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CALCULATIONS OF PROPERTIES OF THE THREE-NUCLEON SYSTEMS WITH LOCAL POTENTIALS

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In the last years the sensible progress in a model-independent description of properties of the three-nuoleon system has been made. The model-independent description is given by the correct dynamical equations - the Faddeev equations, and quite effective methods for the solution of these equations.

Almost all the methods for solving the Faddeev equations can be divided into two groups. The first group contains the methods[1] which enable one to derive the approximate solutions of the exact equations. The second group includes the methods[2] for obtaining the exact solutions of certain approximate equations. The above approximation consists in the replacement of the exaot two-body t-matrix by superposition of the faotorized terms. It means that within the second group methods the approximation of three-body problem is made at the level of the two-partiole problem. This makes it possible to introduce there the oriteria to oharacterize the degree of approximation in a way independent of the quantity calculated.

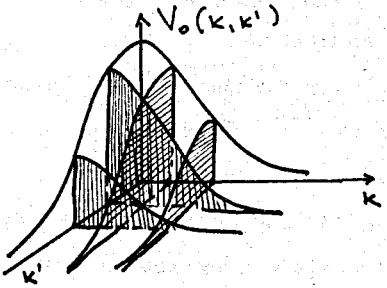
Hereafter one of such possibilities of solving the Faddeev equations based on the so-called Bateman method[3] will be considered The prooedure developed turns out to be rather universal and permits one to solve the Faddeev equations for the local potentials of an arbitrary shape. The method requires no changes if one proceeds from the bound-state description to the scattering problem.

Thus let the l-st partial wave projection of the potential in the momentum representation be expressed as $V_l(\kappa,\kappa')$

$$V_l(\kappa,\kappa') = \frac{1}{2\pi^2}\int_0^\infty j_l(\kappa z) j_l(\kappa' z) V(z) z^2 dz .$$

Without loosing the community in assertions we consider the oase l=0 for the Yukawa potential.

As a point of departure we apply to the plot of the function



Using the Bateman teohnique let us cut the surface $V_0(\kappa,\kappa')$ by the planes whioh are parallel to the ooordinate planes and construct the approximate surface $\tilde{V}(\kappa,\kappa')$ coinoiding with the exact one along the lines of intersections of those planes with the surface $V_0(\kappa,\kappa')$. The function $\tilde{V}(\kappa,\kappa')$ is of the form

$$\tilde{V}(\kappa,\kappa') = \sum_{i,j=1}^{N} \left[d^{-1}\right]_{ij} V_0(\kappa,s_i) V_0(s_j,\kappa') , \qquad (1)$$

where $d_{ij} = V_0(s_i,s_j)$ and s_i are, for the moment, arbitrary parameters.

In such a way we possess the approximate factorized potential $\tilde{V}(\kappa,\kappa')$ which coinoides with local one $V_0(\kappa,\kappa')$ along N lines $V_0(\kappa,s_i)$. Hence, it is also olear that ohoosing number N sufficiently large and plaoing the points s_i in a proper way it is possible to

approximate as good as necessary the local potential $V_0(k,k')$ by the expression (1). It can be shown[4], if one needs, that for the appropriate choice of the points s_i the series (1) converges uniformly. So far for all the short-range potentials the functions $V_\ell(k,k')$ decrease with increasing k and k' the convergence of (1) will be the faster the stronger this decrease. Finally, we shall fix the parameters s_i by minimizing the mean-square deviation χ^2 of the approximate potential from the exact one:

$$\chi^2(s_1 \dots s_N) = \frac{\int\int |V(k,k') - \tilde{V}(k,k')|^2 dk\, dk'}{\int\int V^2(k,k') dk\, dk'} . \qquad (2)$$

The quantity χ^2 can serve for a criterion of calculation accuracy*) since no other approximations are made.

Now, solving the Lippman-Schwinger equation with a potential of the type (1) for the partial t-matrix $t_\ell(k,k',z)$ we find

$$t_\ell(k,k',z) = \sum_{i,j=1}^{N} [C^{-1}(z)]_{ij} V_\ell(k,s_i) V_\ell(s_j,k') , \qquad (3)$$

where

$$C_{ij}(z) = d_{ij} + 8\pi \mu_{12} I_{ij}(z)$$

$$I_{ij}(z) = \int_0^\infty \frac{V_\ell(k,s_i) V_\ell(k,s_j) k^2 dk}{k^2 - 2\mu_{12} z - i\varepsilon}$$

and μ_{12} is the reduced mass of the colliding particles.

