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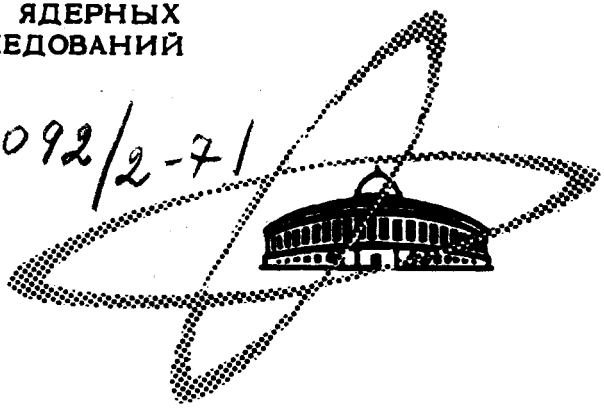

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F. Palumbo

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

NOTE ON THE GARTENHAUS-SCHWARTZ TRANSFORMATION

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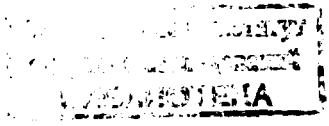

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Палумбо Ф.

Замечание о преобразовании Картенгауза-Шварца

В работе показано, что преобразование Картенгауза-Шварца не устраняет шпурионные эффекты в оболочечной модели.

**Препринт Объединенного института ядерных исследований.
Дубна, 1971**

Palumbo F.

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Note on the Gartenhaus-Schwartz Transformation

It is shown that the Gartenhaus-Schwartz transformation cannot eliminate spurious effects in shell model calculations.

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Dubna, 1971**

The description of electromagnetic interactions of nuclei requires the evaluation of matrix elements with the nuclear wave functions[1]

$$\Psi_{m\underline{K}} = \Psi_{Im} \frac{1}{\sqrt{V}} e^{i\underline{K}\cdot\underline{R}}, \qquad (1)$$

where Ψ_{Im} are intrinsic wave functions, $\underline{K}$ is the momentum of the nucleus, $\underline{R}$ the c.m. position vector and V the normalization volume. Matrix elements of the electromagnetic interaction Hamiltonian H_{int} with wave functions (1) reduce trivially to intrinsic matrix elements

$$\langle \Psi_{m\underline{K}'} | H_{int} | \Psi_{m\underline{K}} \rangle = \delta(-\underline{K}' + \underline{K} + \underline{q}) \int d\underline{r}_1 \cdots d\underline{r}_A \, \delta(\underline{R}) \cdot$$

$$\Psi^*_{Im} \left\{ \sum_{i=1}^{A} \left[\varepsilon_0 e_i e^{i\underline{q}\cdot(\underline{r}_i - \underline{R})} - \underline{\varepsilon} \cdot \frac{e_i}{2m} \left[(\underline{p}_i - \frac{1}{A}\underline{P}) e^{i\underline{q}\cdot(\underline{r}_i - \underline{R})} + \right.\right.\right.$$

$$+ e^{i\underline{q}\cdot(\underline{r}_i - \underline{R})} (\underline{p}_i - \frac{1}{A}\underline{P}) \Big] - i\underline{\varepsilon} \cdot \mu_i \underline{\sigma}_i \wedge \Big[(\underline{p}_i - \frac{1}{A}\underline{P}) e^{i\underline{q}\cdot(\underline{r}_i - \underline{R})} +$$

$$\left. - e^{i\underline{q}\cdot(\underline{r}_i - \underline{R})} (\underline{p}_i - \frac{1}{A}\underline{P}) \Big] \right] - \underline{\varepsilon} \cdot \frac{1}{2mA} (\underline{K} + \underline{K}') \sum_{i=1}^{A} e_i e^{i\underline{q}\cdot(\underline{r}_i - \underline{R})} +$$

$$\left. - i\underline{\varepsilon} \cdot \sum_{i=1}^{A} \mu_i \underline{\sigma}_i e^{i\underline{q}\cdot(\underline{r}_i - \underline{R})} \wedge \frac{1}{A}\underline{q} \right\} \Psi_{Im}, \qquad (2)$$

where $\varepsilon_0, \underline{\varepsilon}$ is the polarization vector of the photon which carries the momentum $\underline{q}$, e_i and μ_i are the charges and magnetic moments of protons and neutrons, $\hbar = c = 1$.

