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Introduction

The experimental confirmation for the collective nature of the nuclear excitation (giant resonance) in muon capture gave the general basis for description of the various aspects of muon-nucleus processes[1]. However, the contribution of such a mechanism to the high energy region of the neutron spectrum is unimportant, while this region claims the particular attention[2,3]. The high energy neutrons in muon capture ($E_n > 15-20$ MeV) appear to relate to a direct emission mechanism. The successive calculations of muon capture in the framework of the direct mechanism are complicated because of the complex nature of both the muon-nucleon Hamiltonian and the final state nuclear wave function. In particular recently it was shown that the relativistic terms of muon-nucleon Hamiltonian are very important in considering the high energy neutrons[4-6]. Therefore it is important to analyse the effects of the various factors on muon capture process with neutron emission.

In the present paper we continue such investigations started earlier[4,5]. We shall discuss some effects of the final state interaction together with the relativistic terms of Hamiltonian on the characteristics of the process with neutron emission

$$\mu^- + (A,Z) \to (A-1, Z-1) + n + \nu . \tag{1}$$

We will concentrate our attention again on the investigations of the general features of the process (1). All calculations are performed for the reaction

$$\mu^- + {}^4He \to {}^3H + n + \nu . \tag{2}$$

It should be noted that in the case of the reaction (2) one can hope that the optical model will describe roughly the region of giant resonance, too, because all the single-particle levels excited in ${}^4He$ are unbound. As to the neutron characteristics of heavier nuclei, one can expect that in the case of the nuclei with closed shells the factors considered will effect similarly.

The General Statement

We use the muon-nucleon Hamiltonian as given by Primakoff[7]

$$H_{eff} = \frac{1}{2}\tau^{(+)}(1 - \vec{\sigma}\vec{\nu}) \sum_{i=1}^{A} \tau_i^{(-)} [G_V\, 1 \cdot 1_i + G_A\, \vec{\sigma}\, \vec{\sigma}_i + G_P\, \vec{\sigma}_i\, \vec{\nu} + \tag{3}$$

$$+ g_V\, \vec{\sigma}\, \frac{\vec{p}_i}{M_N} + g_A\, \vec{\sigma}_i\, \frac{\vec{p}_i}{M_N}]\, \delta(\vec{r} - \vec{r}_i),$$

where $\vec{\nu}$ is the unit vector of neutrino,

$$G_V = g_V\left(1+\frac{E_\nu}{2M_N}\right),\ G_A = g_A - (g_V+g_M)\frac{E_\nu}{2M_N},\ G_P = [(g_P-g_A)-(g_V+g_M)]\frac{E_\nu}{2M_N}$$

and E_ν is its energy. The last two terms in (3) are called relativistic or velocity terms.

The expression for muon capture rate can be written as follows

$$dW = \pi\delta(E_i - E_f)\{\Sigma_1+\Sigma_2+\Sigma_3\}\frac{d\vec{p}\,d\vec{\mathcal{P}}\,d\vec{P}_\nu}{(2\pi)^3}, \tag{4}$$

where E_i and E_f are the energy of initial and final states of the nuclear system respectively, $\vec{p}$ and $\vec{\mathcal{P}}$ are relative and c.m. momenta of neutron and the residual nucleus respectively, and $\vec{p}_\nu$ - neutrino momentum. The quantities Σ_i are equal to

$$\Sigma_1 = G_V^2|M_1|^2 + G_A^2|\vec{M}_2|^2 + G_P(G_P-2G_A)|\vec{\nu}\vec{M}_2|^2 - G_V^2(\vec{\mu}\vec{\nu})|M_1|^2 + G_A^2(\vec{\mu}\vec{\nu})|\vec{M}_2|^2 -$$
$$- G_P^2(\vec{\mu}\vec{\nu})|\vec{\nu}\vec{M}_2|^2 - 2Re\{G_VG_A i[\vec{\mu}\times\vec{\nu}]\vec{M}_2\overset{*}{M}_1 - G_A(G_P-G_A)(\vec{\mu}\vec{M}_2)(\vec{\nu}\vec{M}_2)\}, \tag{5a}$$