The expression (3) is the most convenient for the potentials

*) Note that the actual accuracy of computing some quantities can be and, in fact, is higher than an estimation of (2).

of which the Fourier transforms are expressed in the explicit analytic form (e.g., for superpositions of the Yukawa potentials).

Nevertheless, there exist phenomenological potentials such as, for instance, the Hamada–Johnston potential to which the Bateman method cannot be applied immediately. In this case the functions $V(k,s_i)$ in (1) should be replaced, for example, by the interpolating functions $P(k,s_i)$ /4/ coinciding with $V(k,s_i)$ at the points $s_1, s_2, \dots s_N, s_{N+1}, \dots s_{N+M}$.

Then the expression

$$\tilde{V}(k,k') = \sum_{i,j=1}^{N} [d^{-1}]_{ij} P(k,s_i) P(s_j,k') \qquad (1a)$$

$$d_{ij} = V(s_i,s_j)$$

coincides with the Fourier transform of potential at the points

$$\tilde{V}(s_i,s_j) = V(s_i,s_j) \qquad 1 \le i \le N$$

$$\tilde{V}(s_j,s_i) = V(s_j,s_i) \qquad 1 \le j \le N+M .$$

Just as (1) the expression (1a) converges uniformly.

Next, if the t-matrix is chosen in the form (3), the Faddeev equations for the definite momentum states reduce to the system of the one-dimensional equations. In particular, these were solved for the following problems: three-baryon bound state (H^3, He^3, $_{\Lambda}H^3$) and nd-scattering at zero energy of neutrons. Calculations were performed with two "realistic" potentials describing $^3S_1$ and $^1S_0$ – phase shifts of NN-scattering in the energy range from 0 up to 300 – 400 MeV as well as with a number of potentials without repulsion.

Let us list here the potentials used

$$V_{1m}(r) = V_{0m}\left[e^{-2(r-r_m)/a_m} - 2e^{-(r-r_m)/a_m}\right] \qquad (4)/5/$$

$$m = 1,2;3.$$

Table I. Parameters of the potential (4).

Potential	System	a_0 (fm)	r_0 (fm)	V_{0m} (Mev)	a_m (fm)	r_m (fm)
V_{11}	$^1S_0$, np	-23.678±0.028	2.44±0.11	61.99	0.3957	0.9365
V_{12}	$^1S_0$, nn	-17 ± 1	2.84±0.03	40.38	0.4799	1.0531
V_{13}	$^3S_1$, np	+5.397±0.011	1.727±0.013	119.49	0.3408	0.8668

(a_0, r_0 are the scattering length and effeotive range, respeotively)

$$V_{2m}(r) = \lambda_{1m}\frac{e^{-\mu_{1m}r}}{r} + \lambda_{2m}\frac{e^{-\mu_{2m}r}}{r} \qquad (5)/6/$$

$$m = 1,2,3$$

Table II. Parameters of the potential (5).

Potential	System						
V_{21}	$^1S_0$, np	-23.3	2.8	-2.64	7.39	1.55	3.11
V_{22}	$^1S_0$, np	-23.0	2.6	-3.273	7.333	1.707	3.66
V_{23}	$^3S_1$, np	5.45	1.8	-3.22	7.39	1.55	3.11

In Table III there are presented the calculation results for the doublet scattering length ^2a and the tritium binding energy E_T as functions of the singlet radius r_{os}. These have been computed for the following four types of potentials without repulsion: S – square-well potential, G – the Gaussian potential, E – exponential potential and H – the Hulten potential*).

Table III.

r_{os} (fm)	$^2a(S)$ (fm)	$^2a(G)$ (fm)	$^2a(E)$ (fm)	$^2a(H)$ (fm)	$E_T(S)$ (MeV)	$E_T(G)$ (MeV)	$E_T(E)$ (MeV)	$E_T(H)$ (MeV)
2.5	0.33	–0.08	–0.4		9.15	9.33	9.4	
2.6	0.52	0.11	–0.21		8.93	9.15	9.12	
2.704	0.71	0.285	–0.02	–1.8	8.72	8.96	9.02	10.5
2.8	0.9	0.46	0.17		8.52	8.76	8.83	

Two-particle parameters in Table III are as follows:

a_{os}=–23.714 fm., a_{ot}= 5.4 25 fm., r_{ot} =1.749fm.

