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F.A.Gareev, S.I. Fedotov, V.G.Soloviev

$\Delta N = \pm 2$ MIXING IN ODD-N DEFORMED NUCLEI

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Гареев Ф.А., Федотов С.И., Соловьев В.Г.

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Смешивание компонент с $\Delta N = \pm 2$ в деформированных ядрах с нечетным числом нейтронов

В рамках модели, учитывающей парные корреляции сверхпроводящего типа и взаимодействие квазичастиц с фононами, использующей одночастичные энергии и волновые функции потенциала Саксона-Вудса, изучено $\Delta N = \pm 2$- смешивание в ряде изотопов Sm, Gd и Dy с нечетным числом нейтронов. Показано, что взаимодействия квазичастиц с фононами существенно увеличивают интервал по β_{20}, в котором происходит $\Delta N = \pm 2$ -смешивание, по сравнению с одночастичной моделью.

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Gareev F.A., Fedotov S.I., Soloviev V.G.

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$\Delta N = \pm 2$ Mixing in Odd-N Deformed Nuclei

The $\Delta N = \pm 2$ mixing in a number of the Sm, Gd and Dy isotopes is studied in the framework of the model taking into account the superconducting pairing correlations and the quasiparticle-phonon interaction and using the Saxon-Woods single-particle energies and wave functions. It is shown that at deformations $\beta_{20} = 0.30$-0.33 and $\beta_{40} = 0.04$ it occurs a strong mixing of the following pairs of states: $1/2^{+}$ [400], $1/2^{+}$ [600] and $3/2^{+}$ [402], $3/2^{+}$ [651]. The results of calculations of the energies of these states and the wave function N=4 and N=6 components are in rather good agreement with the corresponding experimental data. It is shown that the quasiparticle-phonon interactions lead to an essential increase of the $\Delta\beta_{20}$ interval in which the $\Delta N = \pm 2$ mixing occurs as compared with the single-particle model.

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1. The Nilsson potential [1,2] is widely used for the description of the average field of deformed nuclei. The single particle states of the Nilsson potential are described with the quantum numbers K^{π} $[N n_z \Lambda]$ (K is the momentum projection on the nuclear symmetry axis, π is the parity, N is the principal oscillator number, n_z is the number of oscillator quanta along the symmetry axis, Λ is the projection of the orbital moment on the nuclear symmetry axis). Between the eigenstates with $\Delta N = 0, \pm 2$ there are non-zero Nilsson potential matrix elements. However, usually the $\Delta N = \pm 2$ matrix elements are neglected.

In the past years the average field of deformed nuclei is successfully described with the Saxon-Woods potential [3-5], the single-particle states of which are characterized by the same quantum numbers K^{π} $[N n_z \Lambda]$. In calculating the single-particle energies and the eigenfunction account is taken of the terms between the states with $\Delta N = \pm 2$.

There is an experimental evidence for the existence, in a number of odd deformed nuclei of the rare-earth region, of the states the wave functions of which contain a considerable mixing of the components with $N=4$ and $N=6$ (see refs. /6-8/). A large mixing of the $\Delta N=\pm 2$ components should be observed in those odd-N deformed nuclei for which the quasi-intersection of the $\Delta N=\pm 2$ levels occurs at their equilibrium deformations and near the Fermi surface energy. The small admixtures of the $\Delta N=\pm 2$ components are actually contained in all eigenfunctions of the Saxon-Woods potential. In ref. /9/ they are used to explain N-forbidden beta transitions.

The analysis of the $\Delta N=\pm 2$ mixing was first performed in works /3,5/ on the basis of the Saxon-Woods wave functions. In ref. /10/ it is indicated that the introduction of the hexadecapole deformation β_{40} to the nucleus shape is important for the $\Delta N=\pm 2$ mixing. In works /11-13/ the influence of the $\Delta N=\pm 2$ interaction on the spectroscopic factors in the (dp) and (dt) reactions is studied on the basis of a simpler model: the expansion of the average field potential in the Taylor series, spherical spin-orbital interaction, etc. In ref. /14/ it is shown that the quasiparticle-phonon interaction affects strongly the $\Delta N=\pm 2$ mixing in odd-A deformed nuclei.

In the present work, the $\Delta N=\pm 2$ mixing is calculated for a number of odd-N deformed nuclei for two pairs of the states: $1/2^+$ [400], $1/2^+$[600] and $3/2^+$ [402], $3/2^+$[651]. The Saxon-Woods single-particle energies and wave functions with the deformed spin-orbital interaction and the hexadecapole deformation β_{40} are used in the calculations. The nonrotational nuclear states are calculated taking into account the quasi-particle-phonon interaction. It is investigated how the mixing of the components $N=4$ and $N=6$ depends on the deformation parameters β_{20} and β_{40} and how this mixing is affected by the quasiparticle-phonon interaction.