$$\Sigma_2 = \frac{2}{M_N}Re\{-G_V g_V(\vec{\nu}\vec{M}_3)\overset{*}{M}_1 + G_A g_V i\vec{\nu}[\vec{M}_2^*\times\vec{M}_3] + (G_P-G_A)g_A(\vec{\nu}\vec{M}_2)\overset{*}{M}_4 +$$
$$+ G_V g_V(\vec{\mu}\vec{M}_3)\overset{*}{M}_1 + G_A g_V i\vec{\mu}[\overset{*}{\vec{M}}_2\times\vec{M}_3] + G_A g_A(\vec{\mu}\vec{M}_2)\overset{*}{M}_4 - G_P g_V([\vec{\mu}\times\vec{\nu}]\vec{M}_3)(\vec{\nu}\overset{*}{\vec{M}}_2) -$$
$$- G_P g_A(\vec{\nu}\overset{*}{\vec{M}}_2)(\vec{\mu}\vec{\nu})M_4\}, \tag{5b}$$

$$\Sigma_3 = \frac{1}{M_N^2}\{g_V^2|\vec{M}_3|^2 + g_A^2|M_4|^2 + g_V^2(\vec{\mu}\vec{\nu})|\vec{M}_3|^2 - g_A^2(\vec{\mu}\vec{\nu})|M_4|^2 \tag{5c}$$
$$-2g_V^2 Re(\vec{\mu}\overset{*}{\vec{M}}_3)(\vec{\nu}\vec{M}_3) + 2g_Vg_A Re\, i([\vec{\mu}\times\vec{\nu}]\overset{*}{\vec{M}}_3)M_4\},$$

where $\vec{\mu}$ is the unit vector in the direction of muon spin, $M_i (i = 1, ...$ $...4)$ are the matrix elements, defined as

$$M_i = \langle \Psi_f | \sum_{k=1}^{A} \tau_k^{(-)} e^{-i\vec{p}_\nu \vec{r}_k} \quad O_i \, \phi_\mu (r_k) | \Psi_i \rangle . \qquad (6)$$

Here Ψ_i and Ψ_f are the nuclear wave functions of the initial and final states respectively,

$$\begin{aligned}
O_1 &= I_k = O_r(0,0) O_\sigma(0,0); \\
O_2 &= \sigma_k = O_r(0,0) O_\sigma(1,m); \; m = \pm 1, 0; \\
O_3 &= p_k = O_r(1,m) O_\sigma(0,0); \; m = \pm 1, 0; \qquad (7) \\
O_4 &= \vec{\sigma}_k \vec{p}_k = \Sigma (-1)^m O_r(1,m) O_\sigma(1,-m); \; m = \pm 1, 0;
\end{aligned}$$

The matrix elements (6) are defined in the same way as in the paper /3/. However, unlike to /3/ we have kept in (4) the term Σ_3. It should be noted that Σ_3 is proportional to M_N^{-2}. To take accurately into account in (4) all terms of the order M_N^{-2} it is necessary to keep them also in Hamiltonian (3). Otherwise we neglect all the terms of the p_l^2/M_N^2 type and partly of the order E_ν^2/M_M^2. It is clear that the contribution of the terms, proportional to E_ν^2/M_N^2 is not greater than 10% compared with that of terms, proportional to E_ν/M_N. When the energy of neutrons increases their contribution decreases. Consequently, it is not necessary to take into account in (4) all the terms of the order E_ν^2/M_N^2 because their contribution in high energy region of the spectrum is unimportant. We consider now the terms of the p_l^2/M_N^2 type in detail. Firstly let us expand the plane wave of neutrino (as in beta decay)

$$e^{-i\vec{p}_\nu \vec{r}} = 1 - i\,\vec{p}_\nu \vec{r} + \ldots \qquad (8)$$

The various terms in (8) correspond to the allowed, first forbidden and so on, transitions. Let us confine ourselves to the case when the main contribution to muon capture comes from the first forbidden transitions (when neglecting the relativistic terms). Such a situation arises in a double magic nuclei and, in particular, in the case of muon capture by ^4He. As to the relativistic terms they correspond in this case effectively to the allowed transitions. This means that their contribution could be of the same order of magnitude as that of the nonrelativistic terms. At the same time the contribution of the terms, proportional to p_i^2/M_N^2 corresponds at least to the second forbidden transitions. Therefore these can be neglected. On the other hand, it is inconsistent to neglect in (4) all the terms of the order M_N^{-2}. The latter would mean that the contribution of the relativistic terms to the capture rate is small compared with that of the nonrelativistic terms (it should be noted that p_i^2/M_N^2 and $\langle|p_i/M_N|\rangle\langle|p_i/M_N|\rangle$ have the different meaning).

