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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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ULTRASONIC ATTENUATION IN DIELECTRIC CRYSTALS IN SECOND SOUND REGION

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ULTRASONIC ATTENUATION IN DIELECTRIC CRYSTALS IN SECOND SOUND REGION

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Поглощение звука диэлектрическим кристаллом в области существования второго звука

Выводится выражение для коэффициента поглощения звука диэлектрическим кристаллом с помощью метода статистического неравновесного оператора Д.Н. Зубарева. В области существования второго звука получено резонансное поглощение энергии падающей волны, что находится в согласии с результатами, полученными с помощью уравнения Больцмана.

Препринт Объединенного института ядерных исследований.
Дубна, 1971

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Ultrasonic Attenuation in Dielectric Crystals in Second Sound Region

A theory of the damping of ultrasonic waves due to the three-phonon process is devaloped by means of the D.N. Zubarev method of non-equilibrium statistical operator. In temperature and frequency ranges where second sound can exist the possibility of resonance between the first and second sound occurs in agreement with results of Guyer.

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1. Introduction

An attenuation of a sound (first sound) is usually considered in the framework of a linear response theory (LRT) /1-4/, which makes it possible to calculate the oscillation amplitude $\langle \vec{u} \rangle$ of ions caused by an external perturbation provided the wave length is much greater than lattice constant and with frequency ω. The system response determines the imaginary part of wave-vector which is equal to the attenuation coefficient. The lifetime of accoustic phonons $\tau(\omega)$ can be expressed by the imaginary part of the mass-operator of the retarted Green function $\langle\langle \dot{\vec{u}}(\vec{r}t) ; \vec{u}(\vec{r}',0) \rangle\rangle$ which is connected with a response of a system. The relation between the attenuation coefficient of a sound $a(\omega)$ and the lifetime of phonons is as follows

$$a(\omega) = \frac{1}{\tau(\omega) v}, \tag{1}$$

where v is the sound velocity.

In the hydrodynamic region, where the wave frequency is much smaller than the phonon collisions frequency, the local equilibrium can be established. For such a case the Boltzmann equation for the phonon gas can be written [5]. The solution of this equation makes it possible to calculate the sound attenuation coefficient. Results obtained in such a way are in agreement with those from the LRT for the hydrodynamic region. With the help of the Boltzmann eq. for the phonon gas [5] Guyer obtained the resonance attenuation of first sound in the second sound region [6].

However, another approach to this problem is possible. Consider the model proposed by Maris [7]. Let us assume that the external sound wave changes the average number of occupation of the accoustic phonons (corresponding to the wave) in comparison with the equilibrium state, whereas all other phonons (thermal phonons) are in equilibrium. Non-equilibrium accoustic phonons can reach an equilibrium state again by a weak interaction with thermal phonons. Such a formulation is equivalent to a typical relaxation problem such as a system in a thermostate. This problem can be solved using the non-equilibrium statistical operator (NSO) method [8,9].

The probability of the accoustic phonon transition and, therefore, the lifetime have been calculated in [7]. In the present paper the lifetime is derived using the kinetic equation, and the obtained formula has the same form as in [7]. The attenuation coefficient can be found from (1). Both, the NSO method and the LRT method give the same results for the attenuation coefficient. In the second sound region the formula for the attenuation coefficient for the Maris model is derived by the NSO method and the form of this formula is the same as that obtained by Guyer [6].

Note that our treatment of the Maris model by the NSO method is equivalent to the replacement of the mechanical perturbation by the thermal one.

