ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
Дубна

E4-5837

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

P. Beregi

ON THE NUMBER OF RESONANCES OF NONLOCAL SEPARABLE POTENTIALS

1971

E4-5837

P. Beregi *

ON THE NUMBER OF RESONANCES OF NONLOCAL SEPARABLE POTENTIALS

Submitted to "Lett. al Nuovo Cimento"

* On leave of absence from the Central Research Institute for Physics, Budapest.

The interest in using nonlocal separable potentials [1,2] has increased recently. This is connected with the fact that with the help of such potentials the Faddeev equations [3] used for solving the quantum-mechanical three-body problems can be reduced to a coupled system of one-dimensional integral equations [2]. However there is an important barrier in solving this system, namely the number of the coupled equations and consequently the dimension of the matrices to be inverted are proportional to the total number $(N+M)$ of terms contained in nonlocal separable potentials [x)]

$$V(k,k') = \lambda^2 \sum_{i=1}^{N+M} c_i \lambda_i^2 g_i(k) g_i(k'), \tag{1}$$

where $c_i = -1$ for $0 \leq i \leq N$, $c_i = +1$ for $N+1 \leq i \leq N+M$, $\lambda_i > 0$ for $1 \leq i \leq N+M$, and the $g_i(k)$ are $N+M$ linearly independent functions fulfilling some simple conditions [4-8]. Of course it is desirable to include into the two-particle system, with the help of minimally possible number of terms, as many physical

x) For convenience we investigate only potentials acting in the relative s-state.

information as one can. Such an information is the number of bound states and that of resonances in the two-particle systems. The number of bound states [x)] $N(b.s.)$ which can be obtained by potential (1) is limited by the number of negative coefficients c_i /4,9/:

$$N(b.s.) \leq N . \tag{2}$$

This means that we cannot choose such functions $g_i(k)$ which give more than N bound states.

The two-particle systems may have also some resonances if the equation

$$D(-k) \equiv \det |M_{ij}(-k)| = 0 \tag{3}$$

is satisfied for $-k = k_0 = k_1 - i k_2$, where $k_1 > k_2 > 0$ and

$$M_{ij}(-k) = \delta_{ij} + 4\pi^2 \lambda^2 c_i c_j \lambda_i \lambda_j \int_0^\infty \frac{g_i(p) g_j(p) p^2 \, dp}{k^2 - p^2 + i\epsilon} . \tag{4}$$

Of course, the investigation of the resonances is very important, too.

The question arises whether we must increase the number of terms in potential (1) for including resonances into the two-particle system or this can be made by a proper choice of the formfactors $g_i(k)$. To answer this question we shall investigate the motion of the poles by changing the strength parameter λ^2. The problem of the pole motion has been investigated for some local potentials. For example, the case of the square-well potential

---

[x)] Here and in the following, except that when we specially emphasize, we are speaking of the bound states of negative energy.

and the case of the screened attractive Coulomb potential have been discussed in /10/ and /11/, respectively. When increasing λ^2 the resonance poles (more exactly they "partners" with $k'_0 = -k^*_0 = -k_1 - i k_2$ /12, 13/) turn into bound states.

We can investigate the problem whether the situation for the case of nonlocal separable potentials is similar to the local one. If all resonance poles become bound states at sufficient large λ^2 then because of the fact that when increasing the strength parameter λ^2 the energy of the bound states does not decrease /5,9/ and due to inequality (2) the total sum of the number of the resonances $(N(r.s.))$ and that of the bound states $(N(b.s.))$ must be limited by

$$N(r.s.) + N(b.s.) \leq N. \tag{5}$$

However, it turns out that for potentials of kind (1) the inequality (5) does not hold. In order to see the breakdown of the inequality (5) we shall investigate some illustrative examples for three simple kinds of interactions.

1. "Repulsive" potentials

$$V(k,k') = \lambda^2 g(k) g(k'), \tag{6}$$

where e.g.

$$g(k) = \frac{1}{k^2 + \beta^2}. \tag{7}$$

The zeros of the determinant (3) for this case can be found from equation

$$1 + \frac{\pi^2 \lambda^2}{\beta(\beta - i k_0)^2}, \tag{8}$$

which gives

$$k_0 = -i\beta \pm \frac{\pi\lambda}{\sqrt{\beta}}. \tag{9}$$

For $\lambda^2 > \frac{\beta^3}{\pi^2}$ we have a resonance pole. Of course, the potential (6) cannot give any bound state. (This situation is similar to the case of the repulsive local potential, e.g. to the case of the square-barrier potential giving no bound state but which can give resonances [10].)

