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Recently [1] in the ^{130}Ba nucleus an isomeric state of an energy of 2.48 MeV has been found. There is some experimental evidence that the isomeric transition is the M2 transition to the 6^+ level of the quasirotational band. Hence we may believe that this state has 8^- spin and parity. The hindrance factor for such a transition is 10^7. The evidence for the existence of a similar 2.34 MeV isomeric state in the ^{132}Ce nucleus [2] permits us to assume that the both states are the two-quasiparticle neutron states. In ref. [1] isomerism of these states was accounted for by the K -forbiddenness. Assuming that the ^{130}Ba nucleus is deformed, i.e, the 6^+ state is the level of the rotational band, and the 8^- isomeric state is a two-quasiparticle one it is easy to find the degree of K -forbiddenness for the M2 transition to be equal to 6.

However, these assumptions are in disagreement with the following experimental results:

1. The ratios of the energies of the 8^+, 6^+, 4^+, 2^+ levels in this nucleus strictly differ from the corresponding ones in strongly deformed nuclei.

2. For strongly deformed nuclei the value of the hindrance factor, calculated per one degree of K -forbiddenness is of 1.5 ÷ 2 orders of magnitude while for ^{130}Ba it is somewhat larger than one order of magnitude.

Another possibility of explaining isomerism is the assumption that this isomeric state is a two-quasiparticle state in the spherical nucleus. Then the 6^+ state would essentially be a six-quasiparticle one and isomerism would arise due to the large difference in the number of quasiparticles for the initial isomeric and the final 6^+ state.

However, it is impossible to explain in such a way the absence of the two-quasiparticle levels below the 8^- state the decays to which would be allowed.

Thus, on the one hand, the absence of the two-quasiparticle levels below 8^- level in ^{130}Ba can be easily seen if this nucleus is assumed to be deformed. On the other hand, this assumption leads to the wrong values of the ratios of quasirotational level energies and to the large value for the hindrance factor.

On Fig. 1 and 2 the ground and excited [n 404↓, n 514↑] state energies are given as functions of the collective β variable. The two-quasiparticle [n 404↑, n 514↓] neutron state is the one of the lowest states in ^{130}Ba. The method of calculation is the same as in ref. /3/.

As may be seen on Fig. 1, the difference between the deformation energies for prolate and oblate shapes of nuclei in the ground state turns out to be small so that we are unable to assign to ^{180}Ba in the ground state some definite equilibrium shape. But at the same time the dependence of the excited state energy (see Fig. 2) on β enables us to assign to it a prolate shape. To describe these properties of ^{180}Ba the following model is just proposed.

Let us assume that:

1. The ground and quasirotational states may be considered in the framework of the Bohr-Mottelson model with γ -independent potential energy /4/ (see Fig. 1).

2. The coupling of nuclear collective motion with quasiparticle excitations leads to an additional dependence of the excited state deformation energies upon γ, such that the 8^- two-quasiparticle state may be considered in the framework of the collective model with the potential energy having a deep minimum at $\gamma = 0$. Then, according to ref. /4/ the quasirotational level wave functions are

$$\Psi(0^+) = N_0^{-1/2} F(\beta)$$

$$\Psi(2^+) = N_2^{-1/2} F(\beta)\, a^*_{2\mu}$$

$$\Psi(4^+) = N_4^{-1/2} F(\beta)\, [a^*_2\ a^*_2\ ]_{4\mu}$$

$$\Psi(6^+) = N_6^{-1/2} F(\beta)\, [[a^*_2\ a^*_2\ ]_4\, a^*_2\ ]_{6\mu},$$

where
$$N_I = 16\pi^2 \frac{(I/2)!}{(I+3)!!}$$

$$a_{2\mu} = D^2_{\mu 0}(\bar{\theta})\cos\gamma + \frac{D^2_{\mu 2}(\bar{\theta}) + D^2_{\mu -2}(\bar{\theta})}{\sqrt{2}}\sin\gamma ,$$

and
$$\int_0^\infty F^2(\beta)\,\beta^4\, d\beta = 1 .$$

Let us assume that the potential energy, being a function of β , has a minimum sufficiently deep to allow the replacement of β^2 by the equilibrium value in the expression for the moment of inertia. Then the energy ratios: $\frac{E(4^+)}{E(2^+)}$; $\frac{E(6^+)}{E(2^+)}$; $\frac{E(8^+)}{E(2^+)}$

for the quasirotational levels turn out to be 2.5; 4.5; 7 correspondingly. These values are in good agreement with the experimental ones: 2.54; 4.48; 6.86; for ^{130}Ba .

