СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
Дубна

E4 - 5767

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

Christian Toepffer

POTENTIAL WELL DEPTH IN THE TWO-CENTER MODEL FOR HEAVY ION SCATTERING

1971

E4 - 5767

Christian Toepffer*

POTENTIAL WELL DEPTH IN THE TWO-CENTER MODEL FOR HEAVY ION SCATTERING

* CERN Travelling fellow.

Permanent address: Institut für Theoretische Physik
Universität Frankfurt, Frankfurt/Main, FRG.

The two-center shell model has recently been often employed in order to explain phenomena in nuclear physics, in which two nuclei interact, such as fission /I-3/, elastic heavy ion scattering /4,5/ and transfer reactions /6,7/. The relative motion of the nuclei takes place on potential energy surfaces (PES) $E(\vec{R})$, which are given by the energy of the nucleons moving in a shell model with two centers in a distance $\vec{R}$. For the calculation of the PES two extreme situations may be considered: Either the relative motion of the nuclei is so slow compared to the internal motion of the nucleons, that the original shells are destroyed and the shells of the compound nucleus are formed, or the relative motion of the nuclei is so fast, that a reversible compression of nuclear matter takes place during the overlap of the nuclei.

The first, so called adiabatic case, in which the volume is conserved throughout the process, is realized (in its time - reversed direction) in nuclear fission. For the case of fast motion one obtains from the analysis of elastic heavy ion scattering PES of the following type /8,9/. For large R one has Coulomb repulsion; near the overlap distance R_0 nuclear forces become important, this leads to an attractive region of the potential until for small distances there exists a strong repulsive core, which is explained by the incompressibility of nuclear matter.

Experiments on O^{16}-O^{16} scattering give for example a value of $E_{exp}(\vec{R}=0) \approx 120$ MeV. This is the energy needed to compress two O^{16} nuclei into the volume of one. This value is in good agreement with predictions from nuclear matter calculations /9,10/.

It seems however, that there is a large disagreement with the nuclear shell model. In the oscillator model the oscillator constant ω is /II/

$$\hbar\omega = \frac{0.86\, A^{1/3} (\hbar c)^2}{Mc^2 \langle r^2 \rangle} = \frac{0.86\, A^{1/3} (\hbar c)^2}{Mc^2 \frac{3}{5} \left(\frac{3}{4\pi} V\right)^{2/3}} \qquad (I)$$

If the volume is conserved this gives the well known $\hbar\omega = 41 A^{-1/3}$ rule. The total energy is in the oscillator model

$$E = \frac{3}{4} \sum \varepsilon_i = \frac{3}{4}\, 0.86\, \hbar\omega\, A^{4/3} = 98 \left(\frac{A}{V}\right)^{2/3} A \quad [MeV], \qquad (2)$$

where the volume V must be given in fm^3. This gives for the compression of the two O^{16} nuclei into the volume of one

$$E_{SM}(0) = (2^{2/3} - 1)\, 26\, A\, [MeV] = 490\ MeV. \qquad (3)$$

Thus the shell model overestimates the height of the core by $E_{SM}(0) - E_{exp}(0) = 370$ MeV. Also for distances $0 < R < R_0$ (Fig. Ib) the PES will turn out too large thereby predicting an attractive dip in the PES, which is too shallow in comparison with experiment /8,9/*).

