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POLARIZATION EFFECTS IN THE ROTATIONAL MOTION OF ODD-MASS NUCLEI.

II. Application of Theory to the Rare-Earth Nuclei

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Introduction

The recent experimental studies of "long" rotational bands (up to the spin value $I \sim 29/2$) in odd-mass deformed nuclei (see, e.g.[1-4]) allow one to extend essentially the understanding of the nature of rotational motion and its connection with the other types of nuclear motion. In particular, the strong nonadiabaticity of the rotational excitations with respect to the intrinsic single-particle motion was found (the Coriolis coupling of single-particle states). A satisfactory description of experimental rotational bands was obtained for a number of nuclei, however, only when using the considerably renormalized (reduced by 20-30%) Coriolis force[1,4].

In the previous paper[5] we have shown that the static renormalization of the single-particle matrix elements $\langle j_t \rangle$ (or the rotational parameter $1/2\mathcal{J}$ entering the Coriolis force) arises from the residual centrifugal and spin-spin interactions between nucleons similarly to the renormalization of the matrix elements $\langle \sigma_t \rangle$ (or spin gyromagnetic ratio g_s) in the magnetic moments. The residual centrifugal interaction

$$H_j = \frac{1}{2\mathcal{J}} \cdot \frac{1}{2} \left[j_+ j_- + j_- j_+ \right] \qquad (1)$$

is contained in the Bohr rotational Hamiltonian /6/ ($\mathcal{J}$ is the effective moment of inertia). The strength parameter of the spin-spin interaction

$$H_{\sigma} = \frac{1}{2} \varkappa [\sigma_{+}\sigma_{-} + \sigma_{-}\sigma_{+}] \qquad (2)$$

is considered to be known from the calculation of magnetic moments ($\varkappa \approx 0.3$ MeV)/7/. The interactions (1) and (2) lead to the following renormalization of the rotational parameter in the Coriolis force (in the quasi-classical approximation)/5/:

$$\frac{1}{2\mathcal{J}} \longrightarrow \frac{1}{2\mathcal{J}} R_j^{\sigma} \quad , \qquad (3)$$

where

$$R_j^{\sigma} = 1 - \frac{2\mathcal{J}_c}{2\mathcal{J}+\mathcal{J}_c} \left[1 + \frac{\varkappa \mathcal{J}}{j^2} \right]. \qquad (4)$$

Here, $\mathcal{J}_c$ is the additional moment of inertia associated with the centrifugal interaction (it coincides in form with the conventional expression in the cranking-model, provided that only the levels of a single j-shell, coupled by the Coriolis force, contribute to $\mathcal{J}_c$).

The present paper is devoted to the application of the theory, developed in/5/, to the analysis of the rotational bands built on the positive parity states in a number of odd-N rare-earth nuclei.

Calculations and Discussion of Results

The most pronounced nonadiabatic effects were found in the rotational bands based on the single-particle levels originating from spherical subshell $i_{13/2}$, which appear near the Fermi surface

in the nuclei with N =89÷107. The calculations for these nuclei were performed using the wave functions and single-particle level scheme in the Saxon-Woods potential/8/. Forty single-particle levels in the vicinity of the Fermi surface were taken into account when solving the conventional equations for Δ and λ (without blocking). The obtained values of Δ and λ are used then in calculating single-quasiparticle spectrum, polarization factor R_j^{σ} (in the quasi-classical approximation) and in diagonalization of the Coriolis force matrix[x]. The states of spherical subshell $i_{13/2}$ and the states $1/2^+$ [400] and $3/2^+$ [402] coupled to them through $\Delta N = 2$ mixing are usually involved in the Coriolis mixing. In all the calculations the pair correlations were taken into account only in the static limit (i.e. Δ and λ are considered to be independent of the rotational frequency), however, the strength parameter of the pairing force G is varied slightly from nucleus to nucleus in order to obtain the best agreement with the experimental data. The effective moment of inertia $\mathcal{J}$ is chosen from the fit of calculations to experimental data, so far as there is no reason to assume that its magnitude should be the same as in the neighbouring even-even nuclei.

