ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E4 - 5836

V.K. Lukyanov

MULTI-CHANNEL ASPECTS IN STRIPPING REACTIONS ON SPHERICAL AND DEFORMED NUCLEI

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

1971

E4 - 5836

V.K.Lukyanov

MULTI-CHANNEL ASPECTS IN STRIPPING REACTIONS ON SPHERICAL AND DEFORMED NUCLEI

Submitted to Symposium on Nuclear Three Body Problem and Related Topics,
Budapest, 8-11. July 1971.

Multi-channel aspects in the A(dp)B reaction, connected with a parting in stripping of the low-lying collective states of the A and B nuclei can be analysed in the framework of the so-called generalized distorted wave Born approximation[1] which, using the zero-range approximation, gives us the following amplitude.

$$T_{dp} = \langle \mathfrak{z}_p \sigma_p ; \Psi^{(-)}_{p+B} \mid \Psi^{(+)}_{d+A} , \mathfrak{z}_d \sigma_d \rangle . \tag{1}$$

Here $\Psi^{(\pm)}$ are the multi-channel entrance and exit wave functions. They may be calculated with the help of the coupled channel method[2], which finds out them as an expansion

$$\Psi^{(+)}_{d+A} = \sum_{\alpha} \Psi^{(+)}_{\alpha}(\vec{r}) \mid \alpha \rangle , \tag{2}$$

where $\mid \alpha \rangle$ denote nuclear state wave functions. Thus, the stripping cross section is

$$\sigma(\vartheta) = \frac{2J_B+1}{2J_A+1} \Big| \sum_{\alpha\beta n} \gamma_n^{\alpha\beta} \, t_n^{\alpha\beta}(\vartheta) \Big|^2 , \tag{3}$$

where $t_n^{\alpha\beta}$ are the kinematical amplitudes. The corresponding spectroscopic amplitudes $\gamma_n^{\alpha\beta}$ are given by relation

$$\gamma_n^{\alpha\beta} = \langle \beta \mid a_n^+ \mid \alpha \rangle . \tag{4}$$

It is found out from eq. (3) that the stripping mechanism can be interpreted as a multi-step one, where there exist additional indirect transitions through the collective states $\mid \alpha \rangle$ and $\mid \beta \rangle$. Otherwise, the result of the standard DWBA-method follows from eq. (3) neglecting all the intermediate transitions, and gives us the only one-step desoription of stripping:

$$\sigma(\vartheta) = \frac{2J_B+1}{2J_A+1} \, S_n^{AB} \, |t_n^{AB}|^2. \tag{5}$$

Here S_n^{AB} is the usual spectroscopic factor.

Unfortunately, the result (3) is in a rather abstract and non-obvious form. Thus, it is not possible to estimate the order of higher stripping steps, while the numerical calculations on the basis of eq. (3) are very complicated. Therefore, to simplify the problem we use later a so-called double-adiabatic approach. It consists of an approximate expansion of the multi-channel wave functions in the following set of partial waves:

$$\Psi_{d+A}^{(+)} = \sum_{\ell_d} \sqrt{\frac{2\ell_d+1}{4\pi}} \, \varphi_{\ell_d}(r, \vec{R}_d) \, Y_{\ell_d 0}(\vec{r}) \, |00A\rangle, \tag{6}$$

$$\Psi_{p+B}^{(-)*} = \sum_{\ell_p m} (-1)^{\ell_p} \varphi_{\ell_p}(r, \vec{R}_p) \, Y_{\ell_p m}(\vec{r}) \, Y_{\ell_p m}^{*}(\vec{k}) \, \langle J_B M_B B|, \tag{7}$$

where

$$|J_B M_B B\rangle = \sum_{\alpha I j \ell M \nu} \gamma_{Ij\ell}^{\alpha B} (IjM\nu|J_B M_B) R_\ell(r) \sum_{m_n \sigma_n} (\ell s_n m_n \sigma_n|j\nu) Y_{\ell m_n}(\vec{r}) |s_n \sigma_n\rangle \cdot |IM\alpha\rangle. \tag{8}$$