The reason for this discrepancy is the following: In compressed nuclear matter the inter-nucleon distanoe becomes smaller, the average movement of the nucleons takes place in more attractive regions of the nucleon-nucleon potential. This must be desoribed in the shell model by a deeper potential well. For total overlap the oscillator potential should be lowered by

---

*) This fact was already noticed in ref. /4/. But unfortunately in this paper the authors employ a constant $\hbar\omega$ thrughout, while eq.(I) gives a $V^{-2/3}$ -dependence under oompression. Thus the compression energy is still underestimated and the attractive part of the potential overestimated.

$$V_0 = \frac{E_{exp}(0) - E_{SM}(0)}{\frac{3}{4}A} = -16.7 \text{ MeV} \qquad (4)$$

to give the correct compression energy (Fig.Ic). Of course a constant V_0 in the infinitively deep oscillator potential has no immediate physical meaning, but it should be noted, that

a) the oscillator is only a convenient approximation to realistic potential wells (dashed-dotted lines in Fig.I) which should become deeper by an amount similar to V_0 in the case of total overlap and

b) it is not sufficient to fit two oscillators together to get the shell model potential in the case of partial overlap. Instead, in the region of compressed nuclear matter the shell model potential becomes lower (Fig.Ib). The exact form of the potential in this case may be obtained either by a more or less heuristic interpolation [I2] or by nuclear matter calculations, in which the variation of the local density is taken account of.

Considering the microscopic Hamiltonian

$$H = \sum_i (T^i + \tfrac{1}{2} V^i) = \sum_i (T^i + \tfrac{1}{2} \sum_j V_{ij}) \qquad (5)$$

one can estimate the variation in the matrix-elements V_{ij} of the nucleon-nucleon interaction upon compression. The kinetic energy is given by the Fermi-gas formula

$$\sum_i T^i = 74 \left(\frac{A}{V}\right)^{2/3} A \qquad (6)$$

which gives for the total overlap of two O^{16} nuclei a change of $\sum_i (\Delta T^i) = +370$ MeV, which must be compensated by a change of $\frac{1}{2}\sum_i (\Delta V^i) = -250$ MeV to give the experimental value of I20 MeV for the compression energy. Now there is with eq.(4).

$$A V_r = A \Delta V^i = A \sum_j \Delta V_{ij} = -16.7 \cdot A \; MeV \quad (7)$$

which gives

$$\overline{\Delta V_{ij}} = \frac{V_c}{A} = -0{,}5 \; MeV \quad (8)$$

as an average value, by which the nucleon-nucleon interaction becomes more attractive when nuclear matter is compressed by a factor 2.

R E F E R E N C E S .

I. N.A.Cherdantsev, V.E.Marshalkin, Bull.Acad.Sci.,USSR,30, (1966), Transl.1967.

2. K.Dickmann, K.Dietrich, Nucl.Phys., AI29, 24I (1969).

3. P.Holzer, U.Mosel, W.Greiner, Nucl.Phys. AI38 , 24I (1969).
D.Scharnweber, U.Mosel, W.Greiner, Phys.Rev.Lett., 24, 60I (1970).
K.Albrecht, D.Scharnweber, U.Mosel, W.Greiner, Phys.Lett., 32B, 229 (1970).
W.Greiner, Contribution to the International Conference on Heavy Ion Scattering, Dubna, (197I), and further references therein.

4. K.Pruess, W.Greiner, Phys.Lett., 33B , I97 (1970).

5. U.Mosel, T.D.Thomas, P.Riesenfeldt, Phys.Lett., 33B, 565 (1970).

6. C.Toepffer, Contribution to the International Conference on Heavy Ion Physics, Dubna (197I).

7. K.Pruess, Effective Schrödinger Equations for the Elastic Interaction of Heavy Ions, preprint, Universität Frankfurt (1971).

8. R.H.Siemssen, J.V.Maher, A.Weidinger, D.A.Bromley. Phys.Rev. Lett., 19, 369 (1967).
R.J.Munn, B.Block, F.B.Malik, Phys.Rev.Lett., 21, 159 (1968).
K.A.Brueckner, J.R.Buchler, M.M.Kelly, Phys.Rev., 173, 944, (1968).