The difficulty with the intrinsic matrix element (2) is twofold. First we do not know any general method for the determination of intrinsic wave functions in terms of intrinsic coordinates; second the matrix element (2) is very complicated in terms of intrinsic coordinates. So, it is desirable to avoid the explicit use of intrinsic coordinates and keep the single particle coordinates. This can be done using factorized wave functions of the form

$$\Psi_n = \Psi_{In}\,\chi(R), \qquad (3)$$

where χ is the same factor for every n. If χ is chosen to be the ground state harmonic oscillator wave function, which we shall call χ_0, wave functions (3) can be easily obtained [2] as expansions in Slater determinants Φ_ν of (arbitrary) single particle wave functions

$$\Psi_n = \sum_\nu a_\nu^{(n)}\,\Phi_\nu. \qquad (4)$$

Then the evaluation of (2) can be done in terms of single particle coordinates, as it has been shown by Tassie and Barker[3].

However in the literature are commonly used wave functions $\tilde{\Psi}_n$ which are not factorized, and then involve unphysical admixtures of intrinsic and c.m. states. This is explicitly shown by their expansion into intrinsic wave functions and (for example) harmonic oscillator wave functions for the c.m.

$$\tilde{\Psi}_n = \sum_{\nu,\mu} C_{\nu\mu}^{(n)}\,\Psi_{I\nu}\,\chi_\mu(\underline{R}). \qquad (5)$$

The terms with $C_{\nu\mu}\neq 0$ for $\mu\neq 0$ give rise to spurious effects. Such effects are not necessarily of order $\frac{1}{A}$, as it is often assumed. They can be quite large on transition amplitudes [4]. In an attempt to prevent spuriosity effects and to relate matrix elements with wave functions (5) to the matrix elements with wave functions (1) Gartenhaus and

Schwartz[5] (G.S.) devised a transformation, defined by the operator $U_\Lambda = e^{-i\frac{\Lambda}{2}(\underline{P}\cdot\underline{R}+\underline{R}\cdot\underline{P})}$, which has the properties

$$\tilde{\Psi}^{\Lambda}_{m} = U_\Lambda \tilde{\Psi}_m = e^{-\frac{3}{2}\Lambda}\tilde{\Psi}_m\left[(\underline{r}_1-\underline{R})+e^{-\Lambda}\underline{R},\cdots(\underline{r}_A-\underline{R})+e^{-\Lambda}\underline{R}\right], \quad (6)$$

$$\tilde{\Psi}^{\Lambda}_{m}\left[\underline{r}_1+\underline{a},\cdots,\underline{r}_A+\underline{a}\right] = U_\Lambda \tilde{\Psi}_m\left[\underline{r}_1+\underline{a}\,e^{-\Lambda},\cdots\underline{r}_A+\underline{a}\,e^{-\Lambda}\right], \quad (7)$$

where $\underline{a}$ is an arbitrary vector.
From (7) G.S. deduce that for $\Lambda\to\infty$ one obtains intrinsic wave functions. In order to simplify the calculations they suggest to use the wave functions $\tilde{\Psi}_m$ with transformed operators,

$$\langle\tilde{\Psi}^{\Lambda}_{m}\,|\,M\,|\,\tilde{\Psi}^{\Lambda}_{m}\rangle = \langle\tilde{\Psi}_m\,|\,U^{+}_{\Lambda}\,M\,U_\Lambda\,|\,\tilde{\Psi}_m\rangle .$$