2. Potentials with "repulsion and attraction"

$$V(k,k') = -\lambda^2 g(k) g(k'), \tag{10}$$

where $g(k)$ changes its sign at some k e.g. [14]

$$g(k) = (k_c^2 - k^2)\,\frac{k^2 + a^2}{k^2 + b^2}\cdot\frac{1}{k^4 + \beta^4}. \tag{11a}$$

The potential (10) with formfactor (11a) can give a bound state of negative energy and a bound state of positive energy [14] (extinct bound state, EBS [15]). By a small change of the parameters the EBS becomes a resonance [16], but this resonance cannot turn into a bound state when increasing λ^2, because we have already the only possible bound state which remains bound state when the interaction strength is increased. In Fig. 1 we illustrate the pole motion for potential (10) with formfactor

$$g(k) = g^{(1)}(k) = \frac{1}{k^2 + \beta_1^2} - \frac{a_2}{k^2 + \beta_2^2}, \tag{11b}$$

where $\beta_2 > \beta_1$, $a_2 > 1$. The potential (10) with formfactor (11b) by a proper choice of the parameters has the same properties as the one-term Tabakin's potential but has a more simple form.

3. At last we investigate in details the potential (10) with formfactors of definite sign e.g. with $g(k) > 0$ for $k \geq 0$.

One can suppose that the resonance poles of such a potential, remembering us the "attractive" one, will turn into bound states when increasing λ^2 . We investigate the pole motion for the following formfactors [x)]

$$g(k) = g^{(2)}(k) = \frac{1}{(k^2+\beta_1^2)(k^2+\beta_2^2)}, \tag{12}$$

$$g(k) = g^{(3)}(k) = \frac{a_1}{k^2+\beta_1^2} + \frac{a_2}{k^2+\beta_2^2}, \tag{13}$$

$$g(k) = g^{(4)}(k) = \frac{a_1}{(k^2+\beta_1^2)^2} + \frac{a_2}{(k^2+\beta_2^2)^2}. \tag{14}$$

The formfactor $g^{(2)}(k)$ is proportional to $g^{(3)}(k)$ if in the latter we put $a_2 = -a_1$ and $g^{(2)}(k)$ gives the formfactor $\frac{1}{(k^2+\beta_1^2)^2}$ /2/ when putting $\beta_2 = \beta_1$.

a, $g(k) = g^{(2)}(k) = \frac{1}{(k^2+1)(k^2+\beta_2^2)}$.

The trajectory of the poles for some β_2 is to be seen in Fig.2.

---

x) In all potentials we can choose $\beta_1 < \beta_2$, $\beta_1 = 1$ and $a_1 = 1$. This means that the strength parameter a_1 is included in λ and that we are working in units β_1 . In order to fulfil the condition $g(k) > 0$ for $k \geq 0$ we have to choose $a_2 \geq -1$.

The potential (10) with $g(k) = g^{(2)}(k)$ gives four physical poles but it does not give any resonance. For the two complex poles we have

$$k_{3,4} \to \pm\sqrt{\beta_1 \beta_2} - i(\beta_1 + \beta_2) \equiv \sqrt{\beta_2} - i(1 + \beta_2) \quad at \quad \lambda \to \infty . \tag{15}$$

b, $\quad g(k) = g^{(3)}(k) = \dfrac{1}{k^2+1} + \dfrac{a_2}{k^2+\beta_2^2}$.

It is easy to solve the equation

$$\dot{D}(-k) \equiv 1 + 4\pi \lambda^2 \int_0^\infty \frac{g^2(p)\, p^2\, dp}{k^2 - p^2 + i\epsilon} = 0 \tag{16}$$

for $\lambda \to \infty$. Two of zeros are given by

$$k_{1,2} \to \pm\, i A \lambda \qquad at \quad \lambda \to \infty , \tag{17}$$

where

$$A = \pi \sqrt{1 + \frac{4 a_2}{1+\beta_2} + \frac{a_2^2}{\beta_2}} \tag{18}$$

and for the two other zeros of $D(-k)$ the following asymptotical expression can be obtained

$$k_{3,4} \to \pm a - i b \qquad for \quad \lambda \to \infty . \tag{19}$$

If the conditions

$$c > 4d \tag{20}$$

and

$$max\{0, \frac{d}{2}[c - d - \sqrt{c^2 - 4cd}]\} \leq a_2 \leq \frac{d}{2}[c - d + \sqrt{c^2 - 4cd}] \tag{21}$$

(where $c = \frac{(\beta_2 - 1)^2}{\beta_2 + 1}$ and $d = \sqrt{\beta_2}$) are fulfilled then $a > b$ and for $\lambda \to \infty$ we have a bound state and a resonance [x)]. In Fig. 3 we illustrate the pole motion for some sets of parameter of $g^{(3)}(k)$.

$$c, \quad g(k) = g^{(4)}(k) = \frac{1}{(k^2+1)^2} + \frac{a_2}{(k^2+\beta_2^2)^2} .$$

The pole motion for a given set of a_2 , β_2 is shown in Fig. 4. One can see that by proper choise of the parameters one bound state (with good asymptotic behaviour) and one resonance can be obtained.

In order to illustrate the possibility of turning the resonance pole into a bound state we present the pole motion in Fig. 4b for the potential (10) with formfactor (14) and $a_2 < -1$.(This means that $g^{(4)}(k)$ changes its sign at some value of k.)