Here the initial ground state $|00A\rangle$ and final state $|J_B M_B B\rangle$ wave functions are included obviously, while the radial functions take into account an additional dependence on internal nuolear variables considered as parameters. Namely,

$$\varphi_\ell(r, \vec{R}) = \exp\left[\Delta(\vec{r}) R \frac{\partial}{\partial R}\right] \varphi_\ell(r, R), \tag{9}$$

where φ_ℓ are the radial functions in a central optical potential with the radius parameter R, and

$$\Delta(\vec{r}) = \sum_{\lambda\mu} \xi_{\lambda\mu} Y^{*}_{\lambda\mu}(\vec{r}) \qquad (10)$$

depends on the nuclear collective variables $\xi_{\lambda\mu}$.
It was shown[3], that expansions given by eqs. (6) and (7) are satisfied within the smallnest of $O(1/(kR)^{1/3})$ if the conditions $kR \gg 1$ and $E \gg E^{\alpha,\beta}_{ex}$ are fulfilled.

Substituting eqs. (6) – (8) into the amplitude (1) and using the relation

$$\langle IM\alpha | \exp[\Delta(\vec{r})(R_p\frac{\partial}{\partial R_p}+R_d\frac{\partial}{\partial R_d})] | 00A\rangle = \sqrt{\frac{4\pi}{2I+1}}\, \hat{A}_I(\alpha A)\, Y^{*}_{IM}(\vec{r}) \qquad (11)$$

it is possible to obtain the multistep cross section

$$\sigma(\vartheta) = \frac{2J_B+1}{2J_A+1} \sum_{Lm} \Big| C \sum_{\ell} \hat{B}^{L}_{\ell}\, \beta^{Lm}_{\ell}(\vartheta) \Big|^2 . \qquad (12)$$

Here the weight- and kinematical factors of the corresponding stripping steps are the following

$$\hat{B}^{L}_{\ell} = \sum_{\alpha I j} \delta^{\alpha B}_{Ij\ell} (-1)^{I+j-J_B}\, \hat{j}\,\hat{\ell}\, (\ell I00|L0)\, W(jJ_B\ell L; I s_n)\, \hat{A}_I(\alpha A), \qquad (13)$$

$$\beta^{Lm}_{\ell} = D_0 \sum_{\ell_p \ell_d} (-i)^{\ell_p} \frac{\hat{\ell}_p \hat{\ell}_d^{2}}{4\pi \hat{L}^{2}} (\ell_p \ell_d m0|Lm)(\ell_p \ell_d 00|L0)\, Y^{*}_{\ell_p m}(\vec{k}) \int r^2 dr\, \varphi_{\ell_p} R_{\ell} \varphi_{\ell_d} . \qquad (14)$$

This latter is in a form very similar to the one given by DWBA-method which therefore can be used here very well as a rather simple numerical method [4]. Our notions are

$$\hat{n} = \sqrt{2n+1} \;;\quad C^2 = \frac{m_p^{*} m_d^{*}}{(2\pi\hbar^2)^2} \frac{k_p}{k_d} \frac{1}{\hat{s}_n^{2}} . \qquad (15)$$

Now we analyze the results (12) – (14) in some details.

1. First of all, the following momentum and parity selection rules for the momentum transfer L

$$\vec{L} = \vec{\ell} + \vec{I} \tag{16}$$

$$\ell + L + I = even \tag{17}$$

hold.

Therefore, for the even intermediate exoitation momentum I the ℓ and L are the same parity, otherwise (I-odd) they are in opposite parities.