It is worthwhile to note that the wave functions (1) have components with different K'_s . For example, the contribution of the $K = 6$ component to the norm of the 6^+ state wave function is 5% (see Appendix 1). It means that in this model the existence of the isomeric state cannot be explained by the K – -forbiddenness alone.

For the collective potential energy, having a deep minimum at $\gamma = 0$, the 8^- two-quasiparticle state wave function is:

$$\sqrt{\frac{17}{24\pi^2\gamma_0^2}}\; F'(\beta)\, e^{-\frac{\gamma^2}{2\gamma_0^2}} \left(D^8_{M8} (a^+_s a^+_{s'})_{K=8} + \right.$$

$$\left. + D^8_{M-8} (a^+_s a^+_{s'})_{K=-8} \right) | 0 > ,$$

where

$$\int_0^\infty F'^2(\beta)\, \beta^4 \, d\beta = 1 .$$

The M2 transition from this state to the 6^+ quasirotational state is forbidden. Indeed, the 8^- two-quasiparticle state has $K = 8$, while the M2 transition operator may change the K -number of the quasiparticle maximum by two units.

The forbiddenness becomes weaker if one takes into account the interaction of quasiparticle excitations with γ -vibrations, the Hamiltonian of which has the form /5/

$$H_{int} = -\hbar\,\omega_0\,\beta_0\,\gamma_0 \sum_{s\sigma, s'\sigma'} f^{(22)}_{s\sigma, s'\sigma'}\, v_{ss'}\, a_{s\sigma}\, a_{s'\sigma'} .$$

Then the weakening of the forbiddenness is due to the appearance of the three-phonon component in the 8^- state wave function. The quasiparticle and collective parts of this component have $K = 2$ and $K = 6$ respectively. The contribution of this component to the norm of the 8^- state wave function calculated with the help of perturbation theory is found to be equal to 10^{-5} (see Appendix 2). Thus, within the above mentioned assumption, the transition $8^- \to 6^+$ turns out to be hindered due to the smallness of: 1) the weight of the three-phonon component in the 8^- state wave function, ii) the contribution of the $K = 6$ component to the 6^+ quasirotational state wave function and III) the value of the overlap integral over the collective parts of the 8^- and 6^+ state wave functions. We have not calculated this integral but to obtain the upper estimate we have put it to be equal to unity.

As a result, the hindrance factor is estimated to be

$$F > \frac{1}{5\cdot 10^{-2}\cdot 2.8\cdot 10^{-5}} > 10^6 .$$

Thus, the model considered permits us to explain the quasirotational band energy ratios, the possibility of the existence of isomeric states and the values of their hindrance factors.

The existence of the isomeric two-quasiparticle levels may probably be related to the fact that below them there are only collective levels, the transitions to which are hindered due to either the K -forbiddenness in strongly deformed nuclei or the above mentioned reasons in transition nuclei or the quasiparticle selection rules in spherical nuclei. Hence the hindrance factor may be considered as a measure of the collectiveness of low -lying states, which is the strongest one in the well-deformed nuclei.

We have not discussed yet the effect of the γ -dependent terms in the potential energy, which are small compared to the deformation energy, but comparable with the first 2^+ excited state energy at large equilibrium β_0 value. The account of these terms would reduce to some extent the weight of the components with large K in the quasirotational wave functions, and the ratios of the quasirotational level energies would be close to the ones for the well-deformed nuclei. However it seems unlikely that these terms may change qualitatively the main results, but the values of the quadrupole moments must be changed essentially. Even a small dependence of the collective potential energy on γ may give rise to large quadrupole moments for the quasirotational levels. This is just observed in experiment.

Furthermore the γ -dependent terms in the Hamiltonian remove the energy level degeneracy which is characteristic of the model with γ -independent potential energy. Hence the collective model with γ -independent potential energy may be used only as a first step approximation.

In conclusion we want to emphasize that the γ -independent potential energy surprisingly well describes the quasirotational level energy ratios for most of the known transition nuclei, as is seen from Table 1. It is unclear how well this model describes other properties of these nuclei.

Appendix 1.