One can introduce now the matrix elements $\mathfrak{M}_i$, related to M_i by the expression:

$$M_i = (2\pi)^3\,\delta(\vec{P}_a - \vec{\mathcal{P}} - \vec{p}_\nu)\,\mathfrak{M}_i \qquad (9)$$

where $\vec{P}_a$ is the momentum of the target nucleus. In the c.m. system $\vec{P}_a = 0$. Taking into account (9) one can integrate the expression (4) over the c.m. coordinate and neutrino energy. As a result we get the following expression:

$$\frac{dW}{dE\, d\Omega\, d\Omega_\nu} = \frac{m^{3/2}\, 2^{1/2}\, M^2}{64\pi^5} \cdot \frac{f^2(E)}{[1+f(E)]} \{ \tilde{\tilde{\Sigma}}_1 + \tilde{\tilde{\Sigma}}_2 + \tilde{\tilde{\Sigma}}_3 \} \qquad (10)$$

where m is the reduced mass of neutron and the residual nucleus, E - their relative energy, $M = M(n) + M(A-1, Z-1)$ is the total mass of neutron and the recoil nucleus, Ω and Ω_ν are the directions of the vectors $\vec{p}$ and $\vec{p}_\nu$ respectively,

$$f(E) = \left(1 + 2\,\frac{\epsilon - E}{M}\right)^{1/2} - 1$$

and

$$\epsilon = M(\mu^-) + M(A, Z) - M(A-1, Z-1) - M(n).$$

Replacing M_i by $\mathfrak{M}_i$ in Σ_i one gets $\tilde{\tilde{\Sigma}}_i$ at the energy $E_\nu = M_i(E)$.

The nuclear final state wave function will be chosen in the following form:

$$\Psi_f(1 \ldots A) = 4\pi \hat{A} \sum_{j\ell} i^\ell\, b_{j\ell}(r) \sum \langle \ell\, m_\ell\, 1/2\, m_s : jM \rangle \langle \ell\, \bar{m}_\ell\, 1/2\, \bar{m} : jM \rangle \cdot$$

$$\cdot\, Y_{\ell m_\ell}(\hat{\vec{r}})\, Y^*_{\ell \bar{m}_\ell}(\vec{n})\, \Psi_{J_0 M_0, T_0 t_0}(1 \ldots A-1)\, \chi_{1/2 m_s}, \qquad (11)$$

where $b_{j\ell}(r)$ is the radial wave function of the neutron and residual nucleus relative motion. It is specified by the angular momentum ℓ of the relative motion and by the total momentum j; $\vec{n}$ is a unit vector $\frac{\vec{p}}{|\vec{p}|}$; $\Psi_{J_0 M_0, T_0 t_0}(1 \ldots A-1)$ is the wave function of the residual nucleus with total momentum J_0, its projection M_0, isospin T_0 and its projection t_0; $\chi_{1/2 m_s}(A)$ is the spin-isospin function of the neutron with m_s being the projection of the spin; $\hat{A}$ is the antisymmetrization operator.

In calculating the nuclear matrix elements (6) the exchange terms are taken into account, too. The muon wave function in K -orbit is taken at $r=0$.

When the final state interaction is neglected $b_{i\ell}(r)$ reduces to the spherical Bessel function $j_\ell(r)$. But if the final state interaction is involved $b_{i\ell}(r)$ can be found by solving the Shroedinger equation with the appropriate potential.

Integrating (10) over the direction of neutrino emission one gets the final expression for $\frac{dW}{dE\,d\Omega}$:

$$\frac{dW}{dE\,d\Omega} = CE^{1/2} \frac{f^2(E)}{[1+f(E)]} \{ \sum_{i=1}^{3} A_i(E) + \sum_{i=1}^{3} B_i(E)\cos\theta \}, \qquad (12)$$

where C is a constant, θ is the angle between muon polarization vector and vector $\vec{p}$.