2. We consider the behaviour of the Saxon-Woods single-particle energies and wave functions near the quasi-intersection of the states $1/2^+[400]$, $1/2^+[600]$ and $3/2^+[402]$, $3/2^+[651]$. As is known, when the $\Delta N = \pm 2$ mixing is taken into account the single-particle levels with the same K^π - values do not intersect. The interval of the closest approach of two such levels is called the quasi-intersection.

We assume that the nucleus shape is described by the function

$$R(\theta,\phi) = R_0(1+\beta_0+\sum_\nu \sum_\mu \beta_{\nu\mu} Y_{\nu\mu}(\theta,\phi)), \tag{1}$$

where R_0 is the radius of spherical nucleus, β_0 is a constant introduced for the nucleus volume to be conserved /10/. The experimental data and calculations show that in the region $153 \leq A \leq 163$ the parameters $\beta_{\nu\mu}$ are as follows:

$$\beta_{20} \approx 0.30, \quad \beta_{40} \approx 0.04, \quad \beta_{60} = \beta_{22} = \beta_{3\mu} = 0.$$

In the case of the Saxon-Woods potential the Schroedinger equation is

$$\{ -\frac{1}{2m}\Delta + V(\vec{r}) + V_{\ell s}(\vec{r}) + V_C(\vec{r}) - E \}\phi(\vec{r}) = 0, \tag{2}$$

where

$$V(\vec{r}) = -\frac{V_0}{1+\exp\{a(r-R(\theta))\}}, \tag{3}$$

$$V_{\ell s}(\vec{r}) = -\kappa(\vec{p}\times\vec{\sigma}, \nabla V(\vec{r})), \tag{3'}$$

$V_C(\vec{r})$ is the Coulomb term. The calculations for the neutron system for $A = 155$ are performed with the following values of the parameters: $R_0 = 1.24 \cdot 10^{-13} A^{1/3}$ cm, $V_0 = 48.2$ MeV, $\kappa = 0.39\ 10^{-26}$ cm^2, $a = 1.8\ 10^{13}$ cm^{-1}. Compared with the results of ref. [15] the parameters have been refined slightly and the accuracy of calculations has been improved (increase of the matrix rank).

Following ref. [5] we write the solution for eq. (2) in the form

$$\phi_q(\vec{r}) = \sum_{n\ell_j} a^q_{n\ell_j} \phi^q_{n\ell_j}(\vec{r}) \tag{4}$$

$E_{n\ell_j}$, $\phi^q_{n\ell_j}$ being the single-particle energy and the eigenfunction of the Schroedinger equation with spherically-symmetric potential $V(r)$ (by $q\sigma$ we denote the quantum numbers of a single-particle state, $\sigma = \pm 1$). We insert eq. (4) into (2) and get

$$(E_{n\ell_j} - E)\, a^q_{n\ell_j} + \sum_{n'\ell'_j} a^q_{n'\ell'_{j'}} (\phi^q_{n\ell_j} | \tilde{\tilde{V}} | \phi^q_{n'\ell'_{j'}}) = 0, \tag{5}$$

where $\tilde{\tilde{V}}$ is the difference between the axial-symmetric and spherically-symmetric Saxon-Woods potentials.

The wave function (4) for a state with positive parity is rewritten in the form

$$\phi_q(\vec{r}) = a^q_{00\,1/2}(N=0)\, \phi^q_{00\,1/2} + \sum_{n\ell_j} a^q_{n\ell_j}(N=2)\, \phi^q_{n\ell_j} + \tag{6}$$

$$+ \sum_{n\ell_j} a^q_{n\ell_j}(N=4)\, \phi^q_{n\ell_j} + \sum_{n\ell_j} a^q_{n\ell_j}(N=6)\, \phi^q_{n\ell_j} + \dots,$$

The normalization condition is

$$\int \phi^*_q(\vec{r})\, \phi_q(\vec{r})(d\vec{r}) = 1 = [a^q_{00\,1/2}(N=0)]^2 + \sum_{n\ell_j} [a^q_{n\ell_j}(N=2)]^2 + \tag{7}$$

$$+ \sum_{n\ell_j} [a^q_{n\ell_j}(N=4)]^2 + \sum_{n\ell_j} [a^q_{n\ell_j}(N=6)]^2 + \dots, =$$

$$= d_0^2(q) + d_2^2(q) + d_4^2(q) + d_6^2(q) + \dots,$$

i.e. the wave functions contain the components with $N = 0,2,4,6...$ though in most cases one of the components $d_N^2(q)$ is predominant. For the states with negative parity the expansion (7) contains the components with $N = 1,3,5, \ldots$