In summary, we have to say that by a proper choice of the formfactors $g_l(k)$ we can put in our two-particle systems

---

x) When $k_1 > \beta_1$, where k_1 is the value of the imaginary part of the bound state pole, the asymptotic behavour of the bound state wave function $\psi(r)$ will be wrong /17, 18/, namely $\psi(r) \sim e^{-\beta_1 r}$ at $r \to \infty$, but in the investigation of the number of the bound states this fact is not taken into account generally and the bound states with wrong asymptotic behaviour are included in this number.

resonances without increasing the number of terms contained in (1) [x)]. Practically this means that the dimension of the matrices to be inverted in solving the three-body problem does not increase and the necessary computer time and the error of the calculations practically do not increase, too. Of course, the properties of the three-body system (e.g. the three-body bound state energy) must be investigated separately for every two-body potential.

References

1. Y. Yamaguchi. Phys. Rev., 95, 1628 (1954).
2. A detailed discussion of the nonlocal separable interaction can be found in A.G. Sitenko and V.E. Kharchenko, Preprint ITF-68-11 (in Russian).
3. L.D. Faddeev, Zh. Eksp. Teor. Fiz., 39, 1459 (1960) (Soviet Phys. - JETP, 12, 1014 (1961)).
4. G.C. Ghirardi and A. Rimini. J. Math. Phys., 5, 722 (1964).
5. G.C. Ghirardi and A. Rimini. J. Math. Phys., 6, 40 (1965).
6. A.V. Rokhlenko. Zh. Eksp. Teor. Fiz., 47, 896 (1964).
7. D. Gutkovski and A. Scalia. J. Math. Phys., 9, 588 (1968); ibid., 10, 2306 (1969).
8. D. Gutkovski and A. Scalia. Nuovo Cimento, 2A, 525 (1971).
9. A.V. Rokhlenko. Zh. Eksp. Teor. Fiz., 50, 93 (1966).
10. H.M. Nussenzweig. Nucl. Phys., 11, 499 (1959).
11. E.M. Ferreira and A.F.F. Teixeira. J. Math. Phys., 7, 1207 (1966).

[x)] It seems to us that resonances can be obtained more easily by those formfactors which change their sign at some k. Two resonances can be obtained e.g. with formfactor $g(k) = \frac{(k^2-a^2)(k^2-b^2)}{(k^2+\beta_1^2)(k^2+\beta_2^2)(k^2+\beta_3^2)}$ which is a superposition of three Yamaguchi formfactors.

12. K.W. Mc Voy. Trieste Lectures 1966 (IAEA, Vienna (1967)).

13. R.G. Newton. Scattering Theory of Waves and Particles, Mc Graw-Hill, New York (1966).

14. F. Tabakin. Phys. Rev., 174, 1208 (1968); ibid., 177, 1443 (1969).

15. M. Stihi. Nuovo Cimento, 60 A, 376 (1969).

16. J.E. Beam. Phys. Letters, 30B, 67 (1969).

17. P. Beregi, I. Lovas and J. Revai. Ann. Phys. (N.Y.), 61, 57 (1970).

18. P. Beregi and I. Lovas. Z. Physik, 241, 410 (1971).

Received by Publishing Department
on May 26, 1971.

