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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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CORRELATIONS BETWEEN QUASIPARTICLE AND COLLECTIVE EXCITATIONS IN NUCLEI

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1. Introduction

The microscopic theory of the collective excitation of nuclei in the usual Random Phase Approximation (RPA) formulation /1/ had been found unsufficient to describe quantitatively the first excited levels. Besides, (RPA) was not able to explain the higher excitations behaviour.

In recent years remarkable progress has been made in deriving the methods removing the (RPA) defects. In these approaches either the equations of motion taking into account the anharmonic corrections /2/ or the boson expansions of the Fermi operators are used /3/. As a rule, (RPA) plays the role of a zero order approximation in all these attempts.

In this paper an optimal zero order boson approximation is proposed. Very important effects which already in the harmonic boson picture lead to the correlations between one-particle and collective excitations will be taken into account. We would like to stress that in many papers, where more accurate anharmonic effects are included, the above effects are disregarded in spite of the fact, that their existence was already indicated earlier /4/.

In 2 the general problem is formulated as a variational principle. The equations for all quantities describing both the one-quasiparticle excitations and the collective ones are derived. In 3 the results are illustrated on the examples.

2. The variational principle

Let us consider the hamiltonian of the N -fermions system as a sum of the one-particle part, pairing and multipole-multipole interactions

$$H = H_0 + H_p + H_{QQ} , \tag{1}$$

$$H_0 = \sum_{jm} (\epsilon_j - \lambda) \, a^+_{jm} \, a_{jm} , \tag{2}$$

where ϵ_j are the one-particle energies, λ is the chemical potential,

$$H_p = -\frac{G}{4} P^+ P , \tag{3}$$

$$P^+ = \sum_{jm} (-1)^{j-m} \, a^+_{jm} \, a^+_{j-m} , \tag{4}$$

$$H_{QQ} = -\frac{1}{2} \sum_{LM} \chi_L \, Q^+_{LM} \, Q_{LM} , \tag{5}$$

$$Q_{LM} = \sum_{j_1 m_1 | j_2 m_2} <1| \, Q_{LM} \, |2> \, a^+_{j_1 m_1} \, a_{j_2 m_2} ,$$

$$Q_{LM}(x) = r^{\ell} \, Y (\theta, \phi) , \tag{6}$$

$$<1| \, Q_{LM} \, |2> = \int \Psi^+_1(x) \, Q_{LM}(x) \, \Psi_2(x) \, dx .$$

We perform now the $u-v$ transformation passing from a, a^+ — fermion particles operators to the c, c^+ -fermion quasiparticle operators

$$a^+_{jm} = u_j c^+_{jm} + (-1)^{j-m} v_j c_{j-m} ,$$

$$a_{jm} = u_j c_{jm} + (-1)^{j-m} v_j c^+_{j-m} , \qquad (7)$$

$$u_j^2 + v_j^2 = 1 .$$

After this transformation our hamiltonian can be written as:

$$H_0 = \sum_j (\epsilon_j - \lambda) [2\Omega_j v_j^2 + \sqrt{2\Omega_j} (u_j^2 - v_j^2) \alpha_j^+ + \sqrt{4\Omega_j} u_j v_j (A_j^+ + A_j)],$$

$\mathcal{N}_j$

$$H_p = -\frac{G}{4} \{ (\sum_j 2\Omega_j u_j v_j)^2 - 4 \sum_{jj'} 2\Omega_j 2\Omega_{j'} u_j v_j u_{j'} v_{j'} \alpha_{j'}^+ +$$

$\mathcal{N}_{j'}$

3-1

$$+ \sum_{jj'} 2\Omega_j \sqrt{4\Omega_{j'}} u_j v_j (u_{j'}^2 - v_{j'}^2)(A_{j'}^+ + A_{j'}) - \sum_{jj'} \sqrt{2\Omega_j} \sqrt{4\Omega_{j'}} u_j v_j \alpha_j^+ (u_{j'}^2 A_{j'} - v_{j'}^2 A_{j'}^+) -$$

$$-2 \sum \sqrt{4\Omega_j} \sqrt{2\Omega_{j'}} (u_j^2 A_j^+ - v_j^2 A_j) u_{j'} v_{j'} \alpha_{j'}^+ +$$

2-2
4-0

$$+ \sum_{jj'} \sqrt{4\Omega_j} \sqrt{4\Omega_{j'}} [\frac{1}{4}(u_j^2 - v_j^2)(u_{j'}^2 - v_{j'}^2)(A_j^+ + A_j)(A_{j'}^+ + A_{j'}) - \frac{1}{4}(A_j^+ - A_j)(A_{j'}^+ - A_{j'}) -$$

$$- \frac{1}{2} \delta_{jj'} (u_j^2 - v_j^2)(1 - \sqrt{2\Omega_j} \alpha_j^+)] +$$

$\mathcal{N}_j$

11-11

$$+ 4 \sum_{jj'} \sqrt{2\Omega_j} \sqrt{2\Omega_{j'}} u_j v_j u_{j'} v_{j'} \alpha_j^+ \alpha_{j'}^+ \} .$$