2. The Model and Attenuation Coefficient

Let us assume that the external incoming wave with the frequency ω , wave vector $\vec{q}$, polarization and mode index j causes the non-equilibrium occupation $\langle n_Q \rangle^t$ of a state $Q=(\vec{q},j)$. Other phonon states are in equilibrium. Let us denote by v the interaction between phonons of state Q and other states.

$$v = a_Q^+ H_Q^+ + a_Q H_Q \ , \tag{2}$$

where

$$H_Q = 3 \sum_{Q_1, Q_2 \neq Q} V(Q_1, Q_2, Q) A_{Q_1} A_{Q_2} \ ,$$

$$A_Q = A_Q^+ , \quad A_Q = a_Q + a_{\bar{Q}}^+ , \quad V(\bar{Q}_1 \bar{Q}_2 \bar{Q}) = V^*(Q_1, Q_2, Q),$$

$\bar{Q} = (-\vec{q}, j)$, $V(Q_1, Q_2, Q)$ is symmetric function of Q_1 , Q_2 , Q_3 . The Hamiltonian of a system has the form

$$H = \omega n_Q + v + H_{ph} \ , \tag{3}$$

where $\hbar = 1$,

$$H_{ph} = \sum_{Q_1 \neq Q} \omega_{Q_1} (n_{Q_1} + \frac{1}{2}) + \sum_{Q_1, Q_2, Q_3 \neq Q} V(Q_1, Q_2, Q_3) A_{Q_1} A_{Q_2} A_{Q_3} \ ,$$

$n_Q = a_Q^+ a_Q$ – is the occupation number operator. Such a Hamiltonian has been used by Opie /10/ to obtain the lifetime and the phonon level shift in an anharmonic crystal.

We shall obtain now the kinetic equation for a system with Hamiltonian (3) using the NSO method /11/. Introduce a quasi-equailibrium statistical operator

$$\rho_\ell (t) = \exp \{ - S(t,0) \} , \tag{4}$$

where

$$S(t,0) = \Omega(t) - f(t) n_Q(0) + \beta H_{ph}(0) \tag{5}$$

is the entropy operator, f is thermodynamic parameter corresponding to n_Q and β is inverse temperature of a medium. All operators are in the Heisenberg picture. The non-equilibrium statistical operator is taken in the form /11/

$$\rho(t) = \exp\{-\widetilde{S}(t,0)\}, \tag{6}$$

where

$$\widetilde{S}(t,0) = \epsilon \int_{-\infty}^{0} dt_1 \, e^{\epsilon t_1} (\Omega(t+t_1) + f(t+t_1) n_Q(t_1) + \beta H_{ph}(t_1)), \quad \epsilon > 0$$

is a quasi-invariant part of the entropy operator. After taking the thermodynamic limit, the next step is to take $\epsilon \to 0$. The parameter $f(t)$ is determined from the condition

$$\langle n_Q \rangle^t = \langle n_Q \rangle_\ell^t ,$$

where

$$\langle \ldots \rangle^t = Tr(\ldots \rho(t)), \quad \langle \ldots \rangle_\ell^t = Tr(\ldots \rho_\ell(t)).$$

Since the following relations take place

$$[H_{ph}, n_Q] = 0 \qquad \langle a_Q \rangle^t = \langle a_Q^+ \rangle^t = 0$$

it follows from /8/ that

$$\rho(t) = Q^{-1} \exp\{-f(t) n_Q - \beta H_{ph} -$$
$$- i \int_{-\infty}^{0} dt_1 \, e^{\epsilon t_1} (f(t+t_1)[n_Q(t_1), V(t_1)] + \beta [H_{ph}(t_1), V(t_1)])\}.$$

The expansion of $\rho(t)$ into a series in a weak interaction gives in a linear approximation

$$\rho(t) = \rho_\ell(t) - i \int_{-\infty}^{0} dt_1 \, e^{\epsilon t_1} [V(t_1), \rho_\ell]. \tag{7}$$

The averaging of the equation of motion for an operator n_Q with $\rho(t)$ in a form as given by (7), enables us to obtain the required kinetic equation

$$\frac{\partial \langle n_Q \rangle^t}{\partial t} = -\int_{-\infty}^{0} dt_1 \, e^{\epsilon t_1} \langle [[n_Q, v], v(t_1)] \rangle_\ell^t \equiv I_C(\langle n_Q \rangle^t). \qquad (8)$$

The calculating of the double commutator gives the collision integral

$$I_C(\langle n_Q \rangle^t) = -\frac{1}{2} \operatorname{Re} \int_{-\infty}^{0} dt_1 \, e^{\epsilon t_1} e^{i\omega t_1} \left(\langle n_Q \rangle^t \langle H_Q^+(t_1) H_Q \rangle - (1 + \langle n_Q \rangle^t) \langle H_Q H_Q^+(t_1) \rangle \right),$$

where $\langle \ldots \rangle = Tr(\ldots Z^{-1} e^{-\beta H_{ph}})$, $Z = Tr(e^{-\beta H_{ph}})$.