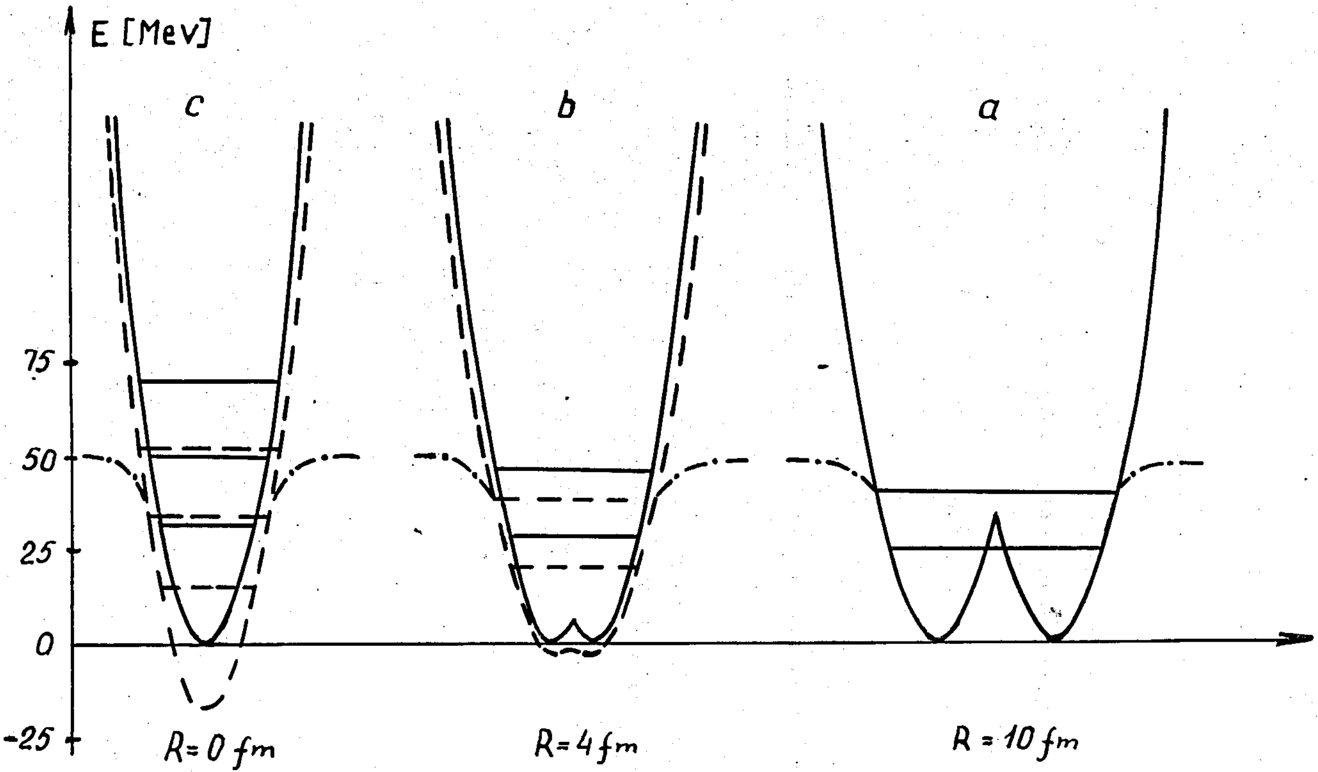
9. W.Scheid, R.Ligensa, W.Greiner, Phys.Rev.Lett., 21, 1479 (1968).
W.Scheid, W.Greiner, Z.f.Physik, 226, 364 (1969).

10. K.A.Brueckner, J.R.Buchler, S.Jorna, R.J.Lombard, Phys.Rev., 171, 1188 (1968).

11. S.A.Moszkovski in Handbuch der Physik, vol.39, ed. S.Flügge, Springer-Verlag, Berlin (1957).

12. T.Morović, Private Communication.

Received by Publishing Department
on April 24, 1971.



F i g.1. Potential wells at different distances R between the two centers before (solid lines) and after (dashed lines) correction of the well depth. The dashed-dotted lines indicate finite wells.