/2^-$	1100		541† 5%,	523†+Q ₁ (22)93%	
$11/2^-$	1130		523†+Q ₁ (22)100%		
$1/2^+$	1140		411†+Q ₁ (22)91%		
$9/2^-$	1145		514†92%,	402†+Q ₁ (32) 4%	
$3/2^+$	1190		411†+Q ₁ (20)100%		
$1/2^-$	1330		541†86%,	411†+Q ₁ (30) 10%	
$9/2^+$	1390		413†+Q ₁ (22)100%		
$3/2^-$	1420		541† 8%,	411†+Q ₁ (30) 87%	
$1/2^-$	1470		550†14%,	532†+Q ₁ (22) 81%,	541†+Q ₁ (22)2%

Table 9

 ^{161}Ho

$K\pi$	Energy, keV		Structure	%
	Experi- ment	Theo- ry		
$7/2^-$	0	0	523†97%	
$3/2^+$	299	260	411†94%,	411†+Q ₁ (22) 3%
$1/2^+$	211	380	411†88%,	411†+Q ₁ (22) 9%
$5/2^+$	700		402†79%, 413†2%,	660†+Q ₁ (22) 8%
$5/2^+$	760	740	413†96%, 402†2%	
$5/2^-$	827	950	532†89%,	411†+Q ₁ (31) 5%, 550†+Q ₁ (22)3%
$7/2^+$	253	1040	404†93%,	651†+Q ₁ (22) 6%
$7/2^+$	1070		413†2%,	411†+Q ₁ (22) 98%
$9/2^-$	1110		514†89%,	402†+Q ₁ (32) 7%
$3/2^-$	1140		541†3%,	523†+Q ₁ (22) 96%
$1/2^+$	1150		420†2%,	411†+Q ₁ (22) 98%
$11/2^-$	1160			523†+Q ₁ (22)100%
$1/2^-$	424	1370	541†72%,	411†+Q ₁ (32) 20%, 411†+Q ₁ (31)5%
$1/2^+$	1400		660†32%,	402†+Q ₁ (22) 58%, 651†+Q ₁ (22)5%
$9/2^+$	1415			413†+Q ₁ (22)100%
$3/2^+$	1420		402†3%	523†+Q ₁ (32) 96%
$1/2^-$	1460		550†2%,	411†+Q ₁ (32) 82%, 532†+Q ₁ (22)10%
$3/2^+$	1465			411†+Q ₁ (20)100%

Table 10

 ^{163}Ho

$K\pi$	Energy, keV		Structure	%
	Experi- ment	Theo- ry		
$7/2^-$	0	0	523†98%	
$3/2^+$	240		4II†95%,	4II†+Q ₁ (22) 2%
$1/2^+$	298	390	4II†91%,	4II†+Q ₁ (22) 8%
$5/2^-$	950		532†91%,	550†+Q ₁ (22) 3%
$5/2^+$	1000		4I3†97%	
$7/2^+$	1010		4I3† 1%,	4II†+Q ₁ (22) 99%
$3/2^-$	1015		54I† 4%,	523†+Q ₁ (22) 95%
$11/2^-$	1040			523†+Q ₁ (22)100%
$1/2^+$	1120		4II†8%,	4II†+Q ₁ (22) 92%
$9/2^-$	1180		5I4†95%,	402†+Q ₁ (32) 2%
$7/2^+$	440	1200	404†91%,	523†+Q ₁ (30) 6%
$3/2^+$	1300			4II†+Q ₁ (20)100%
$1/2^-$	1320		550 †1%,	4II†+Q ₁ (32) 95%
$1/2^-$	1400		550 †16%,	532†+Q ₁ (22) 75%, 4II†+Q ₁ (32) 4%
$3/2^-$	1410		54I †7%,	4II†+Q ₁ (30) 90%
$5/2^+$	1415		402 †34%,	4II†+Q ₁ (22) 58%, 523†+Q ₁ (31) 2%
$17/2^+$	1570			p523†n 642†n 523†