This collision integral determines the lifetime of accoustic phonons

$$\frac{1}{\tau_Q(\omega)} = -\frac{\delta I_C(\langle n_Q \rangle^t)}{\delta \langle n_Q \rangle^t} = \frac{1}{2} \operatorname{Re} \int_{-\infty}^{0} dt_1 e^{(\epsilon + i\omega) t_1} \left(\langle H_Q^+(t_1) H_Q \rangle - \langle H_Q H_Q^+(t_1) \rangle \right). \qquad (9)$$

It is seen from (9) that to obtain the lifetime $\tau_Q(\omega)$ it is necessary to calculate the equilibrium correlation functions

$$\langle H_Q^+(t_1) H_Q \rangle, \quad \langle H_Q H_Q^+(t_1) \rangle.$$

We shall show now that the attenuation coefficient a is connected with the lifetime $\tau_Q(\omega)$. A decrase of a number of phonons in the state Q causes the energy of an accoustic wave to be transferred to a thermostate

$$-\frac{dw}{dt} = \omega_Q I_C(\langle n_Q \rangle^t).$$

For the attenuation coefficient we have

$$a = \frac{-\frac{dw}{dt}}{v w} = \frac{1}{v \, \tau_Q(\omega)},$$

where $w = \langle n_Q \rangle^t \omega_Q$ is a density of the accoustic wave energy.

3. The Relaxation Time

As it follows from (9) the obtaining of the relaxation time is related to a calculation of equilibrium correlation functions.

Let us consider the Bravais lattice for which the following relation holds:

$$V(Q_1, Q_2, 0) = 0. \tag{11}$$

The neglection of U -processes gives

$$\langle A_{Q_1} A^{+}_{Q_2} \rangle = \langle A_{Q_1} A^{+}_{Q_1} \rangle \delta_{Q_1, Q_2}. \tag{12}$$

Let us factorize the correlation functions defining $\tau_Q(\omega)$. From (11) and (12) it follows now

$$\langle H^{+}_{Q}(t_1) H_Q \rangle = 18 \sum_{Q_1, Q_2} |V(\bar{Q}, Q_1, Q_2)|^2 \langle A_{Q_1}(t_1) A^{+}_{Q_1} \rangle \langle A_{Q_2}(t_1) A^{+}_{Q_2} \rangle, \tag{13a}$$

$$\langle H_Q H^{+}_Q(t) \rangle = 18 \sum_{Q_1, Q_2} |V(\bar{Q}, Q_1, Q_2)|^2 \langle A^{+}_{Q_1} A_{Q_1}(t_1) \rangle \langle A^{+}_{Q_1} A_{Q_2}(t_1) \rangle. \tag{13b}$$

Correlation functions $\langle A^{+}_{Q_1} A_{Q_1}(t_1) \rangle$, $\langle A_{Q_1}(t_1) A^{+}_{Q_1} \rangle$ can be expressed in terms of the spectral densities in the following way:

$$\langle A^{+}_{Q_1}(t_1) A_{Q_1} \rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \, J_{Q_1}(\omega) n(\omega) e^{i\omega t_1}, \tag{14a}$$

$$\langle A_{Q_1} A^{+}_{Q_1}(t_1) \rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \, J_{Q_1}(\omega) n(\omega) e^{\beta\omega} e^{i\omega t_1}, \tag{14b}$$

where $n(\omega) = (\exp\beta\omega - 1)^{-1}$, $J_{Q_1}(\omega) = i(D_{Q_1}(\omega + i\epsilon) - D_{Q_1}(\omega - i\epsilon))$, and $D_{Q_1}(\omega) = \langle\langle A_{Q_1}; A^{+}_{Q_1} \rangle\rangle_\omega$ is Fourier transformate of the Green function.