Throughout all the calculations there has been used t-matrix (3) with N=4, the quantity $\sqrt{\chi^2_{N=4}}$ being equal to 6%, but the parameters a_o and r_o calculated differ from the initial ones only by about 1%.

To economize "paper space" we do not present here the results

*)The calculations were based on the separable representation of the two-particle t-matrix by the Bateman method for the S + E – potentials and for the Hulten potential H the expansion (1a) was used.

for the spinless systems calculated by the same method and only point out the rapid convergence of the prooedure as for the soattering length so for the binding energy[7].

Further, Table IV lists the oaloulation results for the doublet soattering length ^2a and the tritium binding energy E_T as well as the experimental data on ^2a and E_T.

Table IV.

Potentials	/7/ $V_{11}V_{13}$	/7/ $V_{12}V_{13}$	/7/ $V_{22}V_{23}$	/7/ $V_{21}V_{23}$	Experimental data
^2a (fm)	0.54	1.33	0.68	1.2	0.7 ± 0.3[8] 0.15 ± 0.05[10] 0.48 ± 0.05[9]
E_T (MeV)	9.12	8.10	8.90	8.56	8.48
r_{os} (fm)	2.44	2.7	2.6	2.8	

From Table IV it can be seen that for the "realistio" potentials the dependence of the binding energy and doublet soattering length on the magnitude of the singlet radius is considerably stronger than on the funotional shape of potential. The above situation is oompletely opposite that for the calculations with the potentials without repulsion (which desoribe the data on "effective range" only), where the shape dependence is more pronounoed (see Table III and Refs.[1,12,13]). If this fact is not aocidental then computing of ^2a and E_T with the "realistio" potentials apparently gives

no possibility to distinguish between various shapes of potentials. If all the results of calculations of E_T and ^2a with the potentials S – H are plotted as $E_T(r_{os})$ and $^2a(r_{os})$ then one obtains the straight lines and it turns out that the most close to the realistic case is just the result calculated with the square-well potential without repulsion.

Table IV indicates also that because of large error the oldest experimental data [8] do not allow to fix the definite variant of calculations. The most consistent with the experiment /9/ are the calculations with $r_{os}=2.4$. However, if the correct experiment is the experiment /10/ then the essential corrections to the present knowledge about the nature of NN-interactions are necessary in order to agree the experimental data with the caloulations. One of the possibilities to obtain the above consistenoy with the experiment /10/ is as follows.

If we take the potential of the form

$$V(r)=\begin{cases}\lambda\delta(r-r_c)+\Psi(r) & , r\le r_c\\ V_1(r) & , r>r_c\end{cases} \qquad (6)$$

then it is easy to see [14] that the phase shift of the scattering on such potential does not depend on Ψ and NN-wave function derived with this potential ($\Psi\neq 0$) has the node $u(r_c)=0$. Choosing the functions Ψ in the form $\Psi^s=V\delta(r-r_1)$, $\Psi^t\equiv 0$ $r_1<r_c$ and the attraction as

$$V_1(r)=\begin{cases}-V_1 & , r_c<r\le r_0\\ 0 & , r>r_0\end{cases} \qquad (7)$$

where V_1^t =48.05 MeV, r_c^t =0.17 fm., r_0^t =1.87 fm,
V_1^s =32.3 MeV, r_c^s =0.177 fm, r_0^s =1.9 fm.

we get the values of the doublet scattering length ^2a and tritium binding energy E_T listed in Table V [x].

Table V.

V	r_1 (fm)	^2a (fm)	E_T (MeV)
10^4	0.06	-7.10^{-3}	
10^4	0.1	0.267	10.5
10^4	0.14	0.5	8.8

Thus, we can conclude that the very strong dependence of the three-body quantities on the off-shell forces Ψ takes place. This is due to the strong dependence of the behaviour of the two-nucleon wave function $u(r)$ on the shape of Ψ at $r < r_c$. Since the behaviour of $u(r)$ at $r < r_c$, at least in the case of singlet scattering, is not known then it is quite possible to introduce the off-shell forces Ψ in singlet state of which the shape and parameters are derived from the three-body problem.

Further, the results of the following calculations: the charge- and magnetic radii of He^3 and H^3 nuclei, the admixture of the mixed symmetry state P_S', the quartet scattering length and axial matrix element $|\int \vec{\sigma}|^2$ for beta-decay of tritium with the potentials (4) and (5)/15/ are presented in Table VI.