Let us calculate the contribution of the K = 6 component to the norm of the 6^+ state wave function.

The normalized 6^+ state wave function has the form:

$$\frac{1}{\sqrt{16\pi^2 \frac{3!}{9!!}}} F(\beta) \sum_{\mu\mu'\mu''} C^{6M}_{4\nu 2\mu''} C^{4\nu}_{2\mu 2\mu'} a^*_{2\mu} a^*_{2\mu'} a^*_{2\mu''} ,$$

where

$$\int_0^\infty F^2(\beta)\beta^4 d\beta = 1.$$

Let us single out from it the component with K = 6

$$\frac{\sin^3\gamma}{\sqrt{16\pi^2 \frac{3!}{9!!}}} F(\beta) \sum_{\mu\mu'\mu''} C^{6M}_{4\nu 2\mu''} C^{4\nu}_{2\mu 2\mu'} \frac{1}{2\sqrt{2}} (D^{*2}_{\mu 2} D^{*2}_{\mu' 2} D^{*2}_{\mu'' 2} +$$

$$+ D^{*2}_{\mu -2} D^{*2}_{\mu' -2} D^{*2}_{\mu'' -2}) = \frac{1}{\sqrt{16\pi^2 \frac{3!}{9!!}}} F(\beta) \frac{D^{*6}_{M6} + D^{*6}_{M-6}}{2\sqrt{2}} \sin^3\gamma .$$

The contribution of this component to the norm of the wave function is equal to

$$\frac{9!!}{16\pi^2\, 3!} \frac{1}{8} \int_0^\infty \beta^4 F^2(\beta) d\beta \int_0^{\pi/3} \sin^3\gamma \cdot \sin^6\gamma \, d\gamma \times$$

$$\times \int d\Omega\, (D^{*6}_{M6} + D^{*6}_{M-6}) \cdot (D^{*6}_{M6} + D^{6}_{M-6}) \approx 0.05 .$$

Appendix 2.

Let us estimate the admixture of the two-quasiparticle and thre phonon state to the 8^- state wave function. For the wave function we have:

$$|8M8;n;s,s'\rangle = \sqrt{\frac{17}{24\pi^2\gamma_0^2}}\, F'(\beta)\frac{1}{\sqrt{n!}}\cdot\left(\frac{\gamma}{\gamma_0}\right)^n \cdot$$

$$\cdot\ \{D^{*8}_{M8}(\alpha^+_s\alpha^+_{s'})_{K=8-2n} + D^{*8}_{M-8}(\alpha^+_s\alpha^+_{s'})_{K=-(8-2n)}\}|0\rangle ,$$

where

$$\int_0^\infty F'^2(\beta)\beta^4\, d\beta = 1$$

M is the momentum projection on the z - axis in the lab. system, n is the number of phonons s, s' are quantum numbers of the quasiparticle states, and $|0\rangle$ stands for the quasiparticle vacuum state.

To choose the dependence of the wave functions upon γ we have assumed that γ vibrations occur in the potential well with a minimum at $\gamma = 0$.

For the Hint matrix elements we have

$$\langle 8M8;n+1;s_1' s_2'|H_{int}|8M8;n;s_1 s_2\rangle = -h\,\omega_0\,\beta_0\,\gamma_0\sqrt{n+1}\ \cdot$$

$$\cdot\{f^{(22)}_{s_1'\sigma_1';s_1\sigma_1} v_{s_1's_1}\delta_{s_2's_2}\delta_{\sigma_2'\sigma_2} + f^{(22)}_{s_2'\sigma_2';s_2\sigma_2} v_{s_2's_2}\delta_{s_1's_1}\delta_{\sigma_1'\sigma_1} -$$

$$-f^{(22)}_{s_1'\sigma_1';s_2\sigma_2} v_{s_1's_2}\delta_{s_1s_2'}\delta_{\sigma_1\sigma_2'} - f^{(22)}_{s_2'\sigma_2';s_1\sigma_1} v_{s_2's_1}\delta_{s_2s_1'}\delta_{\sigma_2\sigma_1'} .$$

It is assumed that the isomeric state is the two-quasiparticle neutron state with quantum numbers:
$|8M8;0;9/2-[514],7/2+[404]\rangle$.

As follows from the calculation of all the three-phonon states, from which transitions to the quasirotational levels are allowed, only the $|8M8;3;5/2-[532],1/2+[400]\rangle$ state gives for the matrix elements, Hint noticeable values.