$\mathcal{N}_j \mathcal{N}_{j'}$

where

$$A_j^{+} = \frac{1}{\sqrt{4\Omega_j}} \sum_m c^{+}_{jm} \; c^{+}_{j-m} \; (-)^{j-m} \; , \tag{10}$$

$$a_j^{+} = \frac{1}{\sqrt{2\Omega_j}} \sum_m c^{+}_{jm} \; c_{jm} \; , \tag{11}$$

$$2\Omega_j = 2j + 1 \; ,$$

$$\begin{aligned} H_{QQ} = & -\frac{1}{2} \sum_{LM11'22'} \chi_L \, Q_L[11'] \, Q_L[22'] \, \big(A^{+}_{LM}[11'] \, A_{LM}[22'] \; + \\ & + (-1)^{L-M} A_{L-M}[11'] \, A_{LM}[22'] + (-1)^{L-M} A^{+}_{LM}[11'] \, A^{+}_{L-M}[22'] + A_{L-M}[11'] A^{+}_{L-M}[22']\big) - \\ & - \frac{1}{2} \sum_{LM11'22'} \chi_L \, Q[11'] \, q_L[22'] \, \big(A^{+}_{LM}[11'] + (-1)^{L-M} A_{L-M}[11']\big) \, a^{+}_{LM}[22'] - \\ & - \frac{1}{2} \sum_{LM11'22'} \chi_L \, Q_L[22'] \, q_L[11'] \, a^{+}_{LM}[11'] \, \big(A_{LM}[22'] + (-1)^{L-M} A^{+}_{L-M}[22']\big) - \\ & - \frac{1}{2} \sum_{LM11'22'} \chi_L \, q_L[11'] \, q_L[22'] \, a^{+}_{LM}[11'] \, a^{+}_{LM}[22'] \end{aligned} \tag{12}$$

(The shell model state (N_1, ℓ_1, j_1, m_1) is simply abbreviated as 1), where

$$Q_L[12] = \frac{1}{\sqrt{2}} (u_1 v_2 + v_1 u_2) \frac{\langle 1 \| Q_L \| 2 \rangle}{\sqrt{2\Omega_L}} \; , \tag{13}$$

$f_\lambda(q q')$

$$q_L[12] = (u_1 u_2 - v_1 v_2) \frac{<1\|Q_L\|2>}{\sqrt{2\,\Omega_L}} . \qquad (14)$$

The matrix element $<1|\,Q_{LM}\,|\,2>$ is written by use of the Wigner-Eckart theorem as

$$<1\,|\,Q_{LM}\,|\,2> \;=\; \frac{<1\|Q_L\|2>}{\sqrt{2\,\Omega_1}} <2,LM\,|\,1> , \qquad (15)$$

and $<2M\,|\,1>$ is a Clebsch-Gordon coefficient $<j_2\, m_2, LM\,|\,j_1\, m_1>$. Operators A, A^+ and a^+ satisfy the commutation relations:

$$[A_1 \; , A^+_{1'}] \;=\; \delta_{11'}\,(1 - \frac{2}{\sqrt{2\Omega_1}}\, a^+_1) , \qquad (16)$$

$$[\sum_{22'} Q_L\,[22']\,A_{LM}[22'] ,\; \sum_{11'} g^a_\lambda\,[11']\,A^+_{\lambda\mu}\,[11']] = \delta_{\lambda L}\,\delta_{\mu M} \sum_{11'} Q_\lambda\,[11']\,g^a_\lambda\,[11] - \qquad (17)$$

$$-2 \sum_{11'L'} \sqrt{4\Omega_L}\,\sqrt{4\Omega_{L'}} <L'M', LM\,|\,\lambda\mu> G^a\,[11',L']\; a^+_{L'M'}[11'] ,$$

$$G^a_{\lambda L}\,[11',L'] \;=\; \sum_2 g^a_\lambda\,[12]\; Q_L\,[21']\; W(11'\lambda L\,;\,L'2) ,$$

and $W(11'\lambda L\,;\,L'2)$ represents a Racah coefficient.

Following the paper [4] our approximation is based on the replacement of the operators of the right hand side in the last relations by their ground state averages:

O

$$< | a_j^+ | > = \sqrt{2\Omega_j}\ \rho_j \quad ,$$

$$< | a_{LM}^+ [11'] | > = \delta_{L0}\ \delta_{11'}\ \sqrt{2\Omega_1}\ \rho_1 \quad . \tag{18}$$

We assume that the approximate commutation relations are:

$$[A_1 \ , \ A_{1'}^+ \ ] = \delta_{11'}(1-2\rho_1) \ , \tag{19}$$

$$[\sum_{22'} Q_L\ [22']\ A_{LM}[22'] \ , \ \sum_{11'} g_\lambda^a\ [11']\ A_{\lambda\mu}^+\ [11']\ ] =$$

$$= \delta_{\lambda L}\ \delta_{\mu M}\ \Sigma\ (1-\rho_{11'})\ Q_\lambda\ [11']\ g_\lambda^a\ [11'] \quad , \tag{20}$$

where

$$\rho_{11'} = \rho_1 + \rho_{1'} \quad .$$

Now we define the bosons

$$B_n^+ = \sum_1 \ (a_{n1}\ A_1^+ - b_{n1}\ A_1\ ) \ , \tag{21}$$

$$B^{+}_{\lambda\mu}[\alpha] = \sum_{11'} (f^{\alpha}_{\lambda}[11'] A^{+}_{\lambda\mu}[11'] - g^{\alpha}_{\lambda}[11'] (-1)^{\lambda-\mu} A_{\lambda-\mu}[11'] , \quad (22)$$

with the following conditions for the coefficients:

$$f^{\alpha}_{\lambda}[11'] = -(-1)^{j_1+j_{1'}+\lambda} f_{\lambda}[1'1] ,$$

$$g^{\alpha}_{\lambda}[11'] = -(-1)^{j_1+j_{1'}+\lambda} g^{\alpha}_{\lambda}[1'1] . \quad (23)$$