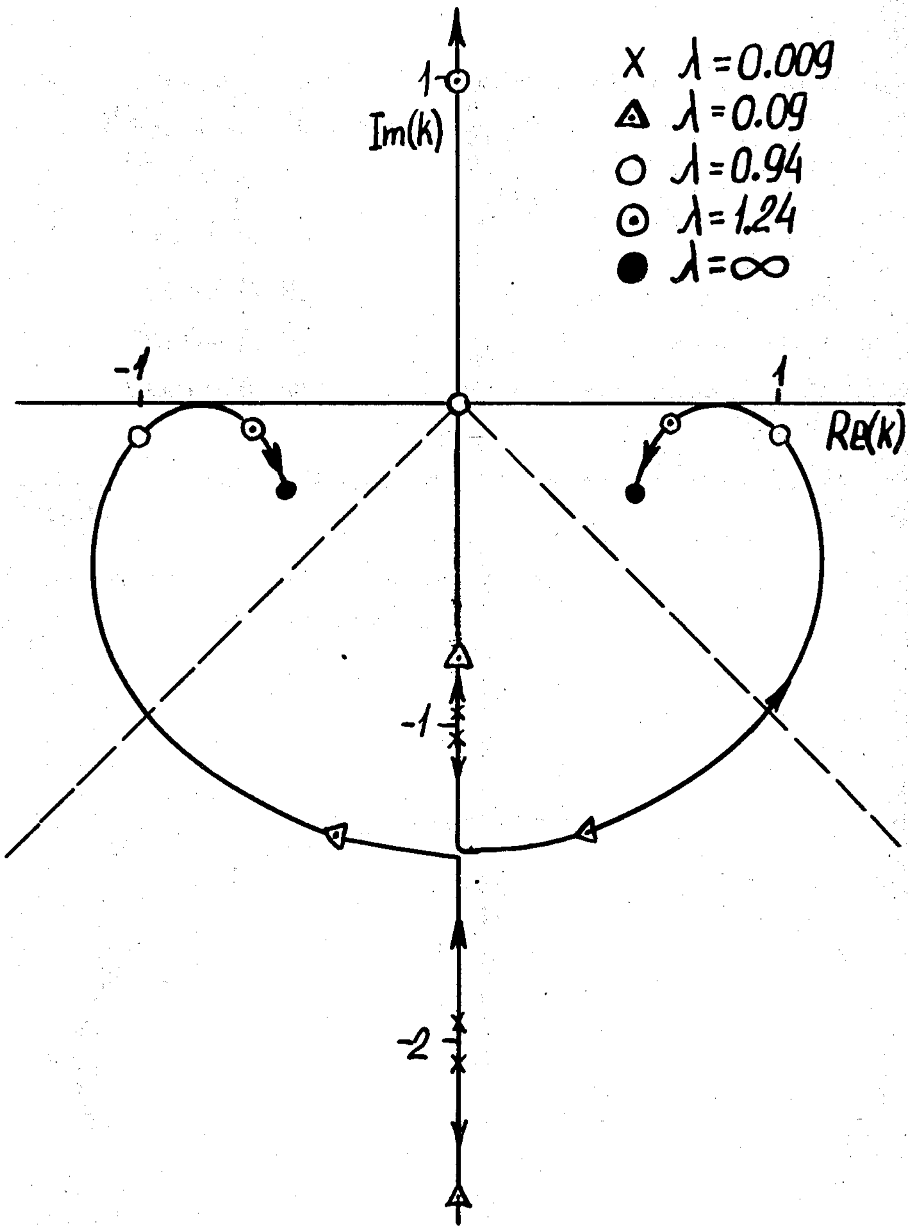

Fig. 1. The pole motion for the potential (10) with formfactor (11b) and $\alpha_2 = -2.83$, $\beta_2 = 2$. The curves in full line are paths described by the poles. The bisectors of the third and fourth quadrants are indicated by the dotted lines. The arrows show the direction of motion of the poles for increasing λ^2 .

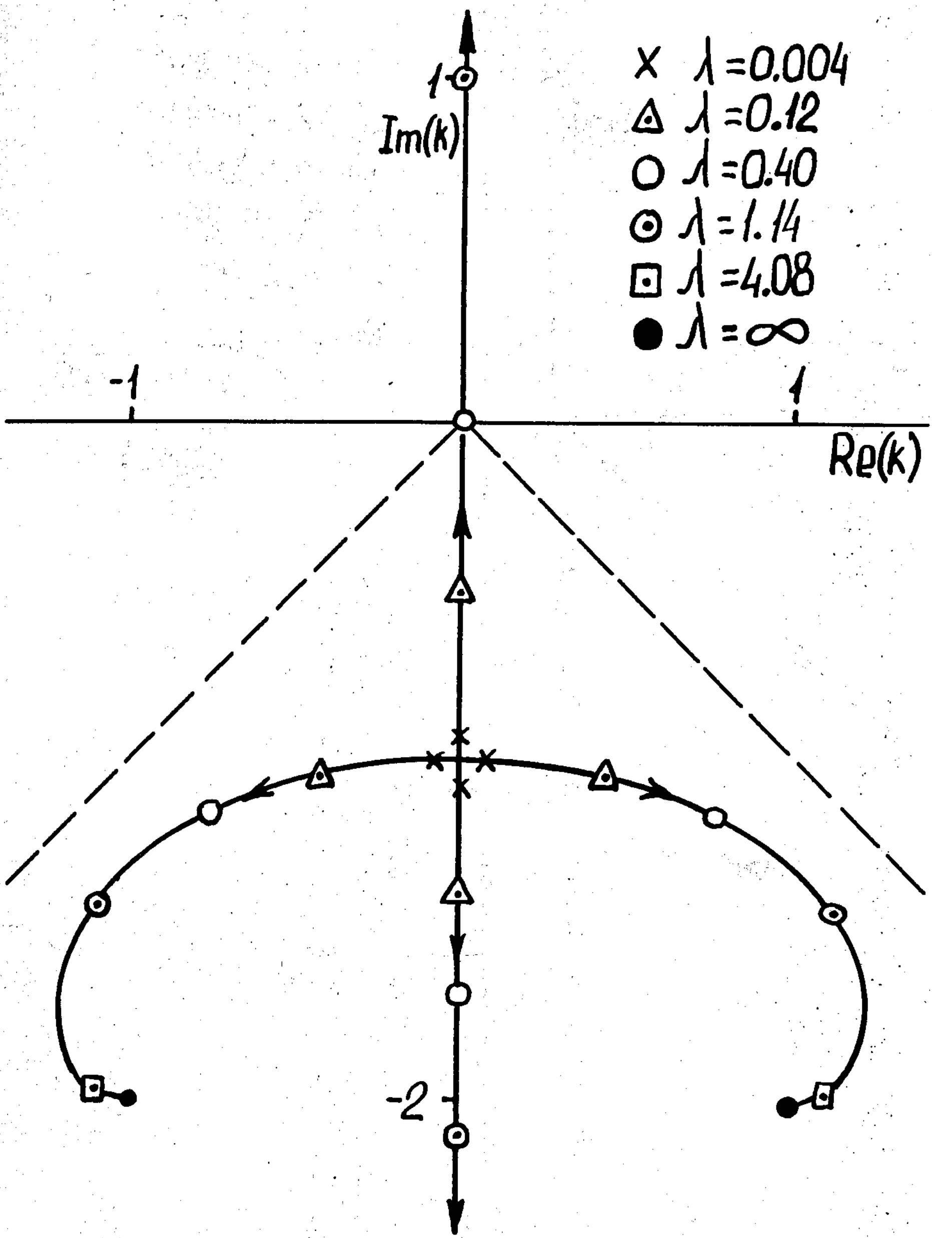

Fig. 2. The pole motion for the potential (10) with formfactor (12).