2. When I=0 (there are not inoluded intermediate excitations) the transferred momentum L coinoides with a captured neutron orbital mōmentum ℓ, and eq. (12) is simplified to the ordinary one-step result

$$\sigma(\vartheta) = \frac{2J_B+1}{2J_A+1} \sum_L \left| \gamma^{AB}_{0J_BL} \right|^2 \left| \sum_m c\beta^{Lm}_L(\vartheta) \right|^2 . \tag{18}$$

The non-vanishing one-step transition $(L=\ell;\ \gamma^{AB}_{0J_BL} \equiv \gamma_L \neq 0)$ may be oalled by <u>allowed</u> one and the corresponding part of the whole amplitude is the <u>principal</u> compoment. In the case of γ_L =0 the one-step transfer $L=\ell$ beoomes <u>forbidden</u> and stripping is going through intermediate oolleotive states ($I \neq 0$), so that the first and the next non-vanished steps are called the two-step and higher ones.

Generally speaking the multi-step cross seotion depends on the all speotroscopic amplitudes γ. They denote (see eq. (4)) the (quasi-) partiole $(\gamma^{\alpha\beta}_{0j\ell})$, (quasi-)particle + one phonon (or "roton") $(\gamma^{\alpha\beta}_{Ij\ell})$ and the other higher components in the whole odd nuoleus wave funotion (8) .

3. It is possible to obtain the two-step oomponents

of the stripping amplitude, which characterizes a virtual transition through the one-phonon (one-roton) intermediate state. For these we retain only the first term on Δ in a perturbation expansion of eq. (11). It gives us

$$\hat{A}_I = a_I \left(R_p \frac{\partial}{\partial R_p} + R_d \frac{\partial}{\partial R_d} \right), \tag{19}$$

where the value a_I is connected with a mass transition I- multipolarity operator $\hat{m}_{IM}$

$$a_I = \frac{\hat{I}\sqrt{4\pi}}{3A\, R_p^I} \langle IO \| \hat{m}_I \| 00 \rangle , \tag{20}$$

$$\hat{m}_{IM} = \sum_{i=1}^{A} r_i^{I} Y_{IM}(\hat{r}_i) . \tag{21}$$

Introducing a dimensionless effective charge e_{ef} and an effective nucleon number q_{ef} we obtain

$$a_I = \left(\frac{q_{ef}}{e_{ef}} \right) \cdot \frac{\hat{I}}{3+I} \mathcal{N}_I^{1/2} \frac{1}{A} , \tag{22}$$

where $\mathcal{N}_I$ is the $\underset{I \to 0}{B}(EI)$ - transition probability in the single particle units.

It is important that the magnitude of a_I can be obtained independently from inelastic scattering data[5] or calculated with the help of the experimental B(EI)-probabilities using $(q_{ef}/e_{ef}) \simeq A/Z$.

In a particular case, neglecting the operator $R_d \frac{\partial}{\partial R_d}$ in eq. (19) and putting the generalized form $-\frac{\partial}{\partial r}\left(U_p^{opt}/U_0 \right)$ instead of $\frac{\partial}{\partial R_p}$ into the radial integral of eq. (14), one obtains the result of previous works[6,7] where the so-called " core-excitation stripping model" was treated in the first order in the exit channel only.

4. To account the all orders in Δ in intermediate transitions one needs to use concrete nuclear models. For instance, basing on the results of ref. [5], the operator can be written in the framework of the surface phonon vibration and rotational models:

a) one-phonon transition $(0 \to I = 2^+, 3^-)$

$$\hat{\mathcal{A}}_I^{vib}(1,0) = b_I \left(R_p \frac{\partial}{\partial R_p} + R_d \frac{\partial}{\partial R_d}\right) F(R) \tag{23}$$

b) two-phonon transition $(0 \to \vec{I} = \vec{I}_1 + \vec{I}_2 \; ; \; I_{1,2} = 2^+, 3^-)$