Using the approximate commutation relations (19), (20) we receive the ortonormality conditions:

$$\sum_{1} (1-2\rho_1)(a_{n1} a_{m1} - b_{n1} b_{m1}) = \delta_{nm} , \quad (24)$$

$$\sum_{n} (1-2\rho_1)(a_{n1} a_{n2} - b_{n1} b_{n2}) = \delta_{12} , \quad (25)$$

$$\sum_{1} (1-2\rho_1)(a_{n1} b_{m1} - a_{m1} b_{n1}) = 0 , \quad (26)$$

$$\sum_{n} (1-2\rho_1)(a_{n1} b_{n2} - b_{n1} a_{n2}) = 0 , \quad (27)$$

$$\sum_{11'} (1-\rho_{11'}) (f^{\alpha}_{\lambda}[11'] f^{\beta}_{\lambda}[11'] - g^{\alpha}_{\lambda}[11'] g^{\beta}_{\lambda}[11'] = \delta_{\alpha\beta} , \quad (28)$$

$$\sum_{a}(1-\rho_{11'})(f_{\lambda}^{a}[11']f_{\lambda}^{a}[22']-g_{\lambda}^{a}[11']g_{\lambda}^{a}[22'])=\frac{1}{2}(\delta_{12}\delta_{1'2'}-(-1)^{j_1+j_2+\lambda}\delta_{12'}\delta_{1'2}$$

$$\sum_{11'}(1-\rho_{11'})(f_{\lambda}^{a}[11']g_{\lambda}^{\beta}[11']-g_{\lambda}^{a}[11']f_{\lambda}^{\beta}[11'])=0, \quad (30)$$

$$\sum_{a}(1-\rho_{11'})(f_{\lambda}^{a}[11']g_{\lambda}^{a}[22']-g_{\lambda}^{a}[11']f_{\lambda}^{a}[22'])=0. \quad (31)$$

Having settled these conditions we can simply perform inverse transformation

$$A_{1}^{+}=\sum(1-2\rho_{1})(a_{n1}B_{n}^{+}+b_{n1}B_{n}), \quad (32)$$

$$A_{\lambda\mu}^{+}[11']=(1-\rho_{11'})\sum_{a}(f_{\lambda}^{a}[11']B_{\lambda\mu}^{+}[a]+(-1)^{\lambda-\mu}g_{\lambda}^{a}[11']B_{\lambda-\mu}[a]. \quad (33)$$

Now we proceed to formulate the problem as a variational principle which is our basic approach. We construct the functional $\mathfrak{L}$ as

$$\mathfrak{L} = \langle \| H \| \rangle - \sum_1 \mu_1 (v_1^2 + u_1^2) -$$

$$- \sum_{a,\lambda,11'} \omega_\lambda^a (1 - \rho_{11'}) (f_\lambda^a [11'] f_\lambda^a [11'] - g_\lambda^a [11'] g_\lambda^a [11']) - \tag{34}$$

$$- \sum_{1n} \omega_0^n (1 - 2\rho_1) (a_{n1} a_{n1} - b_{n1} b_{n1}) ,$$

where $\langle \| H \| \rangle$ is the expectation value of hamiltonian in the ground state. We require

$$\delta \mathfrak{L} = 0 , \tag{35}$$

and we assume the ground state of the system $\| \rangle$ to be a vacuum of bosons (21), (22):

$$B_n \| \rangle = 0 ,$$

$$B_{\lambda\mu} [a] \| \rangle = 0 . \tag{36}$$

We have in the explicite form:

$$\mathfrak{L} = \frac{G}{4} \sum_1 2\Omega_1 + \sum_1 2\Omega_1 (\epsilon_1 - \lambda - \frac{G}{2}) v_1^2 +$$

$$+ \sum_1 (\epsilon_1 - \lambda - \frac{G}{2}) (u_1^2 - v_1^2) 2\Omega_1 \rho_1 - \frac{G}{2} (\sum_1 2\Omega_1 u_1 v_1 (1 - 2\rho_1))^2 -$$

$$- \sum_{1} \mu_1 (u_1^2 + v_1^2) -$$

$$- \frac{G}{4} \sum_{11'} \sqrt{4\Omega_1} \sqrt{4\Omega_{1'}} (1-2\rho_1)(1-2\rho_{1'}) \frac{1}{4} (u_1^2 - v_1^2)(u_{1'}^2 - v_{1'}^2) \sum_n (a_{n1} a_{n1'} + b_{n1} b_{n1'} + a_{n1} b_{n1'} + b_{n1} a_{n1'}) -$$