We consider the mixture of the $N = 4$ and $N = 6$ components for two pairs of states $1/2^+[400]$, $1/2^+[660]$ and $3/2^+[402]$, $3/2^+[651]$ near their quasi-intersections. At the bottom of fig. 1 the behaviour of the $3/2^+[402]$ and $3/2^+[651]$ levels is given as a function of β_{20}, for $\beta_{40} = 0$ and $\beta_{40} = 0.04$. In the quasi-intersection interval the levels cannot be assigned quantum numbers $[N n_z \Lambda]$. The wave function structure before and after quasi-intersection is such as if the intersection has happened. Therefore the upper curves at $\beta_{20} = 0.30$ are assigned the quantum numbers [651] and for $\beta_{20} = 0.33$ - [402]. At the top of fig. 1 the values of d_4^2 and d_6^2 are given for the state $3/2^+[402]$ for $\beta_{40} = 0$ and $\beta_{40} = 0.04$. It is seen that for $\beta_{40} = 0$ these components are mixed in a very narrow interval $\Delta\beta_{20}$ and for $\beta_{40} = 0.04$ this interval becoms wider.

For the other pair of states $1/2^+[400]$ and $1/2^+[660]$ table 1 gives the single-particle energies $E(q)$, $E(q')$, their differences and the components d_4^2 and d_6^2 as functions of β_{20} for $\beta_{40} = 0$ and $\beta_{40} = 0.04$. It is seen from the table that a large mixing of the $N = 4$ and $N = 6$ components occurs in the interval $\Delta\beta_{20} \approx 0.01$ near the quasi-intersection. The mixing interval $\Delta\beta_{20}$ is somewhat enlarged with increasing hexadecapole deformation parameter β_{40}.

The investigations of the solutions of the Schroedinger equation for the Saxon-Woods potential have shown that the quasi-intersections of the levels with the same K^π and the degree of the $\Delta N = \pm 2$ mixing depend strongly on the potential shape, its parameters as well as on the accuracy of solving the equation. Therefore the study of the quasi-intersections will make it possible to improve the shape of the average potential and its parameters.

3. We consider the interaction of quasiparticles with phonons in odd-N deformed nuclei. Following /16/, when the two single particle states q_1 and q_2 with the same K^π are described simultaneously, the wave function is written in the form:

$$\Psi_i(K^\pi; q_1, q_2) = N_i(q_1, q_2)\frac{1}{\sqrt{2}}\sum_\sigma \{ \mathcal{L}_i(q_1) a^+_{q_1\sigma} + \mathcal{L}_i(q_2) a^+_{q_2\sigma} + \tag{8}$$

$$+ \sum_{\lambda\mu i} \sum_q D^{\lambda\mu i}_{q_1 q_2 q\sigma} a^+_{q\sigma} Q^+_i(\lambda\mu) \}\Psi ,$$

where $Q_i(\lambda\mu)$ is the phonon operator of multipolarity $(\lambda\mu)$, $a^+_{q\sigma}$ is the quasiparticle production operator, Ψ is the wave function of the ground state of an even-even nucleus.

Owing to the fact that in the quasi-intersection interval the single-particle states cannot be assigned the asymptotic quantum numbers $[Nn_z\Lambda]$ the quantum numbers of the wave function of the upper levels are denoted by q_a and the lower one – by q_b. Therefore we rewrite the wave function (8) in the form

$$\Psi_i(K^\pi; q_1, q_2) = N_i(q_1, q_2)\frac{1}{\sqrt{2}}\sum_\sigma \{ \mathcal{L}_i(q_a) a^+_{q_a\sigma} + \mathcal{L}_i(q_b) a^+_{q_b\sigma} + \tag{8'}$$

$$+ \sum_{\lambda\mu i} \sum_q D^{\lambda\mu i}_{q_1 q_2 q\sigma} a^+_{q\sigma} Q^+_i(\lambda\mu) \}\Psi$$

and write the normalization condition (8) as

$$N_i^2(q_1,q_2)\{\mathfrak{L}_i^2(q_a)+\mathfrak{L}_i^2(q_b)+\frac{1}{2}\sum_{\lambda\mu i}\sum_{q\sigma}(D^{\lambda\mu ii}_{q_1 q_2 q\sigma})^2\}=1. \tag{9}$$