$$\hat{\mathcal{A}}_I^{vib}(2,0) = b_{I_1} b_{I_2} (1+\delta_{I_1 I_2})^{-1/2} (I_1 I_2 00|I0) \left(R_p \frac{\partial}{\partial R_p} + R_d \frac{\partial}{\partial R_d}\right)^2 F(R) \tag{24}$$

c) rotational trasitions $(0 \to I = 2^+ 4^+ 6^+ \ldots)$

$$\hat{\mathcal{A}}_I^{rot} = \hat{I} \int_0^1 dx \, P_I(x) \, e^{\hat{\delta} P_2(x)}, \tag{25}$$

$$\hat{\mathcal{A}}_0^{rot} = 1 + \frac{1}{10} \hat{\delta}^2 + \ldots \qquad \hat{\mathcal{A}}_2^{rot} = \frac{1}{\sqrt{5}} \hat{\delta} \left(1 + \frac{1}{7} \hat{\delta} + \frac{1}{14} \hat{\delta}^2 + \ldots\right) \quad etc. \tag{26}$$

Here

$$F(R) = \exp \frac{1}{2} \beta_{ef}^2 \left(R_p \frac{\partial}{\partial R_p} + R_d \frac{\partial}{\partial R_d}\right)^2 , \tag{27}$$

$$b_I = \left(\frac{2I+1}{4\pi} \cdot \frac{\hbar}{2 B_I \omega_I} \right)^{1/2} , \tag{28}$$

$$\hat{\delta} = \sqrt{\frac{5}{4\pi}} \, \beta \left(R_p \frac{\partial}{\partial R_p} + R_d \frac{\partial}{\partial R_d}\right) , \tag{29}$$

where $\beta_{ef} = |\langle 0 | \xi^2_{\lambda 0} | 0 \rangle|^{1/2}$ is an effective deformation parameter and the relation $\left(R\frac{\partial}{\partial R}\right)^n = R^n \frac{\partial^n}{\partial R^n}$ must be fulfilled. It was shown[8] that in the case of rotational model the all steps in entrance (n_d) and exit (n_p) channels can be written separately, so that the cross section assumes a more obvious form

$$6^{rot}(\vartheta) = \frac{2J_B+1}{2J_A+1} \sum_{JL} S^{rot}_{J\ell} \sum_m \Big| \sum_{\ell\Lambda} a^2_{\ell\Lambda} (L\tfrac{1}{2}\Lambda\Sigma|J\Omega) c \sum_{n_p n_d n} \hat{B}^{n_p n_d n}_{L\ell\Lambda} \beta^{Lm}_\ell(\vartheta) \Big|^2, \quad (30)$$

where the weight and spectroscopic factors have the forms

$$\hat{B}^{n_p n_d n}_{L\ell\Lambda} = \hat{n}_p \hat{n}_d (n_p n_d 00|n0)^2 (nL0\Lambda|\ell\Lambda)(n\ell 00|L0) \hat{A}^{rot}_{n_p} \hat{A}^{rot}_{n_d} \quad (31)$$

$$S^{rot}_{J\Omega} = (1+\delta_{0K_A}+\delta_{0K_B}) \frac{2J_A+1}{2J_B+1} (J_A J K_A \Omega | J_B K_B)^2 . \quad (32)$$

Note that the well known perturbation result of ref.[9] is obtained from eq. (30) in a particular case, when only $n_{p,d} = 0,2$ are accounted and the first terms in $\hat{S}$ in eq. (26) are kept.

5. Now we estimate the order of higher step contributions (with γ_ℓ) to the allowed transfer (with γ_L). For this we represent qualitatively the kinematical part of an amplitude as follows[10]

$$\beta^{Lm}_\ell \sim e^{-\alpha\vartheta} \cos(kR\vartheta + \vartheta_L) . \quad (33)$$

Then, with the help of eqs. (12) - (22) the ratio of the second to the first stripping steps becomes

$$x = g \frac{kR}{Z} \frac{\hat{I}}{3+I} \mathcal{N}_I^{1/2} \frac{|\gamma_l|}{|\gamma_L|}, \qquad (34)$$

where g is a „correction factor" used here in order to account the vector additional coefficient contribution, etc.