$$- \frac{G}{4} \sum_{11'} \sqrt{4\Omega_1} \sqrt{4\Omega_{1'}} (1-2\rho_1)(1-2\rho_{1'}) \frac{1}{4} \sum_n (a_{n1} a_{n1'} + b_{n1} b_{n1'} - a_{n1} b_{n1'} - b_{n1} a_{n1'}) -$$

$$- \frac{1}{2} \sum_{L a 11'} \chi_L \, 2\Omega_L \, Q_L[11'] \, Q_L[22'] \, (1-\rho_{11'}) \, (1-\rho_{22'}) \times$$

$$\times (f_L^a[11'] f_L^a[22'] + g_L^a[11'] g_L^a[22'] + f_L^a[11'] g_L^a[22'] + g_L^a[11'] f_L^a[22']) - \quad (37)$$

$$- \sum_{La11'} \omega_L^a (1-\rho_{11'}) (-g_L^a[11'] g_L^a[11'] + f_L^a[11'] f_L^a[11']) -$$

$$- \sum_{n1} \omega_0^n (1-2\rho_1) (a_{n1}^2 - b_{n1}^2) .$$

The system of the equations for all the parameters characterizing the quasiparticle and collective excitations:

$$\frac{\partial \mathcal{L}}{\partial u_k} = 0 , \qquad \frac{\partial \mathcal{L}}{\partial v_k} = 0 , \qquad (38)$$

$$\frac{\partial \mathcal{L}}{\partial a_{nk}} = 0 , \qquad \frac{\partial \mathcal{L}}{\partial b_{nk}} = 0 \qquad (39)$$

$$\frac{\partial \mathfrak{L}}{\partial f_L^\alpha [33']} = 0 \;, \qquad \frac{\partial \mathfrak{L}}{\partial g_L^\alpha [33']} = 0 \;, \tag{40}$$

is just set of necessary conditions for $\mathfrak{L}$ to be minimum. As we see $\mathfrak{L}$ depends on the parameters ρ_1 which are the occupation numbers of the quasiparticle levels in the ground state. For these parameters there are additional equations[4]:

$$\rho_1 = \frac{1}{2\Omega_1} \sum_{L\alpha 2} (1-\rho_{12})\, 2\Omega_L \left((f_L^\alpha [12])^2 + (g_L^\alpha [12])^2 - 1 \right) + \frac{1}{2\Omega_1} (1-2\rho_1) \sum_n (a_{n1}^2 + b_{n1}^2 - 1) \;, \tag{41}$$

where

$$\rho_{1k} = \rho_1 + \rho_k \;.$$

Now we reduce the system of equations (38) by the usual method to

$$2R_k u_k v_k - r_k (u_k^2 - v_k^2) = 0 \;, \tag{42}$$

where

$$R_k = (\epsilon_k - \lambda - \frac{G}{2})(1-2\rho_k)\, 2\Omega_k + \frac{G}{4} \sqrt{4\Omega_k}\,(1-2\rho_k) \sum \sqrt{4\Omega_1}\,(1-2\rho_1)(u_1^2 - v_1^2)(a_{n1}a_{nk} + b_{n1}b_{nk} + a_{n1}b_{nk} + b_{n1}a_{nk}), \tag{43}$$

$$r_k = \frac{G}{2} 2\Omega_k (1-2\rho_k) \sum_{1'} 2\Omega_1 u_1 v_1 (1-2\rho_1) +$$

$$+2 \sum \chi_L \langle 1 \| Q_L \| k \rangle \langle 2 \| Q_L \| 2' \rangle (u_2 v_{2'} + v_2 u_{2'})(1-\rho_{k1})(1-\rho_{22'}) \times$$

$$\times (f_L^{a}[1k] f_L^{a}[22'] + g_L^{a}[1k] g_L^{a}[22'] + g_L^{a}[1k] f_L^{a}[22'] + f_L^{a}[1k] g_L^{a}[22']) . \tag{44}$$

The quantities R_k play the role of the renormalized quasiparticle energies, the renormalization being due to pairing vibrations. The quantities r_k are renormalized energy gap, the renormalization being due to the multipole vibrations. Using these formulae we can simply write the formal solutions for u_k , v_k :

$$u_k^2 = \frac{1}{2} \left(1 + \frac{R_k}{\sqrt{r_k^2 + R_k^2}}\right) ,$$

$$v_k^2 = \frac{1}{2} \left(1 - \frac{R_k}{\sqrt{r_k^2 + R_k^2}}\right) . \tag{45}$$

To derive the equations for the amplitudes of collective vibrations we use (41). Generally we get also nonlinear terms, but we will omit them. Thus we have:

$$\frac{\partial \mathcal{L}}{\partial a_{nk}} = (2E_k - \omega_0^n) a_{nk} -$$

$$- \frac{G}{4}\sqrt{4\Omega_k}\,(u_k^2 - v_k^2)\sum_l \sqrt{4\Omega_l}\,(1-2\rho_l)\,(u_l^2 - v_l^2)\,\frac{1}{2}\,(a_{nl}+b_{nl}) -$$

$$- \frac{G}{4}\sqrt{4\Omega_k}\sum_l \sqrt{4\Omega_l}\,(1-2\rho_l)\,\frac{1}{2}\,(a_{nl} - b_{nl}) = 0 , \tag{46}$$

$$\frac{\partial \mathfrak{L}}{\partial b_{nk}} = (2E_k + \omega_0^n)\,b_{nk} -$$