Fig. 2a: $\beta = 1$;

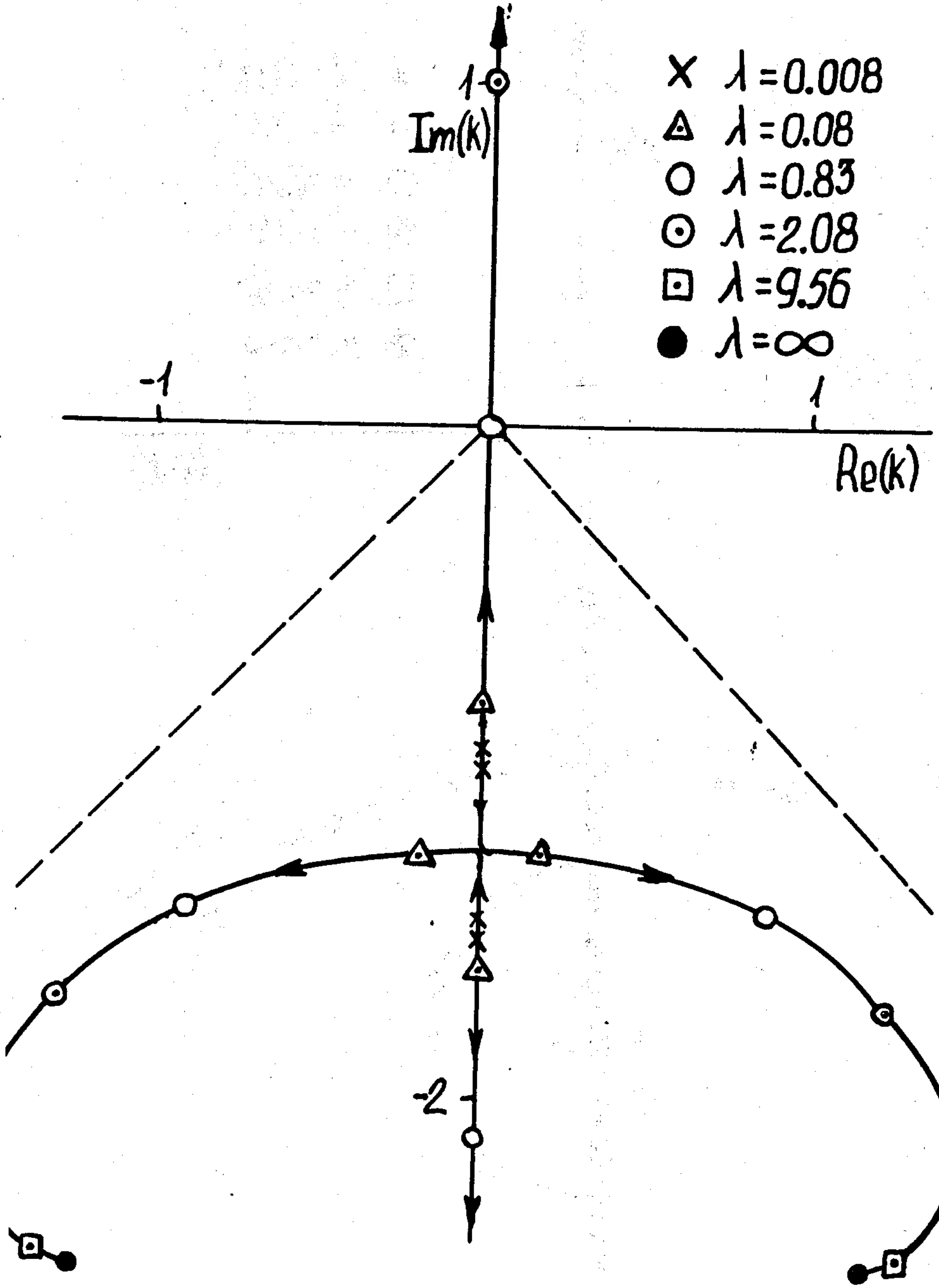

Fig. 2b: $\beta = 1.5$.