The secular equation which defines the energies η_i of the ground and excited states of an odd-N nucleus are of the form

$$\begin{vmatrix} W_i(q_a,q_a)-(\epsilon(q_a)-\eta_i) & W_i(q_a,q_b) \\ W_i(q_a,q_b) & W_i(q_b,q_b)-(\epsilon(q_b)-\eta_i) \end{vmatrix}=0, \tag{10}$$

where $W_i(q_a,q_b)=\frac{1}{4}\sum_{\lambda\mu i}\sum_{q}\frac{v_{q_a q}\,v_{q_b q}}{Y^i(\lambda\mu)}\,\frac{f^{\lambda\mu}(q_a q)f^{\lambda\mu}(q_b q)}{\epsilon(q)+\omega_i^{\lambda\mu}-\eta_i}$,

i is the number of the secular equation root, for the remaining notations see ref. /16/.

In each nucleus the least value of $\eta_1(K_0^{\pi_0})\equiv\eta_F$ is the energy of the ground state, the excited state energies are defined by the differences $\eta_i(K^\pi)-\eta_F$. The quantities $\mathfrak{L}_i(q_a)$, $\mathfrak{L}_i(q_b)$ and $N^{-2}(q_1,q_2)$ are

$$\mathfrak{L}_i(q_a)=1-\frac{W_i(q_a,q_b)}{W_i(q_a,q_a)-(\epsilon(q_a)-\eta_i)},$$

$$\mathfrak{L}_i(q_b)=1-\frac{W_i(q_a,q_b)}{W_i(q_b,q_b)-(\epsilon(q_b)-\eta_i)}, \tag{11}$$

$$N^{-2}(q_1,q_2)=\mathfrak{L}_i^2(q_a)+\mathfrak{L}_i^2(q_b)+\frac{1}{2}\sum_{\lambda\mu i}\sum_{q\sigma}(D^{\lambda\mu ii}_{q_1 q_2 q\sigma})^2. \tag{12}$$

The contributions to the normalization condition (9) of the one-quasiparticle components q_a and q_b are

$$C_{al}^{2} = (N_l(q_1, q_2)\,\mathcal{L}_l(q_a))^2, \tag{13}$$

$$C_{bl}^{2} = (N_l(q_1, q_2)\,\mathcal{L}_l(q_b))^2 .$$

According to eq. (6) the wave function of a single-particle state consists of the sum of terms with different N. We take into account this fact when determining the contribution of the terms with different N to the normalization condition (9). Using expressions (7) and (13) we find that the contribution of the terms with $N = 4$ and $N = 6$ to the wave function normalization (8) is of the form

$$P_{4l}(q_1, q_2) = C_{al}^{2}\, d_4^2(q_a) + C_{bl}^{2}\, d_4^2(q_b),$$
$$P_{6l}(q_1, q_2) = C_{al}^{2}\, d_6^2(q_a) + C_{bl}^{2}\, d_6^2(q_b). \tag{14}$$

Thus, since the quasiparticle-phonon interactions lead to a mixing of the single-particle states they may cause an essential redistribution of the values of the components N and $N \pm 2$ in odd-N nuclei compared with the single-particle model.

4. The energies and wave functions of the first and second nonrotational states are calculated for a number of odd-N deformed nuclei, taking into account the quasiparticle-phonon interaction. The calculations are performed with the single-particle energies and wave functions of the Saxon-Woods potential for $A = 155$ at $\beta_{40} = 0.04$ and at β_{20} in the interval from 0.29-0.34. For each value of β_{20} the phonons $Q_i(\lambda\mu)$ are calculated with the values of the constant $\kappa^{(\lambda)}$ as in ref. /15/. The results of calculations

are given in tables 2 and 3. The energies and the contributions of one-quasiparticle components P_{4i} and P_{6i} are given there for the two first roots. (i = 1,2).

We study the effect of the quasiparticle-phonon interaction on the N = 4 and N = 6 component mixing. To this end we compare table 1 for β_{20} = 0.04 with table 2 and supplement them with the data for β_{20} = 0.29 and 0.34. For the deformation β_{20} = 0.29 the $\Delta N = \pm 2$ mixing is small both in the single-particle model and in the account of the quasiparticle-phonon interaction. The exception is ^{161}Dy in which for β_{20} = 0.29, for the first root $\eta_1(1/2^+)-\eta_F$ = 754 keV, P_{41} = 0.19, P_{61} = 0.41; for the second one $\eta_2(1/2^+)-\eta_F$ = 852 keV, P_{42} = 0.45, P_{62} = 0.15. For the deformation β_{20} = 0.30 the $\Delta N = \pm 2$ mixing is small in the single-particle model, somewhat larger in the nuclei calculated by us and large enough only in ^{161}Dy. For β_{20} = 0.31 the $\Delta N = \pm 2$ mixing is strong in the single-particle model and in all calculated nuclei. For β_{20} = 0.32 the $\Delta N = \pm 2$ mixing in the single-particle model and in a number of nuclei is not large though in ^{157}Gd, ^{159}Gd and ^{159}Dy it is noticeable. At β_{20} = 0.33 this mixing is small in both the single-particle model and in most nuclei, even though it remains considerable in ^{163}Dy.