The energy spectrum $\frac{dW}{dE}$, asymmetry $a(E)$ of angular distribution and capture rate are defined as follows:

$$\frac{dW}{dE} = \int d\Omega \frac{dW}{dE\,d\Omega}, \quad a(E) = \sum_{i=1}^{3} B_i(E) \Big/ \sum_{i=1}^{3} A_i(E), \quad W = \int_0^{E_{max}} dE \frac{dW}{dE}. \qquad (13)$$

It should be noted that $\frac{dW}{dE}$ depends on the relative energy E and describes sufficiently well the neutron spectrum when the residual nucleus mass is large. One can get the neutron spectrum and asymmetry, knowing $\frac{dW}{dE\,d\Omega\,d\Omega_\nu}$ and carrying out some calculations, for example, by the Monte-Carlo method.

Muon Capture by ^4He

To calculate the capture rate for the process (2) one needs to know the wave functions of ^4He and ^3H. In the present paper we confine ourselves to the principal S state.

As was indicated in the paper/8/ the muon capture rate depends weakly on the explicit form of the radial wave function, but significantly on the nuclear r.m.s. radii. We have used the Gaussian form for the radial function

$$\Phi(^4He) = N(^4He)\exp\{-\alpha\sum_{l=1}^{4}(\vec{r}_l - \vec{r}_l)^2\}, \qquad (14)$$

$$\Phi(^3H) = N(^3H)\exp\{-\beta\sum_{l=1}^{3}(\vec{r}_l - \vec{r}_l)^2\}, \qquad (15)$$

where α and β are defined from r.m.s. radii of ^4He and ^3H respectively, $N(^4He)$ and $N(^3H)$ are the normalization constants.

To get M_l one needs to calculate the radial integrals of four types. Their number reduces to two when neglecting the exchange terms.

The final state interaction was taken into account by using the optical potential with surface and volume absorption/9/. Its parameters were chosen to describe the angular distribution of protons on ^3He. (The nuclear system $p + {}^3He$ has the same isospin as the $n + {}^3H$ one). The potential is fitted at the energy E_p=23.5MeV of relative motion. As the potential has been fitted at sufficiently high energies of the incident proton and does not depend on energy, it could be expected that we have overestimated the absorption effect in low-energy region. As to the high energy region $E > E_p$, the underestimation of the absorption will be insignificant, because the new inelastic channels will not be open. So, this optical potential describes better the high energy part of the spectrum than the low-energy one.

The radial wave function was found numerically by solving the Shroedinger equation.

Discussion

The expressions (12) and (13) were used for calculation of the energy spectrum and asymmetry coefficient. The energy spectrum is given on Fig. 1. The final state interaction increases the low energy part and decreases the higher one. Indeed, in the plane wave approximation the neutron and ^3H yield at the relative energy higher than 20MeV is about 30%, whereas when the final state interaction was taken into account this magnitude becomes smaller (about 12%). The integral capture rate in the reaction (2) appears to be the same both in the plane wave approximation and in including final state interaction. This means, that the accurate consideration of the final state interaction (when the energy dependence of the potential is taken into account) will increase the capture rate compared with the plane wave case.

As one would expect the contribution of the relativistic terms of Hamiltonian (3) is important (compare the curves 1 and 3) and is about 1/3 of the total capture rate with one neutron emission. The contribution of the relativistic terms significantly increases when the relative energy E becomes larger (Table 1). Fig. 2 shows the asymmetry coefficient versus energy E. As can be seen the asymmetry essentially is due to the relativistic terms of (3). This means particularly that all conclusions, based on the assumption that the asymmetry coefficient can be represented in factorizable form

$$a(E) = a_H(E)\,\beta_N(E), \tag{16}$$

where $a_H(E)$ is the asymmetry on hydrogen, and $\beta_N(E)$ is the nuclear factor, with $|\beta_N(E)| < 1$, are wrong. Indeed, one can get the expression (16) when neglecting both the relativistic terms in (3) and spin-orbit part of the final state interaction. The fact, that the relativistic terms can change the asymmetry coefficient was mentioned already in the paper /10/, where all estimations of the nuclear matrix elements were made in the framework of Migdal model. An importance of the contribution of the relativistic terms in a direct process of neutron emission was pointed out in papers /2-6/.

The present result is in an agreement with the previous ones. The final state interaction changes significantly the asymmetry coefficient. The change is very essential at high energies (curves 1 and 3). At some energies

$$|a(E)| > |a_H^{max}| = 0,4 .$$

At the present time only the total capture rate in ^4He is measured and its value is equal to 364 ± 46 sec^{-1} /11/. We get for the channel (1) $\Lambda = 220$ sec^{-1} when relativistic terms and the final state interaction are taken into account. As was mentioned above the calculated value is somewhat underestimated.