Taking into account (11), (12), (13) and (14) leads to

$$\frac{1}{\tau_Q(\omega)} = 18\pi \sum_{Q_1, Q_2} |V(\bar{Q}, Q_1, Q_2)|^2 \int_{-\infty}^{\infty} \frac{d\omega_1}{2\pi} \frac{d\omega_2}{2\pi} J_{Q_1}(\omega_1) J_{Q_2}(\omega_2) \times \quad (15)$$

$$\times (1 + n(\omega_1) + n(\omega_2))\, \delta(\omega_1 + \omega_2 - \omega).$$

This expression corresponds to the formula obtained by Kwok [2].

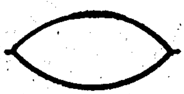
The calculation of the Green function [2,12,13] makes it possible to find the spectral density $J_Q(\omega)$. In the collisionless region $(\omega \tau_Q(\omega) \gg 1)$ the interactions among thermal phonons can be neglected and the lifetime can be obtained using the harmonic spectral density $J_Q^{(0)}(\omega)$ [2,4,7]. This leads to the famous Landau-Rumer formula [14]

$$a \sim \omega T^4 .$$

In the hydrodynamic region $(\omega \tau_Q(\omega) \ll 1)$ it is necessary to take into account the interactions of quasiparticles and the harmonic approximation is not valid. For this region the well known result of Akhiezer [15, 5] is obtained

$$a \sim \omega^2 T^{-1}$$

The dependence of a on the temperature and frequency is very sensitive to both a dispersion of phonons [2] and an aniso-tropy of interaction [4]. The following diagram



corresponds to correlation functions (13). In the collisionless region all lines of a diagram are free, and in the hydrodynamic region it is necessary to take into account corrections to the phonon lines. However, corrections to the vertex of a diagram are neglected. The second sound is completely missing in this approximation. To take into account the influence of the second sound on the first one the summation of the whole class of diagrams must be performed. If ω and $\vec{v}\vec{q}$ are of the order of a small quantity $|V|^2$ then all ladder diagrams

are of the order of 1 /16/. In order to find a sum of these ladder diagrams the results of /16,17/ are used.

Define the Green function of the imaginary time:

$$G(U) = \langle T(H_Q^+(U) H_Q) \rangle , \tag{16}$$

where

$$B(U) = e^{UH_{ph}} B e^{-UH_{ph}}$$

and T – is the imaginary time ordering operator.

Because of

$$G(U+\beta) = G(U) \qquad -\beta < U < 0$$

$G(U)$ can be expanded into the Fourier series

$$G(U) = \sum_{\ell=-\infty} G_\ell \, e^{i\omega_\ell U} \qquad \omega_\ell = \frac{2\pi i \ell}{\beta} \qquad \ell = 0, \pm 1, \pm 2, \ldots ,$$

where

$$G_\ell = \frac{1}{\beta} \int_0^\beta dU \, e^{-\frac{2\pi i \ell U}{\beta}} G(U) = \frac{1}{\beta} \sum_{m,n} \frac{e^{-\beta E_m}}{Z} |\langle m | H_Q^+ | n \rangle|^2 \frac{e^{\beta(E_m - E_n)} - 1}{(E_m - E_n) - i\omega_\ell} , \tag{17}$$

$|m\rangle, |n\rangle$ are eigenfunctions of H_{ph} and E_m, E_n are corresponding to them eigenvalues. Define the function $\mathcal{G}(z)$ of a complex variable z with the help of a condition

$$\mathcal{G}(Z) = G_\ell \qquad \text{when} \qquad z = \omega_\ell .$$

It is possible to obtain $\mathcal{G}(z)$ by the following substitution

$$i\omega_\ell \to z$$

in (17).