To calculate the admixture of this state to the wave function we may resort to perturbation theory since the values of the H_{int} matrix elements are small compared to the energy differences of the mixed states. If we take into account only those intermediate states the contributions of which are large enough the value of this admixture is equal to the product of the three following ratios:

$$\frac{\langle 8M8; 1; 5/2-[532], 7/2+[404] | H_{int} | 8M8; 0; 9/2-[514], 7/2+[404] \rangle}{E(5/2-[532]) + \omega_\gamma - E(9/2-[514])} = -0.04$$

$$\frac{\langle 8M8; 2; 5/2-[532], 3/2+[402] | H_{int} | 8M8; 1; 5/2-[532], 7/2+[404] \rangle}{E(3/2+[402]) + \omega_\gamma - E(7/2+[404])} = 0.30$$

$$\frac{\langle 8M8; 3; 5/2-[532], 1/2+[400] | H_{int} | 8M8; 2; 5/2-[532], 3/2+[402] \rangle}{E(1/2+[400]) + \omega_\gamma - E(3/2+[402])} = 0.44$$

which is equal to 2.8 10^{-6}. Here ω_γ and $E(K^\pi [N n_z \Lambda])$ are the energies of the γ-phonon and the quasiparticle respectively. The quantities ω_γ and γ_0^2 are put to be equal to $\omega_\gamma = 1$ MeV and $\gamma_0^2 = 0{,}1$ which are characteristic of the axial - symmetric nuclei. The single particle levels and the β_0 value were taken from ref. /3/, the matrix elements of the quadrupole operator were taken from ref. /6/.

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Table I

Comparison of the experimental results with the prediction of the unified model with γ-independent potential energy. Figures in columns present experimental [7] energy ratios for quasirotational levels. The last figure of each columns corresponds to the calculated value.

Nucleus	$\frac{E(4^+)}{E(2^+)}$	$\frac{E(6^+)}{E(2^+)}$	$\frac{E(8^+)}{E(2^+)}$	$\frac{E(10^+)}{E(2^+)}$	$\frac{E(12^+)}{E(2^+)}$
^{120}Xe	2.47	4.34	6.52	8.92	
^{122}Xe	2.49	4.39	6.61	9.01	
^{124}Xe	2.48	4.38	6.63		
^{130}Ba	2.54	4.46	6.86		
^{132}Ce	2.64	4.74	7.16		
^{134}Ce	2.56	4.55	6.86		
^{134}Nd	2.68	4.83	7.30		
^{186}Pt	2.56	4.59	7.02	9.71	12.6
^{188}Pt	2.52	4.46	6.70		
Prediction of a model	2.5	4.5	7.0	10.0	13.5