In those nuclei for which the quasi-intersection occurs near the Fermi surface the following particularity is observed for the first root at β_{20} = 0.30 the component P_{61} is predominant, at β_{20} = 0.31 P_{61} and P_{41} become close to each other, at β_{20} = 0.32 P_{61} is larger than P_{41}, at β_{20} = 0.33 the component P_{61} is predominant and at β_{20} = 0.34 of predominance is the component P_{41}. That is as β_{20} increases the components

$N = 4$ and $N = 6$ are mixed. Then the mixing becomes weaker and an exchange of large components between two quasi-intersecting levels follows. This particularity is due to the change of the position of the chemical potential with increasing β_{20}. It occurs in ^{153}Sm, ^{155}Sm, ^{155}Gd and ^{157}Dy. Thus, for $\beta_{20} = 0.34$ in ^{155}Gd $-(\eta_1(3/2^+)-\eta_F) = 105$ keV, $P_{41} = 0.62$, $P_{61} = 0.27$, $\eta_2(3/2^+)-\eta_F = 158$ keV, $P_{42} = 0.34$, $P_{62} = 0.52$; in $^{157}Dy$$-(\eta_1(3/2^+)-\eta_F) = 95$ keV, $P_{41} = 0.91$, $P_{61} = 0.03$, $\eta_2(3/2^+)-\eta_F = 208$ keV, $P_{42} = 0.06$, $P_{62} = 0.74$.

At the deformation $\beta_{20} = 0.34$ in the single-particle model there is no strong $\Delta N = \pm 2$ mixing, and in some calculated nuclei this mixing is considarable. Thus, in ^{163}Dy $\eta_1(3/2^+)-\eta_F = 191$ keV, $P_{41} = 0.67$, $P_{61} = 0.04$; $\eta_2(3/2^+)-\eta_F = 420$ keV, $P_{42} = 0.08$, $P_{62} = 0.59$.

Thus, the interaction of quasiparticles with phonons leads to the $\Delta N = \pm 2$ mixing interval being wider with respect to β_{20}. In the cases when a small component exceeds 0.1 fraction of a large one the mixing interval in the single-particle model $\Delta\beta_{20} = 0.01$, taking into account the quasi-particle-phonon interaction $\Delta\beta_{20} = 0.03$ and for some nuclei much more. If the interval $\Delta\beta_{20}$ was very small as in the Nilsson potential calculations without the account of the quasiparticle-phonon interaction then the probability of experimental observation of the $\Delta N = \pm 2$ mixing would be very small, since it is unlikely that the value of the equilibrium deformation falls just within this narrow interval.

The quasi-particle-phonon interactions lead to the state structure being more complicated with increasing excitation energy, which leads, in turn, to a decrease of the total contribution of two one-quasiparticle components. It should be noted that at an excitation energy of about 1 MeV the $\Delta N = \pm 2$ mixing is considerable.

The comparison of the results of calculations with experiment is somewhat difficult since in the calculations the Coriolis force is disregarded. However, the Coriolis force does not change practically the energy and the wave functions of the nonrotational states $1/2^+$ [400] and $1/2^+$ [660]. According to refs. /17,18/, in nuclei with the neutron number 91 the Coriolis force gives an admixture to the 3/2 $3/2^+$[651] state of about 10% and to the state 3/2 $3/2^+$ [402] of about 1% of the component 3/2 $1/2^+$ [660]. Such admixture leads to a displacement not exceeding 50 keV. Thus, the effect of the Coriolis force to the $\Delta N = \pm 2$ mixing is small for the ground state of rotational bands and it should be taken into account in studying the rotational states.