Note that

$$\lim_{\eta\to 0^+} \frac{\mathcal{G}(\omega+i\delta)-\mathcal{G}(\omega-i\delta)}{2\pi i} = \frac{1}{\beta}\sum_m \frac{e^{-\beta E_m}}{Z}\,|<m|H_Q^+|n>|^2(e^{\beta\omega}-1)\,\delta(E_m-E_n-\omega)= \tag{18}$$

$$\equiv \frac{1}{2\pi}\,\frac{e^{\beta\omega}-1}{\beta}\,J_{H_Q^+H_Q}(\omega).$$

Express now correlation functions $<H_Q^+(t_1)H_Q>$, $<H_QH_Q^+(t_1)>$ by a spectral density $J_{H_Q^+H_Q}(\omega)$

$$<H_Q^+(t_1)H_Q> = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{i\omega t_1}\, J_{H_Q^+H_Q}(\omega), \tag{19a}$$

$$<H_QH_Q^+(t_1)> = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{i\omega t_1}\, J_{H_Q^+H_Q}(\omega)\,e^{\beta\omega}\quad . \tag{19b}$$

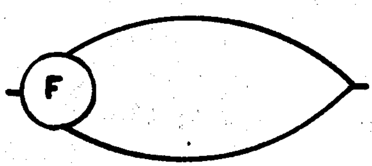
Use of (19) and (9) gives the following formula for the lifetime

$$\frac{1}{\tau_Q(\omega)} = \frac{1}{2}J_{H_Q^+H_Q}(-\omega)(1-e^{-\beta\omega}). \tag{20}$$

The function G_ℓ can be written in the form /17/

$$G_\ell = 9\sum_{Q_1,Q_2,\ell_1} V(\bar{Q},Q_1,Q_2)F_{Q_1Q_2Q}(i\omega_{\ell_1}, i\omega_\ell - i\omega_{\ell_1}). \tag{21}$$

This expression corresponds to a diagram



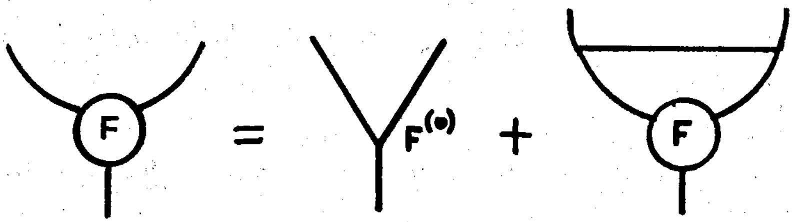
In the hydrodynamic region $F_{Q_1Q_2Q}(i\omega_{\ell_1}, i\omega_\ell - i\omega_{\ell_1})$ is a solution of the following integral equation

$$F_{Q_1Q_2Q}(i\omega_{\ell_1}, i\omega_\ell - i\omega_{\ell_1}) = F^{(0)}_{Q_1Q_2Q}(i\omega_{\ell_1}, i\omega_\ell - i\omega_{\ell_1}) + 36\beta^2 \sum_{Q_3,Q_4,Q_5,\ell_3} V(\bar{Q}_2, Q_4, Q_5)\times \tag{22}$$

$$\times V(\bar{Q}_2, Q_4, \bar{Q}_5)\times D_{Q_1}(i\omega_{\ell_1})D_{Q_2}(i\omega_\ell - i\omega_{\ell_1})D_{Q_5}(i\omega_{\ell_1} - i\omega_{\ell_3})\times$$

$$\times F_{Q_3Q_4Q}(i\omega_{\ell_3}, i\omega_\ell - i\omega_{\ell_3}).$$

This equation can be also presented on the picture



where

$$F^{(0)}_{Q_1Q_2Q}(i\omega_{\ell_1}, i\omega_\ell - i\omega_{\ell_1}) = 2V(Q,\bar{Q}_1,\bar{Q}_2)D_{Q_1}(i\omega_{\ell_1})D_{Q_2}(i\omega_\ell - i\omega_{\ell_1}).$$

here and elsewhere below $Q_1 = \frac{1}{2}Q + Q', Q_2 = \frac{1}{2}Q - Q', j_1 = j_2 = j'$.