$$- \frac{G}{4}\sqrt{4\Omega_k}\,(u_k^2 - v_k^2)\sum_l \sqrt{4\Omega_l}\,(1-2\rho_l)\,(u_l^2 - v_l^2)\,\frac{1}{2}\,(a_{nl}+b_{nl}) + \tag{47}$$

$$+ \frac{G}{4}\sqrt{4\Omega_k}\sum \sqrt{4\Omega_l}\,(1-2\rho_l)\,\frac{1}{2}\,(a_{nl} - b_{nl}) = 0 ,$$

$$\frac{\partial \mathfrak{L}}{\partial f_L^\gamma[33']} = (E_3 + E_{3'} - \omega_L^\gamma)\,f_L^\gamma[33'] -$$

$$- \chi_L\, Q_L[33'] \sum_{11'} Q_L[11'](1-\rho_{11'})\,(f_L^\gamma[11'] + g_L^\gamma[11']) = 0 , \tag{48}$$

$$\frac{\partial \mathfrak{L}}{\partial g_L^\gamma[33']} = (E_3 + E_{3'} + \omega_L^\gamma)\,g_L^\gamma[33'] -$$

$$- \chi_L\, Q_L[33'] \sum_{11'} Q_L[11']\,(1-\rho_{11'})\,(f_L^\gamma[11'] + g_L^\gamma[11']) = 0 , \tag{49}$$

where

$$E_k = (\epsilon_k - \lambda - \frac{G}{2})\,(u_k^2 - v_k^2) + G\,u_k v_k \sum_l 2\Omega_l u_l v_l (1-2\rho_l) . \tag{50}$$

It is simply to check that the equations for f and g are formally identical with the same equations obtained in the papers [4] if we identify E_k with quasiparticle energy. But in our case the parameters u_k and v_k satisfy some other equations. From the equations (48), (49) it follows the secular equation for the multipole excitations

$$1 = 2\chi_L \sum_{11'} \frac{(1-\rho_{11'})\, Q_L^2[11'](E_1+E_{1'})}{((E_1+E_{1'})^2 - (\omega_L^\gamma)^2)} \tag{51}$$

and for the pairing vibrations energy the similar one.

Thus the equations (42), (41), (46), (47), (48), (49) are a coupled system of equations which describes our N -fermion system. Obviously it is not easy to resolve it in the general case. We can hope only on some iteration procedure. In the next paragraphe we will give some simple examples by which we will demonstrate the corrections characteristic of our method.

3. Examples

a) Simplified equations for u_k and v_k.

Let us neglect, for the sake of simplicity, the second terms in the formulae for R_k and r_k. In this case we put

$$\Delta = G \cdot \sum_1 2\Omega_1 u_1 v_1 (1-2\rho_1) \tag{52}$$

and the equation for the Δ gets the form

$$\frac{2}{G} = \sum_1 \frac{\Omega_1(1-2\rho_1)}{\sqrt{\Delta^2+(\tilde{\tilde{\epsilon}}_1-\lambda)^2}} , \quad \tilde{\tilde{\epsilon}}_1 = \epsilon_1 - \frac{G}{2} . \tag{53}$$

From the condition

$$<|N|> = N = \sum_1 (2\Omega_1 v_1^2 + 2\Omega_1(u_1^2-v_1^2)\rho_1) \tag{54}$$

we can also determine λ :

$$\Omega - N = \sum_1 \frac{\Omega_1(\tilde{\tilde{\epsilon}}_1-\lambda)(1-2\rho_1)}{\sqrt{\Delta^2+(\tilde{\tilde{\epsilon}}_1-\lambda)^2}} . \tag{55}$$

We see, in this crude approximation, that the coupling with the vibrations leads to the blocking type corrections in the energy gap equation, namely the energy gap decreases. This effect is large enough in nuclei in which there are strongly collective low lying states, and small when such states are absent.

b) Pairing vibrations

From the equations (46) we have

$$\begin{aligned} (2E_k-\omega_0^n)\, a_{nk} - \frac{G}{4}\sqrt{4\Omega_k}\,(u_k^2-v_k^2)\,\Phi_n - \frac{G}{4}\sqrt{4\Omega_k}\,\Psi_n &= 0 , \\ (2E_k+\omega_0^n)\, b_{nk} - \frac{G}{4}\sqrt{4\Omega_k}\,(u_k^2-v_k^2)\,\Phi_n + \frac{G}{4}\sqrt{4\Omega_k}\,\Psi_n &= 0 . \end{aligned} \tag{56}$$

$$a_{nk} = \frac{G\sqrt{4\Omega_k}\,[(u_k^2-v_k^2)\Phi_n+\Psi_n]}{4(2E_k-\omega_0^n)} , \quad b_{nk} = \frac{G\sqrt{4\Omega_k}\,[(u_k^2-v_k^2)\Phi_n-\Psi_n]}{4(2E_k+\omega_0^n)} , \tag{57}$$

$$\Phi_n = \sum \frac{\sqrt{4\Omega_1}}{2} (1-2\rho_1)(u_1^2 - v_1^2)(a_{n1} + b_{n1}) , \qquad (58)$$

$$\Psi_n = \sum \frac{\sqrt{4\Omega_1}}{2} (1-2\rho_1)(a_{n1} - b_{n1}) . \qquad (59)$$