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x) It should be noted that the potential (7) provides too low value of $r_{0S} = 2.15$ fm.

Table VI.

Potentials	$V_{11}V_{13}$	$V_{21}V_{23}$	Experimental data
$P_{S'}$	4.7%	2%	
^4a (fm)	6.35	6.37	6.45±0.05
$R_m(H^3)$ (fm)	1.52	1.71	1.7 ±0.05
$R_m(He^3)$ (fm)	1.48	1.70	1.74±0.05
$\Delta=4R^2_{ch}(He^3)-R^2_{ch}(H^3)$	10.24	11.24	11.1±0.58
$\lvert S(\vec{\sigma})\rvert^2$	2.63	2.83	2.84±0.06 /16/

Aside from the quartet soattering length whioh is the same for both potentials all the given results testify to the potential (5) with the value τ_{os} =2.8 fm. The reasons of suoh a differenoe between the potentials (4) and (5) are as follows: First, the repulsion in the singlet part of (4) is not enough that gives rise to inoreased values of the binding energy and deoreased values of the electromagnetio radii. Seoond, the attraotion in the triplet part of (4) is rather overrated beoause of whioh the value of $P_{S'}$ is rather higher and the value of axial matrix element differs from that of experiment for beta-decay. We also picture in Fig.I the dependenoe of ratio of charge form faotors of nuolei H^3 and He^3 on the momentum transfer squared under condition that the neutron form faotor is zero. Both the ourves relate to the calculations with the potential (5) but the solid ourve was found with the rough acoount of the Coulomb interaction in He^3 nucleus. The prooedure was as follows. As the protons in He^3 interaot in the singlet state so we have assumed the Coulomb potential to be responsible only for the deorease of the singlet

attraction. The decrease of attraction was chosen in such a way that the Coulomb energy E_c =0.76 MeV was reproduced.

The "body" form factors $F_0(q^2)$ and $F_L(q^2)$ were calculated by using the potentials (4) and (5) without the Coulomb interaction[15], too. The curve for calculations with the potential (4) is higher than the experimental one, the disagreement increases with the growth of q^2 which, in turn, shows the nonsufficient repulsion in the potential (4). As soon as the experimental data [19] on the electric form factor of He^3 in the range $8 \leq q^2 \leq 20$ fm^{-2} became available we computed this form factor with the potential (5). Our calculations (as in the paper[20]) do not result in the experimental minimum on the curve $|F_{ch}(q^2)|$ at $q^2 \sim 10$ fm^{-2}. This indicates probably the stronger repulsion at short distances than in the potential (5). The above assertion has been confirmed by the calculations of form factors of He^3 and T with the Hamada-Johnston potential [24] providing the minimum required experimentally.

So, if for the moment we do not take into consideration the recent experimental data on the doublet scattering length then we can say that the potential (5) with r_{os}=2.8 is the most preferable. The bound state wave functions obtained by using this potential apparently reveal in the most adequate way the low-energy properties of the system. This has "inspired" us to compute the reaction of μ-capture on He^3 nucleus. If the nuclear wave functions are known the probability of this reaction depends only on one unknown variable - the weak coupling constant g_p of the induced pseudoscalar interaction. By using the experimental value of probability equal to 1468 sec^{-1} for the pseudoscalar coupling constant we get

$$\frac{g_p}{g_A} = 5.33$$

Finally, we briefly review the problem of hypertritium. There exists the opinion (based on the calculations of light hypernuclei by the variational method[17]) that the parameters of ΛN-interaction employed to obtain the binding energy of light hypernuclei are somewhat different from those of ΛN-potential describing the elastic scattering of Λ-partioles on protons at low energies. By the above method we have calculated[18] the binding energy of ${}_{\Lambda}H^{3}$ with the potentials of ΛN- and NN-interactions of the form (4). The parameters of ΛN-potentials were fixed in such a way that the binding energy of ${}_{\Lambda}H^{3}$ and the cross section of Λp-scattering were reproduced simultaneously. It turned out that the agreement of these data was possible only for the repulsion radius (i.e. the radius at which a potential changes its sign) 0.55 or 0.65 fm (for the potentials of two types).