Table 11

 ^{165}Tm

K	Energy, keV		Structure	%			
	Experi-ment	Theo-ry					
$1/2^+$	0	0	$411\uparrow$	96%			
$7/2^-$	149	300	$523\uparrow$	94%, $411\uparrow+Q_1(32)$	3%		
$9/2^-$		550	$514\uparrow$	94%, $402\uparrow+Q_1(32)$	3%		
$7/2^+$	69	630	$404\uparrow$	96%			
$3/2^+$		660	$411\uparrow$	84%, $523\uparrow+Q_1(32)$	7%, $411\uparrow+Q_1(22)$	6%	
$5/2^+$		990	$402\uparrow$	42%, $413\uparrow$	2%, $411\uparrow+Q_1(22)$	49%, $514\uparrow+Q_1(32)$	3%
$3/2^-$		1010	$541\uparrow$	3%, $523\uparrow+Q_1(22)$	96%		
$11/2^-$		1060		$523\uparrow+Q_1(22)$	100%		
$3/2^+$		1100	$411\uparrow$	4%, $411\uparrow+Q_1(22)$	94%		
$5/2^+$		1100	$402\uparrow$	44%, $413\uparrow$	3%, $411\uparrow+Q_1(22)$	46%, $514\uparrow+Q_1(32)$	3%
$7/2^+$		1250	$413\uparrow$	1%, $411\uparrow+Q_1(22)$	98%		
$1/2^+$		1320		$411\uparrow+Q_1(22)$	100%		
$1/2^-$		1340	$541\uparrow$	89%, $411\uparrow+Q_1(30)$	8%		
$5/2^-$		1400	$532\uparrow$	75%, $411\uparrow+Q_1(31)$	14%, $411\uparrow+Q_1(32)$	3%	
$3/2^+$		1470	$651\uparrow$	11%, $404\uparrow+Q_1(22)$	86%, $523\uparrow+Q_1(32)$	2%	
$17/2^+$		1960	$p523\uparrow n642\uparrow n523\uparrow$				

Table 12

 ^{167}Tm

K	Energy, keV		Structure	%		
	Experi-ment	Theo-ry				
$1/2^+$	0	0	$411\uparrow$	97%		
$7/2^-$	293	360	$523\uparrow$	96%, $411\uparrow+Q_1(32)$	2%	
$9/2^-$		560	$514\uparrow$	96%, $402\uparrow+Q_1(32)$	2%	
$7/2^+$	179	600	$404\uparrow$	97%, $651\uparrow+Q_1(22)$	2%	
$3/2^+$	471	670	$411\uparrow$	82%, $411\uparrow+Q_1(22)$	11%, $523\uparrow+Q_1(32)$	4%
$5/2^+$		820	$402\uparrow$	12%, $413\uparrow$	3%, $411\uparrow+Q_1(22)$	84%
$3/2^-$		900	$541\uparrow$	3%, $523\uparrow+Q_1(22)$	97%	
$11/2^-$		940		$523\uparrow+Q_1(22)$	100%	
$3/2^+$		990	$411\uparrow$	10%, $411\uparrow+Q_1(22)$	89%	
$5/2^+$		1000	$402\uparrow$	74%, $411\uparrow+Q_1(22)$	13%, $660\uparrow+Q_1(22)$	4%
$7/2^+$		1140	$413\uparrow$	1%, $411\uparrow+Q_1(22)$	98%	
$1/2^+$		1200		$411\uparrow+Q_1(22)$	100%	
$1/2^-$	172	1290	$541\uparrow$	90%, $411\uparrow+Q_1(30)$	8%	
$5/2^-$	1527	1510	$532\uparrow$	81%, $514\uparrow+Q_1(22)$	6%, $411\uparrow+Q_1(32)$	5%
$5/2^+$	1581	1620	$413\uparrow$	94%, $411\uparrow+Q_1(22)$	3%	
$19/2^+$		2110	$p523\uparrow n523\uparrow n633\uparrow$			