To investigate the dependence of the asymmetry coefficient on the imaginary part of the potential we have calculated the above characteristics of the process (2) neglecting the absorption. The results are given in Figs 3 and 4. Comparing this result with that where absorption is taken into account one concludes that qualitatively the asymmetry coefficient remains unchanged. Consequently, the accurate consideration of the final state interaction will increase the total capture rate compared with the value obtained. As to the asymmetry coefficient it will change only slightly.

From an analysis of the experimental data it follows that in the neutron-tritium scattering the spin-orbital interaction is extremely small. This does not hold, in general, for heavier nuclei. For this reason we have calculated the spectrum and asymmetry for our process by using the potential with an arbitrary strong spin-orbital term. One must remember, of course, the purely qualitative character of such a study since the spin-orbital interaction introduced does not depend on energy. The results of the above calculations are presented on Figs 3 and 4.

All our results are given for the pseudoscalar constant $g_P = 8 g_A$. However this constant has the energy dependence of the form /12/

$$g_P = g_A \frac{2m_\mu M_N}{m_\pi^2 + q^2}, \qquad (17)$$

where m_μ and m_π are muon and pion masses respectively, q - transferred four-momentum. Such a dependence means that g_P increases from $7g_A$ as $E_\nu \approx E_{max}$ to $\approx 30 g_A$ at $E_\nu \approx 0$ .The straightforward calculations show, however, that both the energy spectrum and asymmetry coefficient change only slightly when $g_P = 8g_A$ is replaced by (17). The asymmetry depends slightly, in general, on pseudoscalar constant.

Conclusion

The present results show that the energy spectrum and asymmetry coefficient at high energies are very sensitive both to the details of muon-nucleon Hamiltonian (the relativistic terms of Hamiltonian) and to the nuclear problem (final state interaction). Therefore in interpreting the experimental data one needs to be careful. Of course some additional calculations are required. Of some interest are the investigations of the short range correlations in nuclei and the other components of the ground state wave function. At the same time accurate experimental data are necessary. The present data do not provide the unambiguous picture /13-15/.

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Table 1

Energy of the relative motion, MeV	20	40	60	70
Contribution of the non-relativistic terms to the energy spectrum, (in %)	71	54	26	18

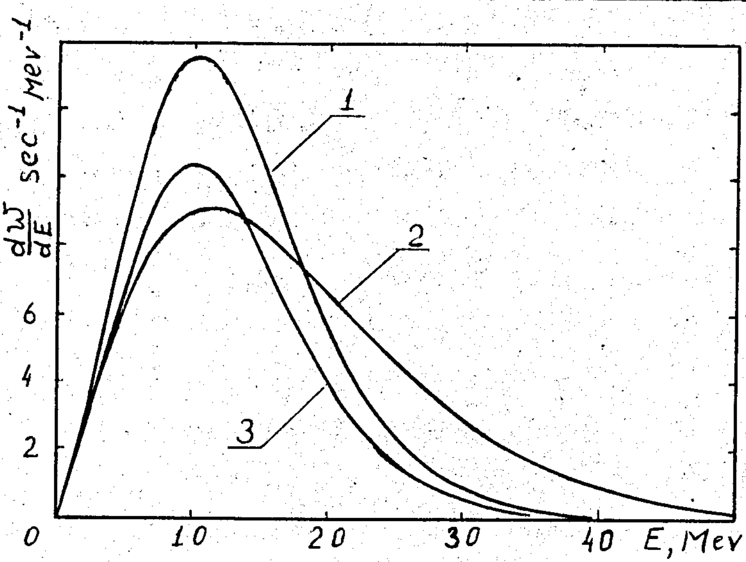

Fig. 1. The energy spectrum versus relative energy E. 1. The relativistic terms and final state interaction are included. 2. Plane wave approximation with relativistic terms. 3. The relativistic terms are neglected, the final state interaction is included.