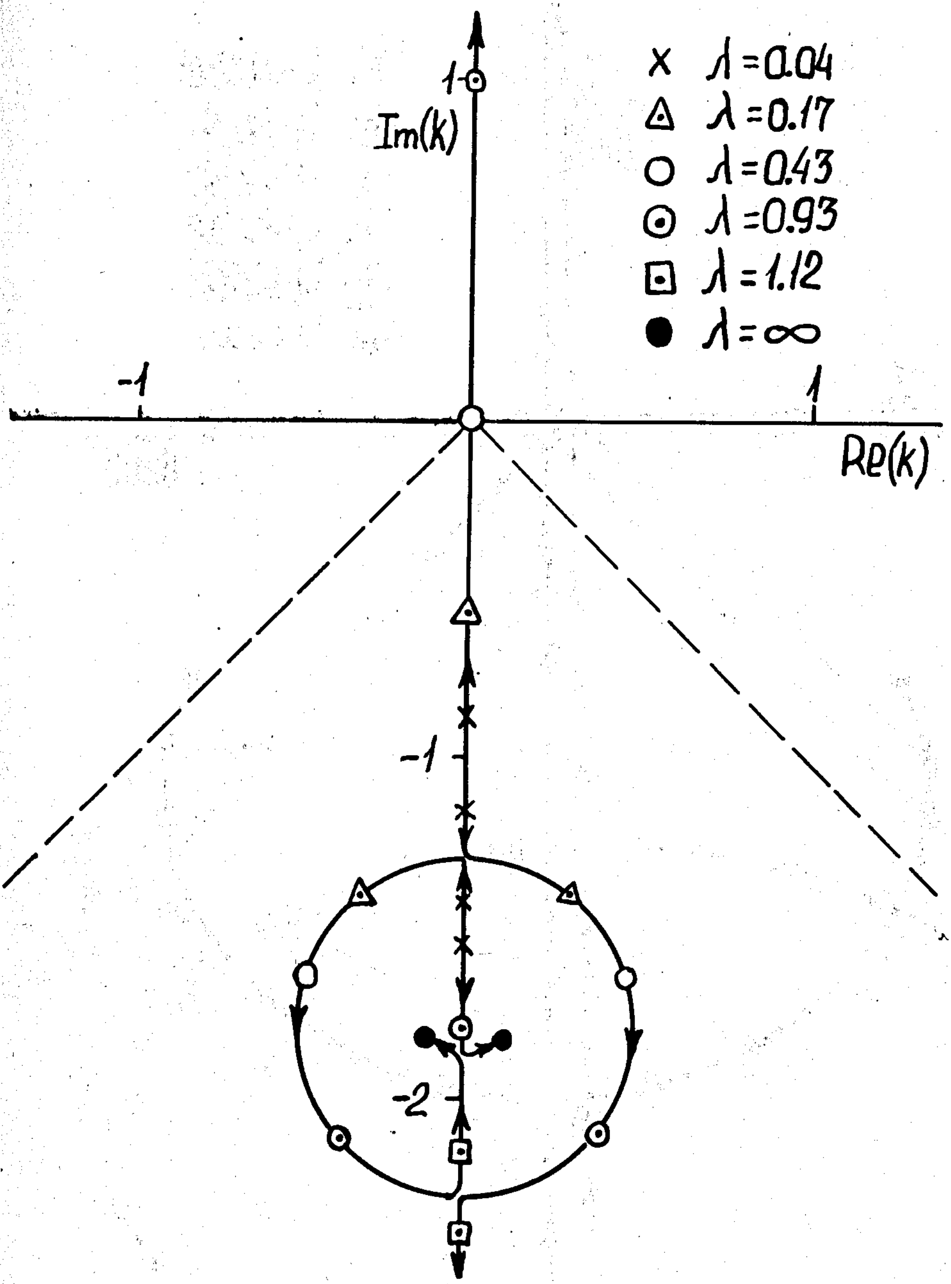

Fig. 3. The pole motion for the potential (10) with formfactor (13).
Fig. 3a: $\alpha_2=-0.5$; $\beta_2=1.5$