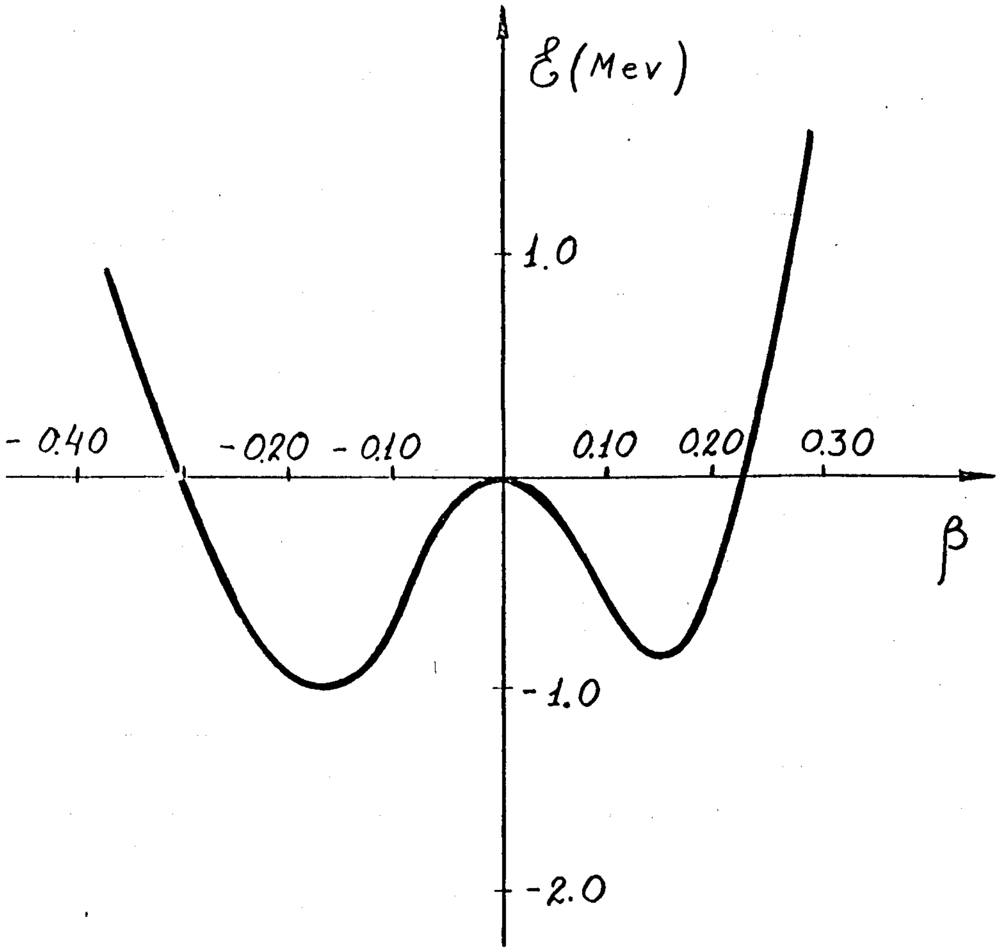

Fig. 1. The ground state energy of the ^{130}Ba nucleus as a function of β .

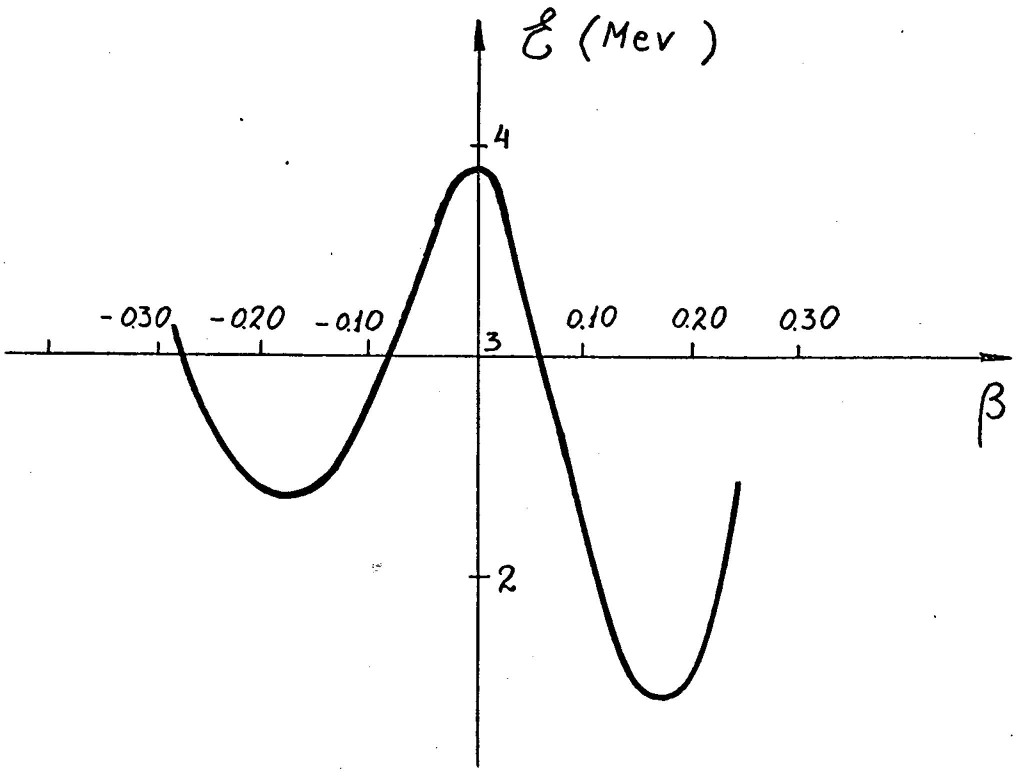

Fig. 2. The energy of one of the lowest two-quasiparticle neutron states for the ^{130}Ba nucleus as a function of β .