The results of calculation of the relative components with $N = 4$ and $N = 6$ is in satisfactory agreement with experimental data for the following deformations: for $K^\pi = 1/2^+$ and $3/2^+$ states in ^{153}Sm for $\beta_{20} = 0.310$ in ^{155}Gd for $\beta_{20} = 0.305$; for $K^\pi = 1/2^+$ states in ^{159}Gd for $\beta_{20} = 0.305$, in ^{161}Dy for $\beta_{20} = 0.305$, in ^{163}Dy for $\beta_{20} = 0.300$; for $K^\pi = 3/2^+$ states in ^{157}Dy for $\beta_{20} = 0.315$, in ^{159}Dy for $\beta_{20} = 0.320$, in ^{161}Dy for $\beta_{20} = 0.315$, in ^{163}Dy for $\beta_{20} = 0.335$. That is, the relative values of the components P_{41} and P_{61} are correctly described at β_{20} deformations somewhat larger than the equilibrium deformations of the neighbouring even-even nuclei. At $\beta_{20} = 0.30$ for $K^\pi = 3/2^+$ states in ^{159}Dy, ^{161}Dy and ^{163}Dy the calculated values of P_{41} and P_{61} for the lower state well describe the observable components of the upper state and vice versa.

The calculated values of the energies of the first and second $K^\pi = 1/2^+$ and $3/2^+$ states are somewhat smaller than the experimental ones. In some nuclei this difference is not large:, in others, like ^{157}Gd, ^{159}Dy it is noticeable. The calculated energy differences between the first and second $K^{\pi} = 3/2^{+}$ or $1/2^{+}$ states desribe rather well (within the accuracy of 10-40 keV) the appropriate experimental data. The exception is the splitting energies in ^{153}Sm.

The results of the present calculations for $K^{\pi} = 1/2^{+}$ and $3/2^{+}$ states differ noticeably from those obtained in ref. /15/. This difference is due to the fact that here the Saxon-Woods potential patameters are slightly altered (which leads to a small change in the energy and the structure of other states) and the calculations of the corresponding single-particle states are performed more accurately (which is essential for states near their quasi-intersections).

5. On the basis of the calculations performed we may draw the following conclusions concerning the mixing of states $1/2^{+}[400]$, $1/2^{+}[660]$ and $3/2^{+}[402]$, $3/2^{+}[651]$ in the Sm, Gd and Dy isotopes with an odd number of neutrons.

1. The $N = 4$ and $N = 6$ components of the Saxon-Woods wave functions are strongly mixed in the interval $\Delta\beta_{20} = 0.01$ near the quasi-intersection of levels, the latter being essentially larger than the mixing interval of the Nilsson wave functions.

2. The mixing interval $\Delta\beta_{20}$ increases slightly with increasing hexadecapole deformation parameter.

3. The study of the behaviour of the single-particle levels near their quasi-intersection makes it possible to improve the shape and the parameters of the average field potential.

4. The quasi-particle-phonon interactions lead to a broadening of the $\Delta N = \pm 2$ mixing interval up to $\beta_{20} = 0.03$. This fact makes it possible to observe experimentally the $\Delta N = \pm 2$ mixing effect.

5. The quasi-particle-phonon interactions lead to the change of the $N = 4$ and $N = 6$ components as compared with the single-particle model, in a number of nuclei this change being cardinal.

6. The calculations performed with the Saxon-Woods single-particle energies and wave functions, taking into account the superconducting pairing correlations and the quasi-particle-phonon interactions give a rather good agreement of the experimental data on the $\Delta N = \pm 2$ mixing and on the energies of these states.

For a further progress in the $\Delta N = \pm 2$ mixing studies a larger amount of experimental information is needed.

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Table I

$N = 4$ and $N = 6$ Component Mixing Near the Point of Quasi-Intersection of $1/2^+[660]$ and $1/2^+[400]$ States in the Neutron Scheme with $A = 155$

β_{20}	0.300	0.305	0.310	0.312	0.315	0.320	0.325	0.330
				$\beta_{40} = 0.04$				
$E(\varphi)$, MeV	-8.524	-8.552	-8.549	-8.528	-8.485	-8.403	-8.317	-8.226
$d_4^2(\varphi)$	0.098	0.190	0.628	0.796	0.903	0.950	0.961	0.965
$d_6^2(\varphi)$	0.856	0.766	0.345	0.184	0.080	0.035	0.024	0.020
$E(\varphi')$, MeV	-8.772	-8.695	-8.649	-8.649	-8.661	-8.691	-8.725	-8.751
$d_4^2(\varphi')$	0.937	0.844	0.409	0.240	0.132	0.086	0.074	0.071
$d_6^2(\varphi')$	0.052	0.138	0.558	0.718	0.822	0.866	0.876	0.878
$E(\varphi)-E(\varphi')$	0.248	0.143	0.100	0.121	0.176	0.288	0.408	0.525
				$\beta_{40} = 0$				
$E(\varphi)$, MeV	-8.228	-8.266	-8.295	-8.295	-8.267	-8.188	-8.103	-8.012
$d_4^2(\varphi)$	0.058	0.078	0.220	0.454	0.810	0.927	0.944	0.949
$d_6^2(\varphi)$	0.902	0.880	0.743	0.516	0.172	0.057	0.039	0.034
$E(\varphi')$, MeV	-8.549	-8.465	-8.389	8.370	-8.369	-8.399	-8.434	-8.468
$d_4^2(\varphi')$	0.954	0.934	0.795	0.560	0.206	0.086	0.070	0.065
$d_6^2(\varphi')$	0.027	0.048	0.184	0.412	0.754	0.868	0.885	0.888
$E(\varphi)-E(\varphi')$	0.321	0.199	0.094	0.075	0.102	0.211	0.331	0.456