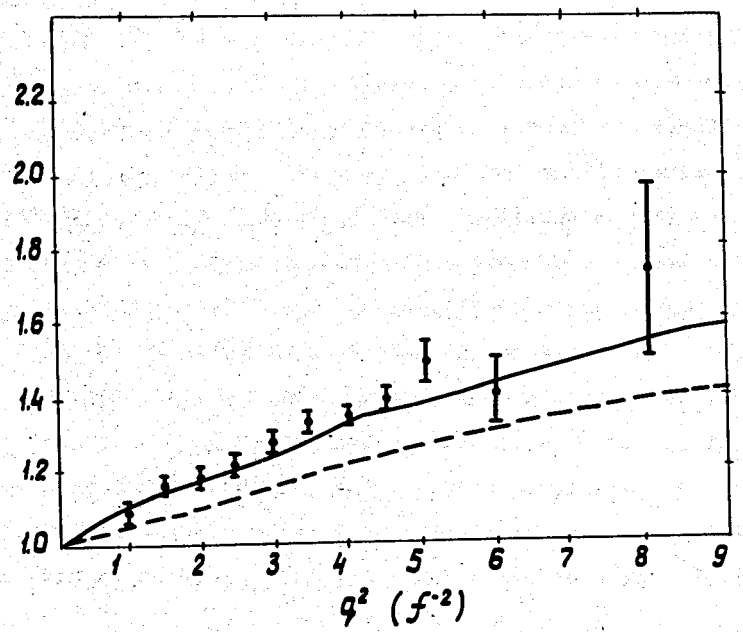

Fig. 1

REFERENCES

1. J.A.Tjon. Phys.Rev., D7, 2109(1970);
   Y.E.Kim. J.Math.Phys., 10, 1491(1969).
2. V.N. Efimov. Preprint JINR P-2890, Dubna,1966.
   G.Sitenko, V.F.Kharchenko, N.M.Petrov. Phys.Lett.,28B, 308(1968);
   V.B.Belyaev, J.Wrzecionko. Preprint JINR P4-4144, Dubna 1968.
3. H.Bateman. Proc.Roy.Soc., A100, 441(1922);
   L.V.Kantorovich, V.I.Crylov. Priblijennye methody vysshego analysa,
   Gostekhizdat, M. 1950;
   B.Akhmadkhodzaev, V.B.Belyaev, J.Wrzecionko. Yadernaya Fizika,
   11, 5(1970).
4. V.B. Belyaev, A.L. Zubarev. JINR Communications, P4-5510,
   Dubna (1970); Yadernaya Fizika, 14, No 8 (1971).
5. G.Darewych, A.E.S.Green. Phys.Rev., 164, 1324(1967).
6. R.A.Malfliet and J.A.Tjon. Nuol.Phys., A127, 161(1969).
7. B.Akhmadkhodzaev, V.B.Belyaev, J.Wrzecionko. JETP (Letters), 9,
   692(1969);
   V.B.Belyaev, J.Wrzecionko, A.L.Zubarev. Preprint JINR P4-5000,
   Dubna 1970; Yadernaya Fizika, 12, 923(1970).
8. D.G.Hurst, J.Aloock. Can.J.Phys., 29, 26(1951).
9. L.Koster and H.Ungerer. Z.Phys., B219, N3(1969).
10. W.T.Van Oers and I.D.Seagrave. Phys.Lett., 24B, 562(1967).
11. A.M.Badalyan et al. Yadernaya Fizika, 8, 313(1968).
12. L.P.Kok. Factorability and the Nuolear Three-Body Problem. Nova
    Press, Groningen 1969.
13. G.Sitenko, V.F.Kharohenko. Preprint ITPh 69-71, Kiev, 1969.
14. V.B.Belyaev, A.L.Zubarev. Preprint JINR P1-5345, Dubna, 1970.

15. B.Akhmadkhodzaev, V.B.Belyaev, J.Wrzecionko. Preprint JINR P4-4986, Dubna 1970.

16. R.C.Salgo, H.H.Staub. Nuol.Phys., A138, 417(1969).

17. R.C.Herndon, Y.C.Tang. Phys.Rev., 165, 1093(1968); 153, 1091(1967).

18. B.Akhmadkhodzaev, V.B.Belyaev, J.Wrzecionko. JETPh (Letters), 10, 557(1969).

19. J.S.McCarthy et al. Phys.Rev.Lett., 25, 884(1970).

20. T.A.Tjon, B.F.Gibson and J.S.O'Connell. Phys.Rev.Lett., 25, 540(1970).

21. M.A.Hennell and L.M.Delves. Preprint of University of Liverpool, England 1970.

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