Table 13

 ^{171}Tm

K π	Energy, keV		Structure	%
	Experi- ment	Theo- ry		
1/2 ⁺	0	0	4II†97%	
7/2 ⁻	425	360	523†97%	
9/2 ⁻		550	5I4†97%	
7/2 ⁺	635	560	404†96%,	65I†+Q ₁ (22) 3%
3/2 ⁺	676	680	4II†88%,	4II†+Q ₁ (22) 7%
5/2 ⁺	913	930	402†70%,	4II†+Q ₁ (22) 18%, 660†+Q ₁ (22) 4%
3/2 ⁻		1015	54I† 4%,	523†+Q ₁ (22) 95%
1/2 ⁺		1040		4II†+Q ₁ (20) 100%
11/2 ⁻		1070		523†+Q ₁ (22) 100%
5/2 ⁺		1090	402†16%, 4I3† 6%,	4II†+Q ₁ (22) 75%
3/2 ⁺		1100		4II†+Q ₁ (22) 93%
7/2 ⁺		1250	4I3† 2%	4II†+Q ₁ (22) 98%
1/2 ⁻		1260	54I†86%,	4II†+Q ₁ (30) 11%
1/2 ⁻		1460		4II†+Q ₁ (3I) 100%
9/2 ⁺		1440		523†+Q ₁ (3I) 100%
5/2 ⁺		1445		523†+Q ₁ (3I) 100%
5/2 ⁻		1500	532†84%,	4II†+Q ₁ (3I) 8%, 550†+Q ₁ (22) 4%
5/2 ⁺		1530	4I3†26%	4II†+Q ₂ (22) 70%
5/2 ⁺		1640	4I3†63%	4II†+Q ₂ (22) 29%

 ^{169}Lu

K π	Energy, keV		Structure	%
	Experi- ment	Theo- ry		
7/2 ⁺	0	0	404†97%,	402†+Q ₁ (22) 2%
9/2 ⁻		60	5I4†96%,	402†+Q ₁ (32) 1%
1/2 ⁺		220	4II†90%,	4II†+Q ₁ (22) 6%
5/2 ⁺		540	402†87%,	5I4†+Q ₁ (32) 4%, 660†+Q ₁ (22) 4%
3/2 ⁺		730	4II†39%,	4II†+Q ₁ (22) 56%
7/2 ⁻	493	740	523†93%,	4II†+Q ₁ (32) 3%
1/2 ⁻	30	980	54I†94%,	532†+Q ₁ (22) 2%
5/2 ⁺		1000	4I3† 6%,	4II†+Q ₁ (22) 93%
3/2 ⁺		1050	402†10%,	404†+Q ₁ (22) 89%
11/2 ⁺		1130		404†+Q ₁ (22) 100%
1/2 ⁺		1270		4II†+Q ₁ (20) 100%
5/2 ⁻		1300	532† 2%,	5I4†+Q ₁ (22) 97%
7/2 ⁺		1305		404†+Q ₁ (20) 100%
13/2 ⁻		1310		5I4†+Q ₁ (22) 100%
3/2 ⁻		1340	54I†12%,	523†+Q ₁ (22) 85%
9/2 ⁻		1430		5I4†+Q ₁ (20) 100%
11/2 ⁻		1450		523†+Q ₁ (22) 100%
1/2 ⁺		1510	660†30%,	402†+Q ₁ (22) 63%, 402†+Q ₁ (22) 2%
3/2 ⁻		1530	532†59%,	4II†+Q ₁ (3I) 22%, 54I†+Q ₁ (22) 9%
1/2 ⁻		1550	530† 7%,	4II†+Q ₁ (30) 85%, 4II†+Q ₁ (3I) 7%
19/2 ⁻		1720		p404n 523m 633†

Table 15

 ^{173}Lu

K π	Energy, keV		Structure %
	Experi-ment	Theo-ry	
7/2 ⁺	0	0	404†99%
9/2 ⁻		40	5I4†97%, 402†+Q ₁ (32) 2%
1/2 ⁺	425	310	4II†98%
5/2 ⁺	357	600	402†84%, 5I4†+Q ₁ (32) 14%
7/2 ⁻		720	523†93%, 4II†+Q ₁ (32) 6%
1/2 ⁻	I28	I030	54I†97%, 4II†+Q ₁ (3I) 1%
1/2 ⁺		I070	4II†+Q ₁ (20)100%
3/2 ⁺		I080	4II†67%, 523†+Q ₁ (32) 27%, 4II†+Q ₁ (22) 2%
1/2 ⁻		II60	530†1%, 4II†+Q ₁ (3I) 98%
3/2 ⁻		II70	532†2%, 4II†+Q ₁ (3I) 98%
7/2 ⁺		II80	404†+Q ₁ (20)100%
3/2 ⁻		I270	4II†+Q ₁ (32)100%
5/2 ⁻		I280	532†3%, 4II†+Q ₁ (32) 92%, 404†+Q ₁ (3I) 4%
5/2 ⁻		I290	404†+Q ₁ (3I) 95%, 4II†+Q ₁ (32) 4%
5/2 ⁺		I330	642†2%, 4II†+Q ₁ (22) 97%
3/2 ⁺		I330	4II†+Q ₁ (22)100%
7/2 ⁺		I390	5I4†+Q ₁ (3I)100%
3/2 ⁻		I395	532†5%, 404†+Q ₁ (32) 95%
II/2 ⁻		I400	404†+Q ₁ (32)100%
3/2 ⁺		I450	404†+Q ₁ (22)100%
II/2 ⁺		I460	404†+Q ₁ (22)100%
9/2 ⁺		I500	404†1%, 523†+Q ₁ (3I) 99%
1/2 ⁻		I620	4II†+Q ₁ (30)100%