Using the simplified expression (52) for the Δ we receive the following equations

$$\sum_1 \frac{\Omega_1 (1-2\rho_1)((\omega_0^n)^2 - 4\Delta^2)}{E_1 (4E_1^2 - (\omega_0^n)^2)} \Phi_n + \sum_1 \frac{\Omega_1 (1-2\rho_1)(\tilde{\tilde{\epsilon}}_1 - \lambda) 2\omega_0^n}{E_1 (4E_1^2 - (\omega_0^n)^2)} \Psi_n = 0 , \qquad (60)$$

$$\sum_1 \frac{\Omega_1 (1-2\rho_1)(\tilde{\epsilon}_1 - \lambda) 2\omega_0^n}{E_1 (4E_1^2 - (\omega_0^n)^2)} \Phi_n + \sum_1 \frac{\Omega_1 (1-2\rho_1)(\omega_0^n)^2}{E_1 (4E_1^2 - (\omega_0^n)^2)} \Psi_n = 0 . \qquad (61)$$

Hence it is easy to get the secular equation:

$$(\omega_0^n)^2 \left\{ \left(\sum_1 \frac{\Omega_1 (1-2\rho_1)}{E_1 (4E_1^2 - (\omega_0^n)^2)} \right) \left(\sum_1 \frac{\Omega_1 (1-2\rho_1)((\omega_0^n)^2 - 4\Delta^2)}{E_1 (4E_1^2 - (\omega_0^n)^2)} \right) - \right. \qquad (62)$$

$$\left. - \left(\sum_1 \frac{\Omega_1 (1-2\rho_1) 2 (\tilde{\tilde{\epsilon}}_1 - \lambda)}{E_1 (4E_1^2 - (\omega_0^n)^2)} \right)^2 \right\} = 0 .$$

This equation differs from the usual one[4] in two points: 1°. There appear corrections of $(1-2\rho_l)$ type and 2°. E_k (playing here the role of one-quasiparticle energy) differs from the analogous term in the formulae for u_k and v_k by the renormalization. (The spurious states here also appear in connection with $\omega_0^0 = 0$ solution). In order to get the solutions for Φ_n, Ψ_n we use the condition of the normalization (24) and the result is:

$$\Phi_n = \frac{2}{G}\Big(\sum_k 2\Omega_k(1-2\rho_k)\Big\{\frac{4E_k\omega_0^n(u_k^2-v_k^2)^2}{(4E_k^2-(\omega_0^n)^2)} +$$

$$+\frac{4E_k\omega_0^n\left(\sum_1 \frac{2\Omega_1(1-2\rho_1)(\tilde{\tilde{\epsilon}}_1-\lambda)\omega_0^n}{E_1(4E_1^2-(\omega_0^n)^2)}\right)^2}{(4E_k^2-(\omega_0^n)^2)^2\left(\sum_1\frac{\Omega_1(1-2\rho_1)(\omega_0^n)^2}{E_1(4E_1^2-(\omega_0^n)^2)}\right)^2} - \tag{63}$$

$$-\frac{2(u_k^2-v_k^2)(4E_k^2+(\omega_0^n)^2)\left(\sum_1\frac{2\Omega_1(1-2\rho_1)(\tilde{\tilde{\epsilon}}_1-\lambda)\omega_0^n}{E_1(4E_1^2-(\omega_0^n)^2)}\right)}{(4E_k^2-(\omega_0^n)^2)\left(\sum_1\frac{2\Omega_1(1-2\rho_1)(\omega_0^n)^2}{E_1(4E_1^2-(\omega_0^n)^2)}\right)}\Big\}\Big)^{-1/2},$$

$$\Psi_n = \frac{2}{G}\Big(\sum_k 2\Omega_k(1-2\rho_k)\Big\{\frac{4E_k\omega_0^n}{(4E_k^2-(\omega_0^n)^2)^2} + \tag{64}$$

$$+\frac{4E_k\omega_0^n(u_k^2-v_k^2)^2\left(\sum_1\frac{2\Omega_1\omega_0^n(1-2\rho_1)(\tilde{\tilde{\epsilon}}_1-\lambda)}{E_1(4E_1^2-(\omega_0^n)^2)}\right)^2}{(4E_k^2-(\omega_0^n)^2)^2\left(\sum_1\frac{\Omega_1(1-2\rho_1)((\omega_0^n)^2-4\Delta^2)}{E_1(4E_1^2-(\omega_0^n)^2)}\right)} -$$

$$-\frac{2(u_k^2-v_k^2)(4E_k^2+(\omega_0^n)^2)\left(\sum_1\frac{2\Omega_1(1-2\rho_1)(\tilde{\tilde{\epsilon}}_1-\lambda)\omega_0^n}{E_1(4E_1^2-(\omega_0^n)^2)}\right)}{(4E_k^2-(\omega_0^n)^2)^2\left(\sum_1\frac{\Omega_1(1-2\rho_1)((\omega_0^n)^2-4\Delta^2)}{E_1(4E_1^2-(\omega_0^n)^2)}\right)}\Big\}\Big)^{-1/2}.$$

c) One level model

Let us take into account, as the last unfilled shell, the isolated level. In this model our formulae are simplified:

$$1 = \frac{G}{2} \frac{(j+\frac{1}{2})(1-2\rho_j)}{E_j} , \quad E_j = \sqrt{(\tilde{\tilde{\epsilon}}_j-\lambda)^2 + \Delta^2} , \tag{65}$$

$$N = (j + \frac{1}{2}) - (j + \frac{1}{2}) \frac{\tilde{\tilde{\epsilon}}_j - \lambda}{E_j} (1-2\rho_j) , \tag{66}$$

$$1 = 2\chi \frac{(1-2\rho_j) \; Q^2 [jj] \; 2E_j}{4E_j^2 - \omega^2} , \tag{67}$$

$$Q[jj] = \frac{1}{\sqrt{2}} 2 u_j v_j \frac{<j||Q||j>}{\sqrt{5}} = \frac{\Delta}{\sqrt{2} E_j \sqrt{5}} <j||Q||j> , \tag{68}$$

$$\rho_j = \frac{5(2E_j - \omega)^2}{2(2j+1)\,\omega\, 2E_j} . \tag{69}$$

we assume $2j + 1 = 2\Omega_j = 40$, $N = 18$. The $B(E2)$ transitions will be also calculate. The reduced $E2$ transition probability is given by the expression

$$B(E2;\, 2 \to 0) = \sum_{\mu=-2}^{2} |<|Q_{2\mu} B^{+}_{2\mu}|>|^2 . \tag{70}$$

Here we have taken into account the $L = 2$ vibrations only. Solving this system numerically we obtain the results which are shown on the figures 1 and 2. As we can see in figures 1 and 2 the K.Hara's corrections in the secular equations are partly cancelled by the corrections $(1 - 2\rho_i)$ in the energy gap and (68), (69) equation. Owing to the both corrections the result is close to the pure (RPA) calculations.

4. Conclusions

A. A complete system of coupled equations which is necessary for describing the nuclear excitations has been obtained from the variational principle without using equations of motion.

B. The expressions playing the role of the one-quasiparticle energies are different in the secular equations and in the equations for u_k and v_k. In the latter case these expressions are being changed: 1° by one-particle energy renormalization which depends upon pairing vibration only, 2° by the energy gap renormalization which is induced by the multipole vibrations.

C. The appearance of the factors $(1-2\rho_i)$ in the energy gap equations means that in this approach some blocking effect of the one particle levels is taken into account.

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References

1. M. Baranger. Phys.Rev., 120, 957 (1960).
   T. Tamura, T. Udagawa. Progr. Theor. Phys., 26, 947 (1961).
2. G. Fano, J. Sawicki. Nuovo Cim., 25, 586 (1962).
   T. Tamura, T. Udagawa. Nucl. Phys., 53, 33 (1964).
   T. Tamura, T. Udagawa. Phys.Rev., 150, 783 (1966).
   A.K. Kerman, A. Klein. Phys.Rev., 132, 1326 (1964).
   G. Do Dang, A. Klein. Phys.Rev., 133, B257 (1964).
   A.K. Kerman, A. Klein. Phys.Rev., 140, B245 (1965).
   D.J. Rowe. Rev.Mod.Phys., 40, 153 (1968).
3. S.T. Beliajev, V.G. Zelevinsky. Nucl.Phys., 39, 582 (1962).
   T. Marumori, M. Yamamura, A. Tokunaga. Progr.Theor.Phys., 31, 1009 (1964).
   B. Sorensen. Nucl.Phys., A97, 1 (1967).
   B. Sorensen. Progr.Theor.Phys., 39, 1468 (1968).
4. K. Hara. Progr.Theor.Phys., 32, 88 (1964).
   K. Ikeda, T. Udagawa, H. Yamamura. Progr.Theor. Phys., 33, 22 (1965).

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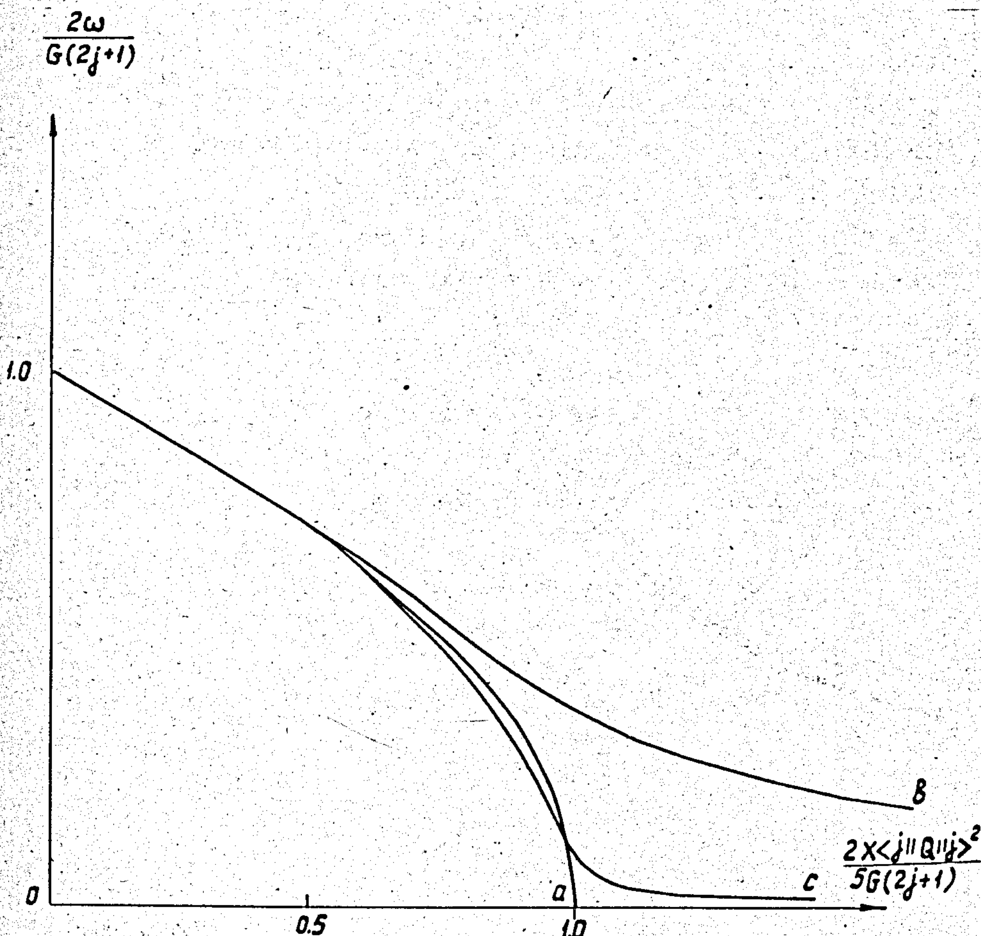