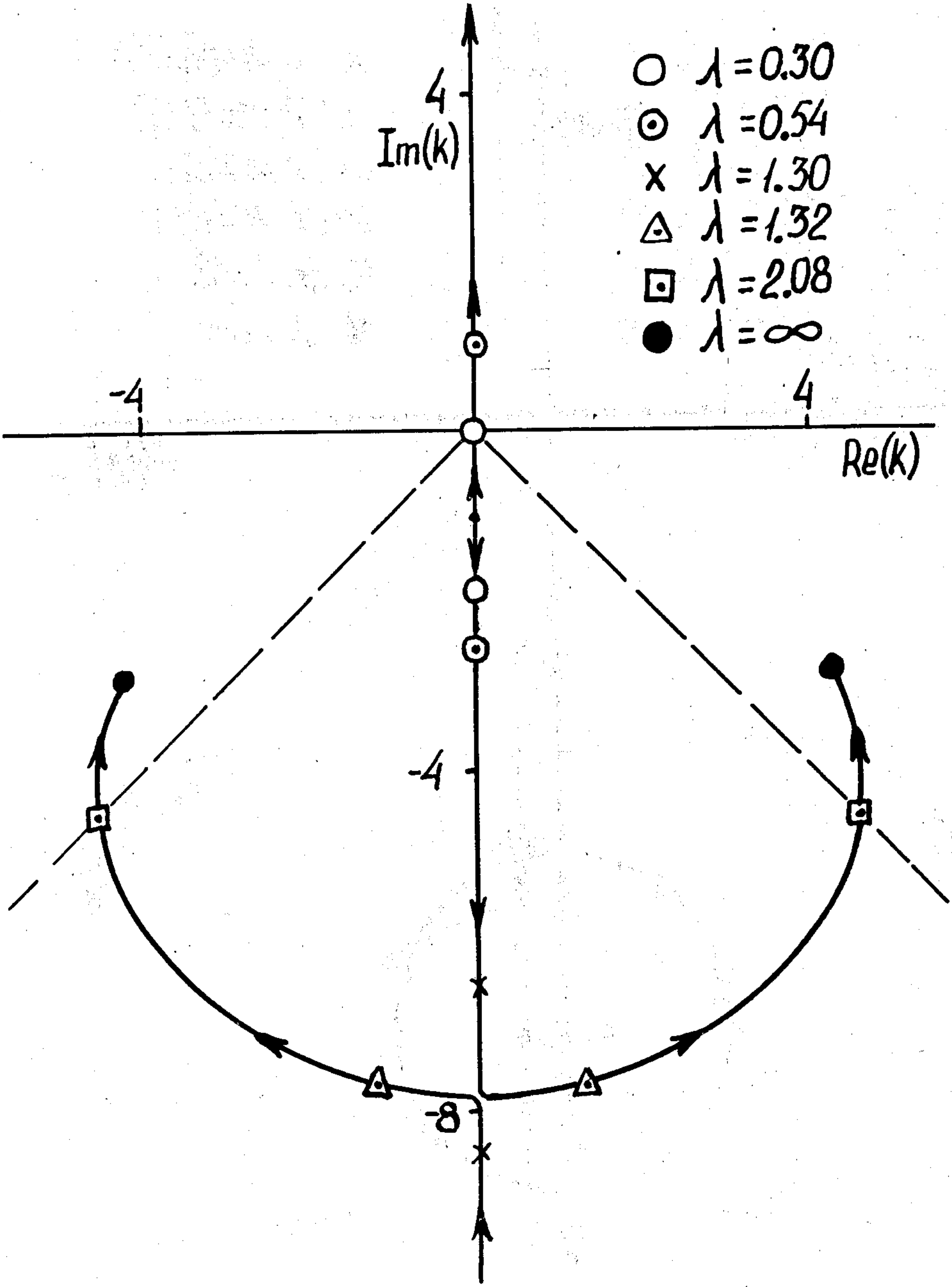

Fig. 3b: $\alpha_2 = 50$, $\beta_2 = 40$.