Table 2

N = 4 and N = 6 Component Mixing Near the Point of Quasi-Intersection of 1/2⁺ [400] and 1/2⁺ [660] States at β_{40} = 0.04

Nuclei		$\beta_{20}=0.30$			$\beta_{20}=0.31$			$\beta_{20}=0.32$			$\beta_{20}=0.33$		
		$Q_i(1/2^+)-Q_F$ keV	P_{4i} %	P_{6i} %	$Q_i(1/2^+)-Q_F$ keV	P_{4i} %	P_{6i} %	$Q_i(1/2^+)-Q_F$ keV	P_{4i} %	P_{6i} %	$Q_i(1/2^+)-Q_F$ keV	P_{4i} %	P_{6i} %
^{153}Sm	1	54	9	67	58	38	35	45	10	61	98	7	62
	2	736	73	6	325	42	34	338	74	6	432	81	3
^{155}Sm	1	401	8	68	447	41	35	454	15	61	496	5	51
	2	783	72	4	654	41	36	622	70	10	567	62	20
^{155}Gd	1	120	8	64	114	37	34	132	10	60	163	7	62
	2	342	67	4	259	41	33	280	73	6	257	80	3
^{157}Gd	1	480	8	68	498	40	33	436	29	44	437	75	4
	2	634	66	4	586	37	34	493	50	23	619	8	64
^{159}Gd	1	713	7	62	737	40	30	715	41	28	670	68	4
	2	894	63	4	866	33	34	796	32	36	806	8	60
^{161}Gd	1	812	**65**	38	817	32	18	725	56	4	690	60	2
	2	1052	54	3	922	23	28	878	6	37	844	4	42
^{157}Dy	1	201	15	62	131	35	32	123	9	56	185	7	56
	2	254	61	13	243	40	32	245	71	5	251	77	3
^{159}Dy	1	469	7	65	448	38	30	495	45	25	490	71	2
	2	516	59	5	520	33	32	540	28	39	530	6	61
^{161}Dy	1	624	45	23	539	37	26	453	63	2	365	66	1
	2	780	27	36	736	28	31	753	5	51	781	4	49
^{163}Dy	1	783	5	33	588	29	24	450	11	43	374	16	41
	2	1024	54	3	833	30	27	672	50	8	586	47	12

T a b l e 3

N=4 and N=6 Component Mixing Near the Point of Quasi-Intersection of $3/2^+[402]$ and $3/2^+[651]$ States at $\beta_{20} = 0.04$

Nuclei	i	β_{20}=0.30			β_{20}=0.3I			β_{20}=0.32			β_{20}=0.33		
		$\varepsilon_i(K_i^+)-\varepsilon_F$ keV	P_{4i} %	P_{6i} %	$\varepsilon_i(K_i^+)-\varepsilon_F$ keV	P_{4i} %	P_{6i} %	$\varepsilon_i(K_i^+)-\varepsilon_F$ keV	P_{4i} %	P_{6i} %	$\varepsilon_i(K_i^+)-\varepsilon_F$ keV	P_{4i} %	P_{6i} %
^{153}Sm	I	46	6	64	6	22	62	I0	35	48	46	I5	65
	2	543	84	4	264	66	20	245	58	32	2I5	79	I2
^{155}Sm	I	227	7	77	2II	23	60	2I3	40	44	260	39	45
	2	6I8	79	4	54I	66	20	476	5I	36	376	54	34
^{155}Gd	I	3I	8	79	0	24	60	0	35	47	8	I6	64
	2	225	75	5	220	64	2I	2I2	56	32	I57	79	I2
^{157}Gd	I	265.	9	75	205	26	53	2I2	43	38	24I	60	23
	2	527	72	6	480	58	25	436	45	40	447	29	53
^{159}Gd	I	47I	7	67	4I5	24	50	3I7	4I	33	468	58	28
	2	378	7I	5	739	56	23	67I	4I	40	640	25	52
^{161}Gd	I	772	8	66	638	27	44	52I	47	I5	438	66	7
	2	98I	67	6	934	50	27	900	29	50	747	II	62
^{157}Dy	I	98	II	78	75	29	53	5I	40	40	85	23	54
	2	I65	74	8	I88	57	27	I83	50	37	I98	70	I9
^{159}Dy	I	235	9	65	240	28	46	240	42	33	243	64	I7
	2	44I	69	8	420	52	27	397	40	40	390	23	56
^{161}Dy	I	556	27	5I	49I	42	33	422	56	I8	343	74	3
	2	657	5I	28	648	43	37	628	24	55	627	I4	70
^{163}Dy	I	753	I0	65	543	22	43	322	3I	34	232	24	40
	2	976	64	8	845	50	22	680	40	29	43I	47	2I