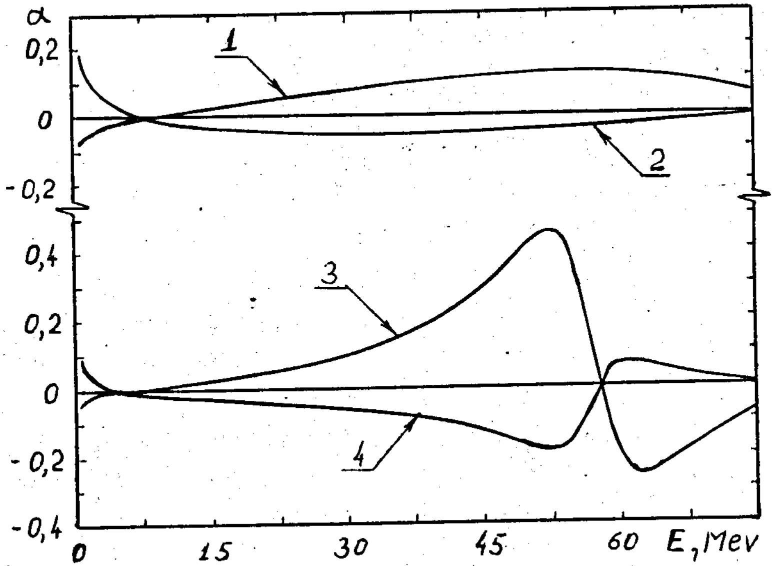

Fig. 2. Asymmetry of angular distribution versus relative energy. 1. Plane wave approximation with relativistic terms. 2. Plane wave approximation without relativistic terms. 3. The relativistic terms and final state interaction are included. 4. The relativistic terms are neglected, final state interaction is included.

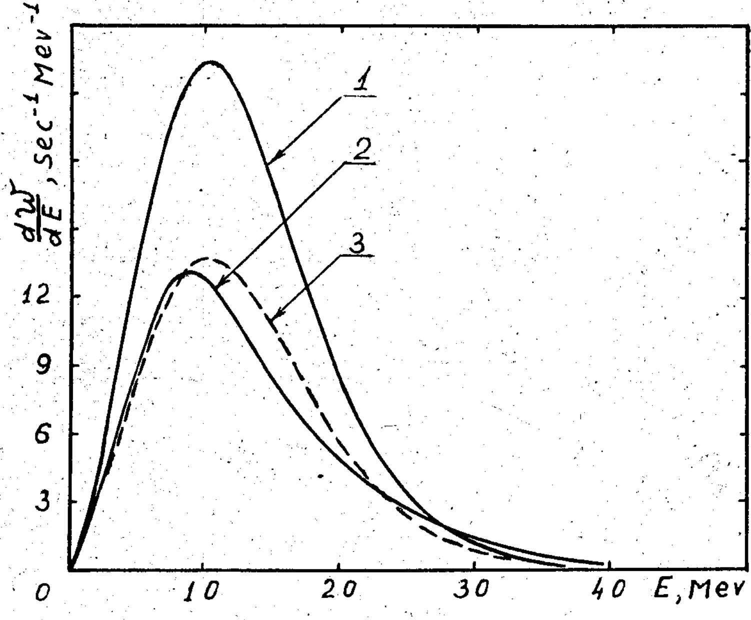

Fig. 3. The energy spectrum versus relative energy E. The optical potential is taken: 1. Without absorption. 2. With strong spin-orbital part. 3. From the experimental data.

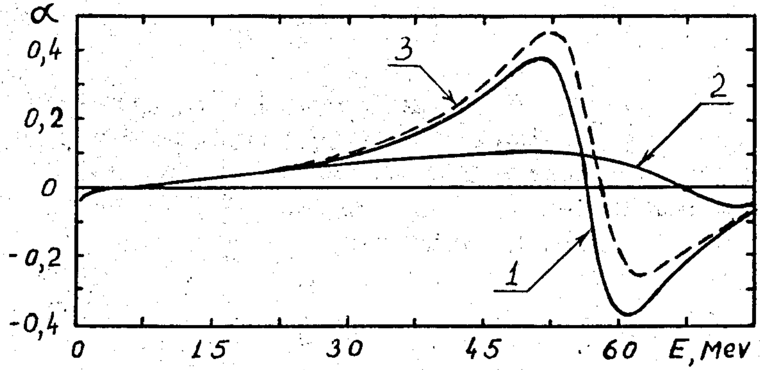

Fig. 4. Asymmetry of angular distribution versus relative energy E. The captures are the same as in Fig. 3.