Table 16

 ^{175}Lu

K π	Energy, keV		Structure %
	Experi-ment	Theo-ry	
7/2 ⁺	0	0	404†99%
9/2 ⁻	396	I00	5I4†99%
1/2 ⁺		310	4II†97%
5/2 ⁺	343	700	402†96%, 660†+Q ₁ (22) 2%
7/2 ⁻		850	523†99%
1/2 ⁻	358	I000	54I†97%
3/2 ⁺		II80	4II†69%, 4II†+Q ₁ (22) 29%
3/2 ⁻		I280	4II†+Q ₁ (32)100%
5/2 ⁻		I280	4II†+Q ₁ (32)100%
19/2 ⁺	I40I	I300	p404†n5I2†n5I4†
1/2 ⁺		I340	4II†+Q ₁ (20)100%
2I/2 ⁻		I360	p5I4†n5I2†n5I4†
7/2 ⁺		I380	404†+Q ₁ (20)100%
II/2 ⁻		I390	404†+Q ₁ (32)100%
3/2 ⁻		I400	404†+Q ₁ (32)100%
9/2 ⁻		I470	5I4†+Q ₁ (20)100%
5/2 ⁺		I490	5I4†+Q ₁ (32)100%
13/2 ⁺		I490	5I4†+Q ₁ (32)100%
1/2 ⁻		I520	4II†+Q ₁ (30)100%
3/2 ⁻		I600	532†9%, 4II†+Q ₁ (3I) 90%
3/2 ⁻		I680	532†82%, 4II†+Q ₁ (3I) 10%

^{177}Ta

K^π	Energy, keV				Structure	%
	Experiment	Theory				
$7/2^+$	0	0	404†	99%		
$9/2^-$	74	-60	5I4†	99%		
$5/2^+$	7I	I80	402†	95%		
$I/2^-$	2I7	480	54I†	98%	$532 + Q_1(22)$	I%
$I/2^+$		6I0	4II†	92%	$4II + Q_1(22)$	6%
$3/2^-$		I020	532†	9I%	$54I + Q_1(22)$	6%
$3/2^+$		II30	402†	I5%	$404† + Q_1(22)$	83%
$II/2^-$		I200			$404† + Q_1(32)$	I00%
$3/2^-$		I200			$404† + Q_1(32)$	I00%
$7/2^+$		I2I0			$404† + Q_1(20)$	I00%
$5/2^+$		I220			$5I4† + Q_1(32)$	I00%
$I3/2^+$		I220			$5I4† + Q_1(32)$	I00%
$I/2^+$		I230	660†	36%	$402† + Q_1(22)$	58%, $402† + Q_1(22)$ 3%
$3/2^+$		I250	4II†	43%	$4II† + Q_1(22)$	56%
$9/2^-$		I260			$5I4† + Q_1(20)$	I00%
$7/2^-$		I270	523†	98%		
$5/2^-$		I300	532†	I%,	$5I4† + Q_1(22)$	99%
$I/2^-$		I390	530†	90%,	$532† + Q_1(22)$	7%
$23/2^+$		I460			$p5I4†n5I2†n624†$	
$2I/2^-$		I470			$p5I4†n5I2†n5I4†$	
$I/2^+$		I490	400†	37%,	$402† + Q_1(22)$	58%, $402† + Q_1(22)$ 2%
$2I/2^-$		I5I0			$p404†n5I2†n624†$	