We want to show that their conclusion is unjustified[6], and the above transformation cannot prevent spurious effects. Indeed eq. (6) which defines $\tilde{\Psi}^{\Lambda}_{m}$ is meaningless for $\Lambda\to\infty$, and (as G.S. advise), in the calculations $\tilde{\Psi}^{\Lambda}_{m}$ must be used with finite Λ, and the limit $\Lambda\to\infty$ must be taken in the final results. But if this is done, one obtains for the matrix element of any intrinsic operator M_I exactely the same result as if one uses simply the wave functions $\tilde{\Psi}_m$, i.e.

$$\langle\tilde{\Psi}_m\,|\,M_I\,|\,\tilde{\Psi}_m\rangle = \sum_{\nu\nu'\mu}\left(C^{(m)}_{\nu'\mu}\right)^{*} C^{(m)}_{\nu\mu}\,\langle\Psi_{I\nu'}\,|\,M_I\,|\,\Psi_{I\nu}\rangle, \quad (8)$$

both with the direct and inverse G.S. transformation. In the case of the inverse transformation the result is trivial, because U_Λ commutes with M_I for each value of Λ, so that

$$\lim_{\Lambda\to\infty} \langle \tilde{\Psi}^{\wedge}_m | M_I | \tilde{\Psi}^{\wedge}_m \rangle = \lim_{\Lambda\to\infty} \langle \tilde{\Psi}_m | U^{+}_{\Lambda} M_I U_{\Lambda} | \tilde{\Psi}_m \rangle = \langle \tilde{\Psi}_m | M_I | \tilde{\Psi}_m \rangle .$$

For the direct transformation let us use the expansion (5) for the $\tilde{\Psi}_m$

$$U_{\Lambda} \tilde{\Psi}_m = \sum_{\nu\mu} C^{(m)}_{\nu\mu} \bar{\Psi}_{I\nu} U_{\Lambda} \chi_{\mu} = \sum_{\nu\mu} C^{(m)}_{\nu\mu} \bar{\Psi}_{I\nu} \chi^{\wedge}_{\mu} ,$$

where $\chi^{\wedge}_{\mu}$ are the harmonic oscillator wave functions with parameter $e^{\wedge} R_0$, if the χ_{μ} have parameter R_0 . So

$$\lim_{\Lambda\to\infty} \langle \tilde{\Psi}^{\wedge}_m | M_I | \tilde{\Psi}^{\wedge}_m \rangle = \lim_{\Lambda\to\infty} \sum_{\nu\nu'\mu\mu'} \left(C^{(m)}_{\nu\mu} \right)^{*} C^{(m)}_{\nu'\mu'} \cdot$$

$$\cdot \langle \bar{\Psi}_{I\nu} | M_I | \bar{\Psi}_{I\nu'} \rangle \langle \chi^{\wedge}_{\mu} | \chi^{\wedge}_{\mu'} \rangle = \sum_{\nu\nu'\mu} \left(C^{(m)}_{\nu\mu} \right)^{*} C^{(m)}_{\nu'\mu} \langle \bar{\Psi}_{I\nu} | M_I | \bar{\Psi}_{I\nu'} \rangle ,$$

which completes the proof of our statement.

The G.S. transformation oan be used and has been used [7] in order to obtain the definition of intrinsic operators, a task which is not trivial if relativistic effects are taken into acoount. However, as we have shown, it cannot eliminate spurious components in the wave functions, while a correct calculation must be done using <u>both</u> intrinsic operators and intrinsic wave functions.

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References and footnotes

(1) There are only trivial changes in what follows if the plane wave is replaced by any superposition of plane waves.

(2) J.P.Elliott and T.H.Skyrme, Proc.Roy.Soc., A232, 561(1955).
F.Palumbo, Nucl.Phys., A99, 100 (1967)
F.Palumbo, JINR Preprint E4-5890, Dubna (1971)

(3) L.J.Tassie and F.C.Barker, Phys.Rev., 111, 940 (1958)

(4) F.Palumbo and D.Prosperi, Nucl.Phys., A115, 296 (1968)

(5) S.Gartenhaus and C.Schwartz, Phys.Rev., 108, 482 (1957)

(6) This fact has been already noted in the second of refs.(2)

(7) F.E.Close and L.A.Copley, Oxford preprint 9/70.

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