An Ansatz for the solution of (22) is

$$F_{Q_1Q_2Q}(\omega' + i\eta; \omega - \omega' + i\eta) = X_{QQ'}(\omega,\omega')\frac{4\pi}{\beta^2}(\delta(\omega' - \omega_Q) - \delta(\omega' + \omega_{Q'})), \quad (23a)$$

$$F_{Q_1Q_2Q}(\omega' - i\eta, \omega - \omega' - i\eta) = U_{QQ'}(\omega,\omega')\frac{4\pi}{\beta^2}(\delta(\omega' - \omega_{Q'}) - \delta(\omega' + \omega_{Q'})) . \quad (23b)$$

For even and odd combination of solutions of (23) under transformation $\vec{q}' \to -\vec{q}'$

$$u^{(\pm)}_{Q'}(Q,\omega) = \frac{1}{2}(U_{QQ'}(\omega,\omega_{Q'}) \mp U_{QQ'}(\omega,-\omega_{Q'}))$$

$$x^{(\pm)}_{Q'}(Q,\omega) = \frac{1}{2}(X_{QQ'}(\omega,\omega_{Q'}) \mp X_{QQ'}(\omega,-\omega_{Q'}))$$

the Boltzmann-like equation is obtained.

$$-i\omega\begin{Bmatrix} x^{(+)}_{Q'}(Q,\omega) \\ x^{(-)}_{Q'}(Q,\omega)\end{Bmatrix} n_{Q'}(1+n_{Q'}) + i\vec{v}_{Q'}\vec{q}\begin{Bmatrix} x^{(-)}_{Q'}(Q,\omega) \\ x^{(+)}_{Q'}(Q,\omega)\end{Bmatrix} n_{Q'}(1+n_{Q'}) +$$

$$+\begin{Bmatrix} -V(Q,\bar{Q}_1,\bar{Q}_2) \\ 0 \end{Bmatrix} n_{Q'}(1+n_{Q'}) = \sum_{Q''} P_{Q'Q''}\begin{Bmatrix} x^{(+)}_{Q''}(Q,\omega) \\ x^{(-)}_{Q''}(Q,\omega)\end{Bmatrix} , \quad (24)$$

$$i\omega \begin{Bmatrix} u^{(+)}_{Q'}(Q,\omega) \\ u^{(-)}_{Q'}(Q,\omega) \end{Bmatrix} n_{Q'}(1+n_{Q'}) - i\vec{v}_{Q'}\vec{q} \begin{Bmatrix} u^{(-)}_{Q'}(Q,\omega) \\ u^{(+)}_{Q'}(Q,\omega) \end{Bmatrix} n_{Q'}(1+n_{Q'}) +$$

$$+ \begin{Bmatrix} -V(Q,\bar{Q}_1,\bar{Q}_2) \\ 0 \end{Bmatrix} n_{Q'}(1+n_{Q'}) = \sum_{Q''} p_{Q'Q''} \begin{Bmatrix} u^{(+)}_{Q'}(Q,\omega) \\ u^{(-)}_{Q'}(Q,\omega) \end{Bmatrix} ,$$

where $v_{Q'} = \dfrac{\partial \omega_{Q'}}{\partial q'}$

$$\sum_{Q''} p_{Q'Q''} f_{Q''} = 72 \sum_{Q''Q'''} [\, |V(Q';Q'';\bar{Q}''')|^2 (1+n_{Q'})(1+n_{Q''}) n_{Q'''} \delta(\omega_{Q'} +$$

$$+\omega_{Q''} - \omega_{Q'''})(f_{Q'''} - f_{Q''} - f_{Q'}) + \frac{1}{2} |V(\bar{Q}';Q'',Q''')|^2 (1+n_{Q'}) n_{Q''} n_{Q'''} \times$$

$$\times \delta(\omega_{Q'} - \omega_{Q''} - \omega_{Q'''})(f_{Q'''} + f_{Q''} - f_{Q'})].$$

Substituting (23) into (21) after performing the summation over the discrete frequencies and analytic continuation we obtain the following relations

$$G(\omega+i\epsilon) = 36 i\omega \sum_{Q'} V(\bar{Q},Q_1,Q_2) n_{Q'} (1+n_{Q'}) x^{(+)}_{Q'} (Q,\omega), \tag{26}$$

$$G(\omega-i\omega) = 36 i\omega \sum_{Q'} V(\bar{Q},Q_1,Q_2) n_{Q'} (1+n_{Q'}) u^{(+)}_{Q'} (Q,\omega). \tag{27}$$