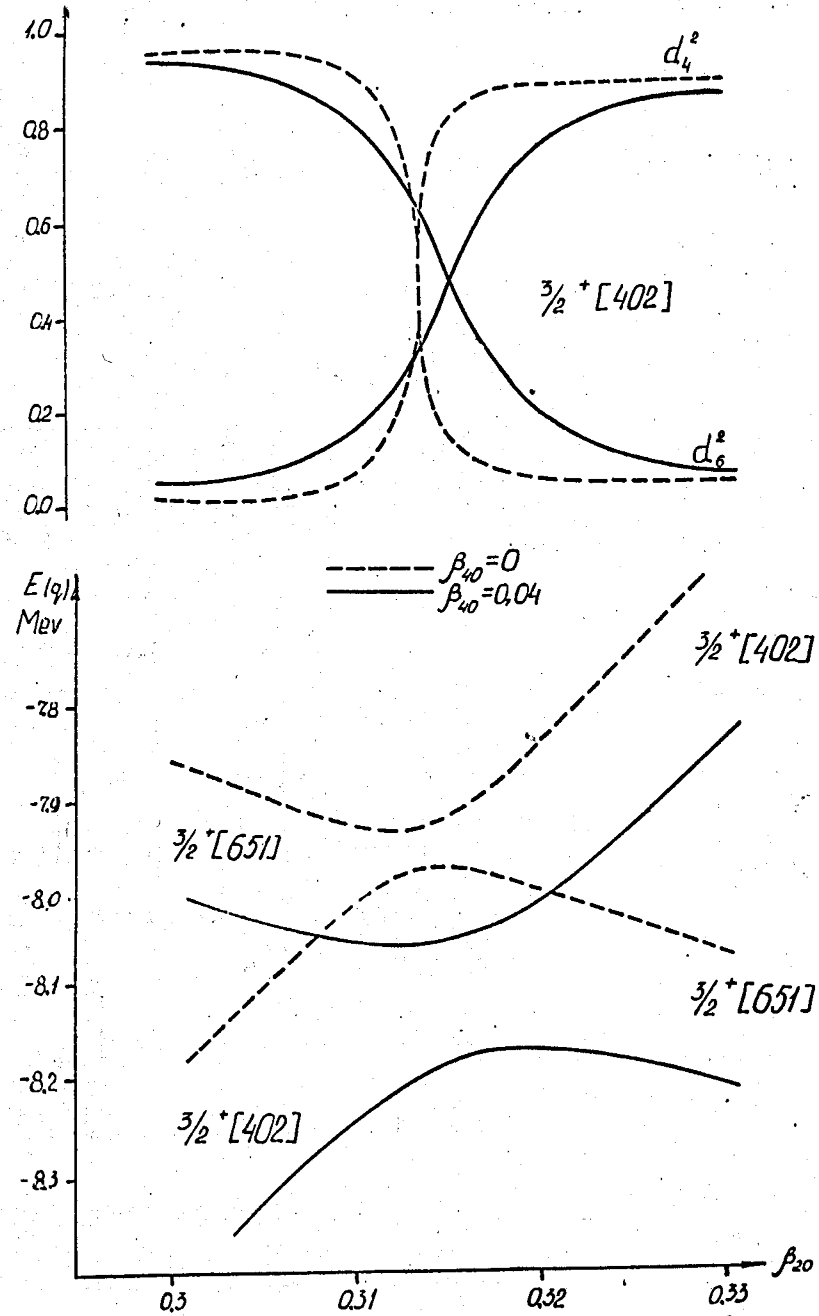

Fig. 1. β_{20} dependence of the components d_4^2 and d_6^2 for the $3/2^+$[402] state (upper part of figure) and the position of the single-particle levels $3/2^+$[402] and $3/2^+$[651] (lower part of figure) in the region of their quasi-intersection for β_{40}= 0 and β_{40}= 0.04.