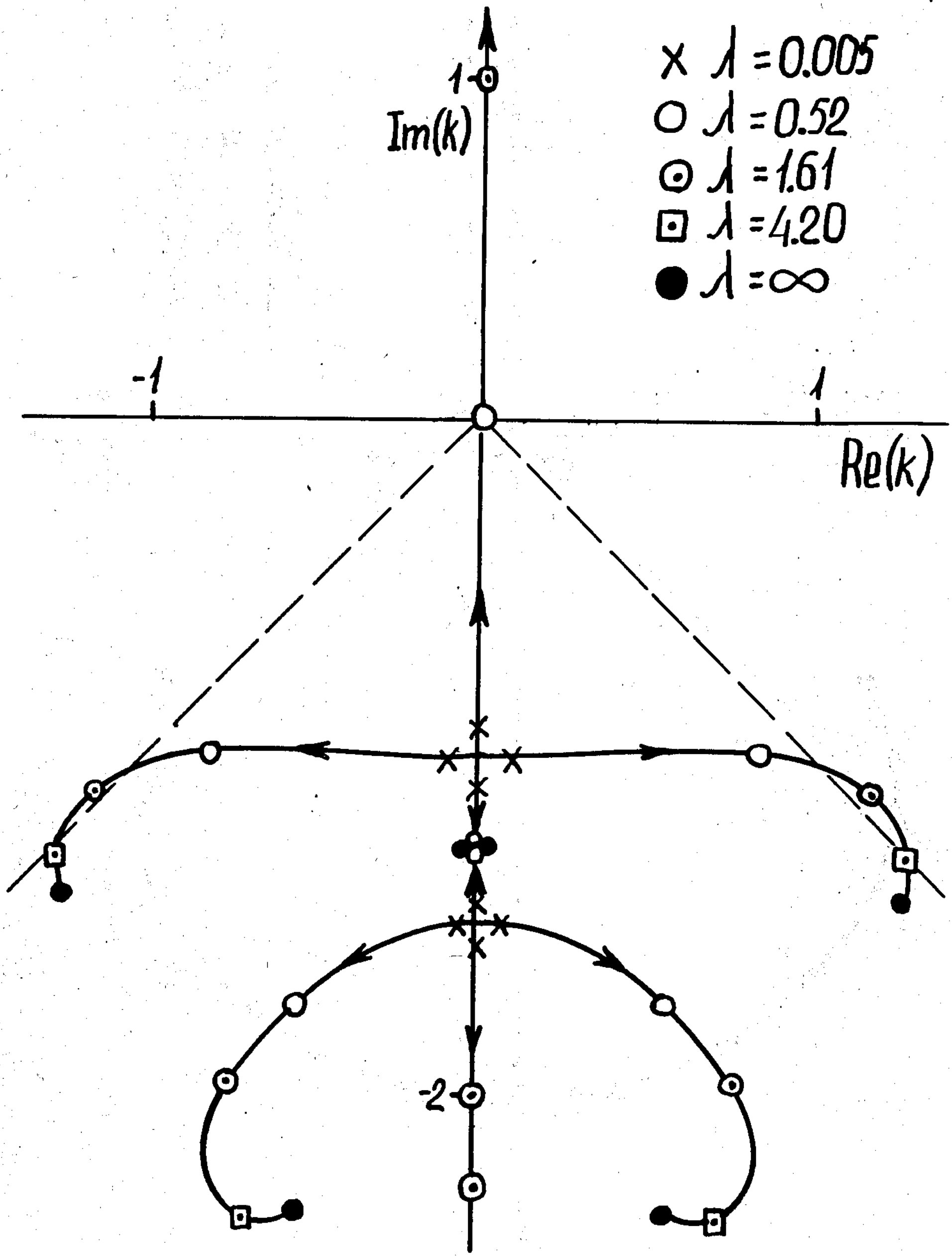

Fig. 4. The pole motion for the potential (10) with formfactor (14).
Fig. 4a: $a_2 = -1$, $\beta_2 = 1.5$;

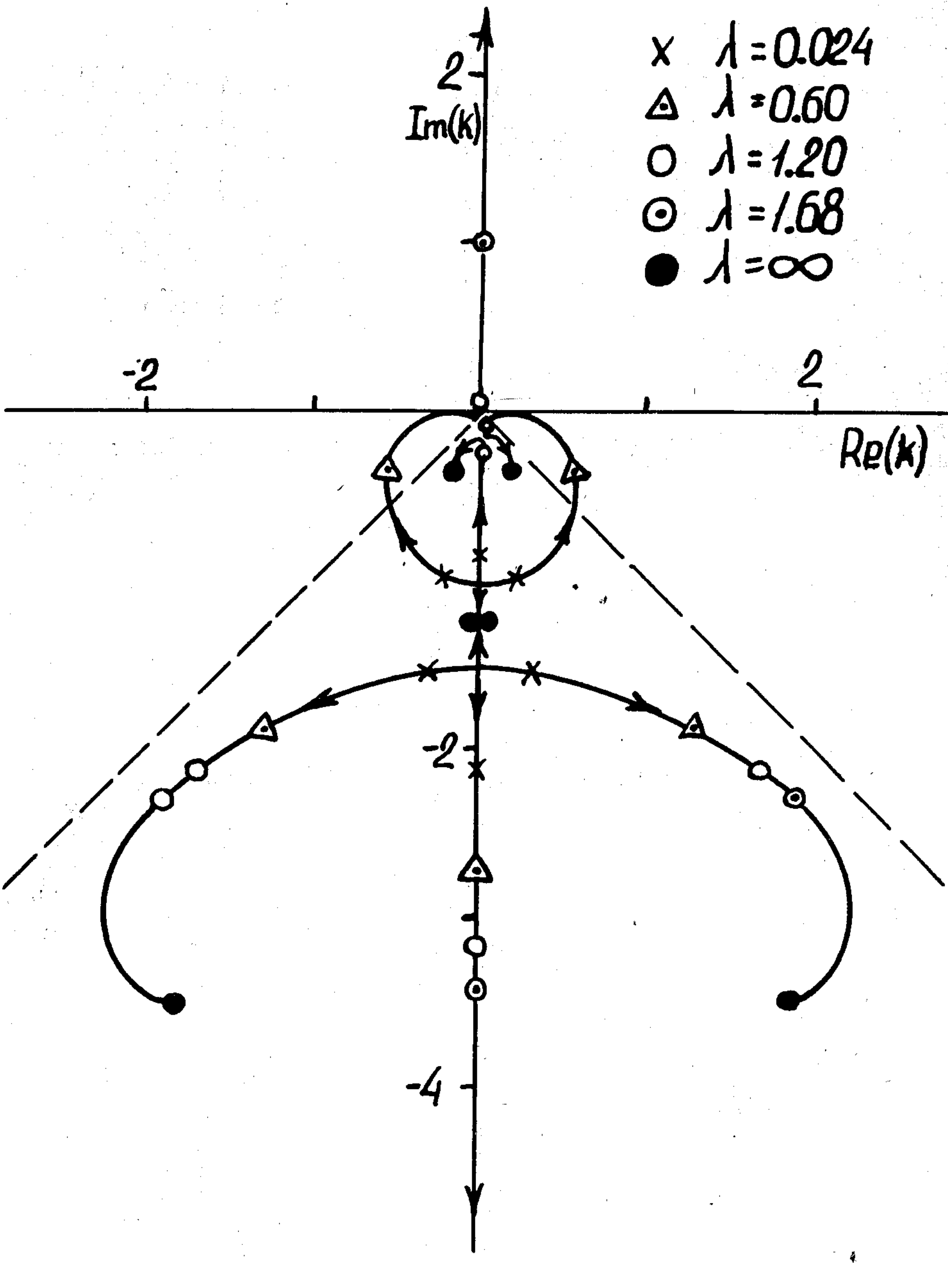

Fig. 4b: $\alpha_2 = -5$, $\beta_2 = 1.5$.