For spherical nuclei one must take into account the phonon intermediate excitations. Supposing $kR \sim 10$, $I=2$, $Z=20$, $\mathcal{N}_I \sim 4 \div 10$, $|\gamma_l|/|\gamma_L| \sim 1 \div 10^{-1}$ and $g_{vib} \sim 0.5$ we obtain

$$x_{vib} \sim 0.3 \div 10^{-2}. \qquad (35)$$

For deformed nuclei it is necessary to take into consideration the different assymptotical exponential behaviour of the bound-state Nilsson-type spherical components. Assuming the surface character of the stripping reactions we account this fact, putting $g_{rot} \sim exp(\Delta\varkappa_{lL} R)$, where $\Delta\varkappa_{lL} = \sqrt{\frac{2mE_l}{\hbar^2}} - \sqrt{\frac{2mE_L}{\hbar^2}}$ is a difference between the magnitudes of the bound-state moments of the corresponding compbnents. Then, expressing the B(E2) probability as a function of the deformation parameter β we obtain

$$x_{rot} \sim \frac{\beta kR}{2\sqrt{5\pi}} \cdot \frac{|a_{l\Lambda}^{a}|}{|a_{L\Lambda}^{a}|} \cdot exp(\Delta\varkappa_{lL} R). \qquad (36)$$

It is known that in the odd light-weight deformed nuclei (A ~ 25) the low-lying states are formed from the closely disployed sphericel components, i.e. $\Delta\varkappa_{lL} \simeq 0$ and $g_{rot} \simeq 1$. Thus, for $kR \sim 10$, $\beta \sim 0.3$ and $|a_l|/|a_L| \sim 0.2 \div 1$

one obtains

$$x_{rot} \sim 0.5 \div 10^{-1} . \tag{37}$$

As to the heavy deformed nuclei it is clear from eq. (36) that in different cases the g_{rot} - factor may intensify or relax the higher step contributions depending on the fact which components must be necessarily taken into account in a process.

Thus, we find out that the higher steps can play an important role in the allowed transitions both in angular distributions and absolute cross sections. This thesis is confirmed by exact numerical calculations given e.g. in refs.[11,12]. It is clear that the forbidden transitions are the pure two- or higher step processes.

References:

1. S.K.Penney, G.R.Satchler. Nucl.Phys. 53 (1964) 145.
2. T.Tamura. Rev.Mod.Phys. 37 (1965) 678.
   S.I.Drozdov. Journ.of Nucl.Phys. 1 (1965) 407.
3. E.V.Inopin, A.V.Shebeko. Journ.of Nucl.Phys. 6 (1967) 279.
4. R.N.Bassel et al. Oak Ridge Nat.Lab.Tech.Report No. ORNL-3240, 1962.
   K.A.Gridnev et al. Preprint 2458, Dubna, 1965.
5. E.V.Inopin, A.V.Shebeko. JETP 51 (1966) 1761.
6. B.Kozlowsky, A. de-Shalit. Nucl.Phys. 77 (1966) 215.
7. I.E.McCarthy, Dipti Pal. Phys.Rev. C1 (1970) 2000.
8. V.K.Lukyanov. Phys.Lett. 34B (1971) 354.
9. P.J.Iano. N.Austern, Phys.Rev. 151 (1966) 853.
10. V.K.Lukyanov, I.Z.Petkov. Phys.Lett. 28B (1969) 368.
11. N.K.Glendenning. Proceedings Int.Conf. on Prop. of Nucl.States, Montreal, 1969.
12. H.Schulz et al. Nucl.Phys. A159 (1970) 324.

Received by Publishing Department
on May 26, 1971.