The solution of (24a) and (24b) according to /6,16,18/ and taking into account (18) gives an expression for the spectral density

$$J_{H^+_Q H_Q}(\omega) = \frac{36\,\omega^2 \mu^2 \beta^3 |D_0|^2}{(e^{\beta\omega}-1)} \frac{\tau^{-1}}{(\omega^2 - \frac{c^2 q^2}{3})^2 + \frac{\omega^2}{\tau^2}}, \tag{28}$$

where

$$D_0 = \sum V(\bar{Q},Q_1,Q_2) n_{Q'} (1+n_{Q'}) \omega_{Q'}, \tag{29}$$

$\mu^2 = \frac{k}{\Omega C_v}$, k is the Boltzmann constant, C_v the specific heat at constant volume, $\frac{C}{\sqrt{3}}$ is the second sound velocity, $\frac{1}{\tau} = \frac{1}{\tau_z} + \frac{3}{5}\omega^2 \tau_n$ is the inverse of the relaxation time, τ_z and τ_n are the impuls non-conserving and impuls conserving processes relaxation time respectively /6, 18/, Ω is volume of the system. Eq. (28) is valid provided that

$$\omega\tau_z \gg 1 \gg \omega\tau_n \,. \tag{30}$$

This is a region where the second sound appears /18/.

We shall calculate now $|D_0|^2$ for the Ziman model /19/ for which the matrix elements for cubic-anharmonic interaction are of the form

$$V(Q, Q_1, Q_2) = \frac{i}{3!} \frac{1}{\sqrt{\Omega\rho^3}} \frac{A_{QQ_1Q_2} q q_1 q_2}{\sqrt{8\omega_Q \omega_{Q_1} \omega_{Q_2}}},$$

where $q = |\vec{q}|$, ρ is the crystal density and $A_{QQ_1Q_2}$ depends on the angles only. We obtain

$$|D_0|^2 = \frac{1}{8(3!)^2} \hat{A}^2 \frac{q}{v^5} k^2 T^4 C_v^2 \Omega \,, \tag{31}$$

where $\hat{A}$ is angular average and $v = \frac{\omega}{q}$ is the first sound velocity.

The final result is obtained after substituting (28) and (31) into (20)

$$\alpha = \frac{\frac{1}{8\rho v^2} \bar{\gamma}^2 q \omega^3 C_v T \frac{1}{\tau}}{(\omega^2 - \frac{c^2}{3} q^2)^2 + \frac{\omega^2}{\tau^2}} = \frac{\frac{3}{8\tau^2 \rho v^2} \bar{\gamma}^2 \omega^2 P^2 \Delta^2}{(1-P^2)^2 + \Delta^2}, \tag{32}$$

where $P = \frac{\frac{C}{\sqrt{3}}}{v} \equiv \frac{1}{\sqrt{3}} R$ is a ratio of the second sound velocity to the first sound velocity, $\bar{\gamma}^2 = \frac{\hat{A}^2}{\rho v^4}$.

From (32) it follows that the resonance attenuation of the first sound appears when velocities of the first and second sound are equal.

Introduction of

$$\gamma^2 = \frac{9}{8r^2}\gamma^{-2}$$

leads to the Guyer formula /6/ for the resonance attenuation of the first sound. This difference between our and Guyer formulas is not surprising if one remembers only that Guyer's approach was phenomenological one whereas our considerations are microscopic.

3. Conclusions

It has been shown in the paper that the NSO method enables us to obtain the attenuation coefficient of first sound in a simple, direct manner. Performing of exact calculations of the equilibrium correlation functions leads to the resonance attenuation of a first sound by a second one in the hydrodynamic region where the second sound can propagate. Our results are in a good agreement with those obtained by other authors.

The attenuation coefficient can be changed by taking into account the higher order anharmonic interactions. This problem will be considered in the forthcoming paper.

Acknowledgments

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