Fig. 1. The energy of the first collective excitation plotted as a function of the coupling constant χ of a long range force: a) in the (RPA) approximation, b) in the K. Hara approximation, c) in our method approximation.

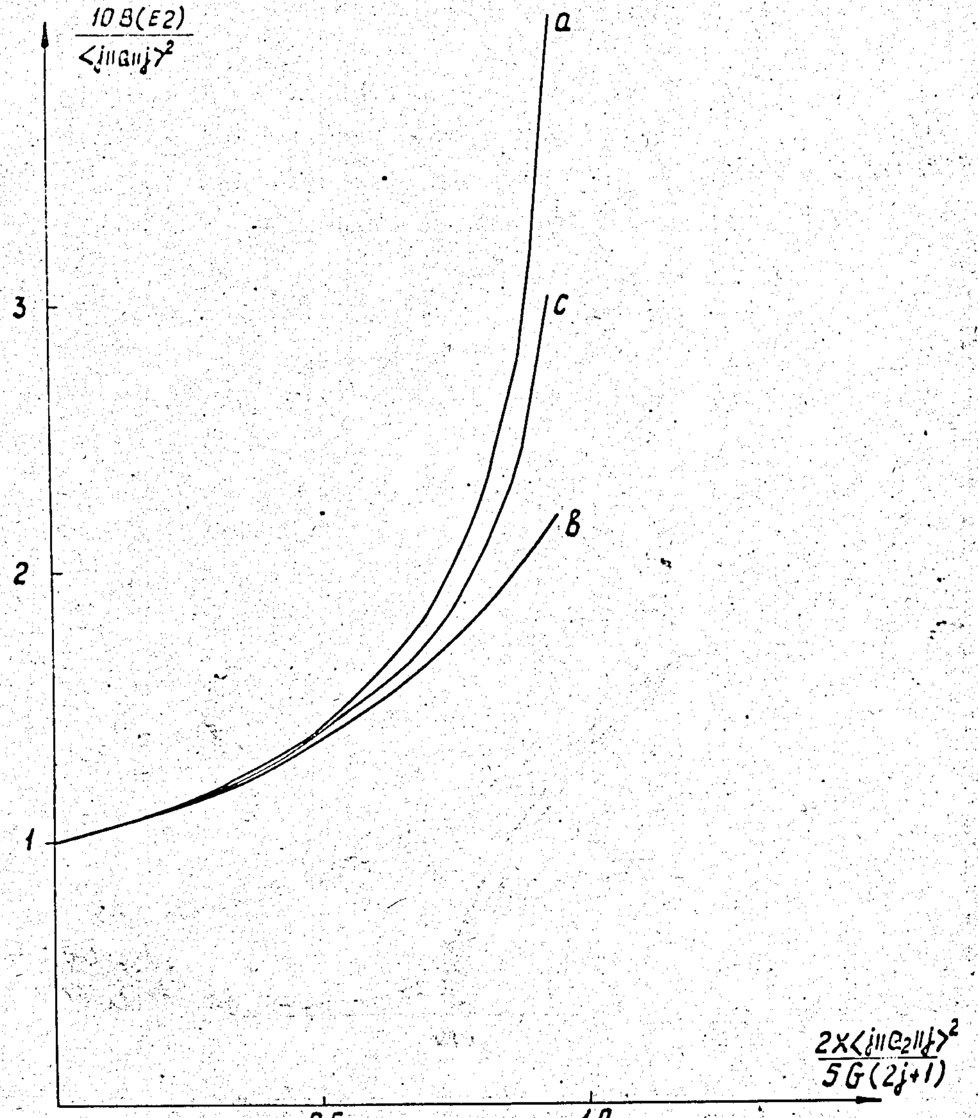

Fig. 2. Dependence of the reduced B(E2) transition probability on the coupling constant χ in the cases a), b), c).