Besides the energies,there were calculated also the intraband E2-transition probabilities and the deviations from the Alaga rule for branching ratio $B(E2,I\to I-2)/B(E2,I\to I-1)$ were investigated. In calculations a simple formula for $B(E2)$ was used in which only the terms proportional to the total quadrupole moment of the nucleus Q_0 were conserved (the latter is assumed to be approximately the same for all the single-particle states):

[x] All the necessary expressions and equations are given in ref./5/.

I	^{161}Dy Exp./2/	Theor. (a)	Theor. (b)	Theor. (c)	Theor. (d)
33/2	2235	2368	2359	2316	2344
31/2	2162	2178	2158	2163	2163
29/2	1693	1770	1765	1731	1752
27/2	1602	1597	1583	1582	1583
25/2	1222	1263	1261	1236	1251
23/2	1119	1109	1100	1095	1096
21/2	826	845	844	829	838
19/2	719	711	706	700	701
17/2	508	515	515	506	511
15/2	407	403	400	396	397
13/2	267	267	267	264	266
11/2	184	182	181	179	180
9/2	100	98	98	98	99
7/2	44	43	42	42	42
5/2	0	0	0	0	0

Fig. 1. Ground state rotational band in ^{161}Dy . Calculations are performed with parameters: a) δ = 0.275; κ = 0; $1/2\mathcal{J}$ = 11.8 keV; Δ = 0.43 MeV. b) δ = 0.275; κ = 0.3 MeV; $1/2\mathcal{J}$ = 11.6 keV; Δ = 0.50 MeV. c) δ = 0.302; κ = 0; $1/2\mathcal{J}$ = 12 keV; Δ = 0.50 MeV. d) δ = 0.302; κ = 0.3 MeV; $1/2\mathcal{J}$ = 12 keV; Δ = 0.58 MeV.

$$B(E2, I\to I') \sim e^2 Q_0^2 \left| \sum_K C_K^I C_K^{I'} \langle IK20 | I'K \rangle \right|^2, \quad (5)$$

where C_K^I are the K-mixing amplitudes.

The calculations were carried out for Gd, Dy, Er, Yb and Hf isotopes at different values of deformation parameter δ, both with and without account of the spin polarization. The stability of the results obtained against small variations of δ, Δ, $\mathcal{J}$ and $\mathcal{H}$ was checked.

Below we discuss briefly the results obtained.

1. The sensitivity of calculations to the choice of the parameters is demonstrated for the nucleus ^{161}Dy in Fig.1 (the experimental data are taken from ref./2/). In all the cases shown the parameters $1/2\mathcal{J}$ and Δ were chosen from the fitting of calculations to experimental data. The preference was given to the best description of the low-energy part of the rotational band, since for high spin states the dynamical effects arising from the dependence of $\mathcal{J}$ and Δ on the rotational frequency (CAP effects), will play a certain role. It turned out that usually the position and the order of the lowest levels within rotational bands depend essentially on the choice of Δ and are less sensitive to the variation of the effective moment of inertia $\mathcal{J}$. The latter can be chosen reliably only from the energies of rotational levels with high spin $(I \sim 17/2)$, the variation of $1/2\mathcal{J}$ within 0.2 keV weakly affecting the results.

The calculations of rotational energies in ^{161}Dy, performed for δ=0.275 (cases (a) and (b)) and 0.302 (cases (c) and (d)), give practically the same results, though the wave functions differ considerably in these cases[x/]. The value of the effective moment of inertia slightly changes with changing deformation. The effect of the spin polarization is practically reduced to the renormalization of Δ (usually at $\varkappa \neq 0$ the energy gap is by 80-100 keV greater than at $\varkappa = 0$).

Nonadiabatic effects in the ground state rotational band of the ^{161}Dy are strong. It turned out to be impossible to describe it by means of the phenomenological Bohr formulae with A, B and A_5 parameters obtained by Bunker and Reich from the experimental energies of lowest levels[/9/] (already at $I = 19/2$ the deviation of calculated energy from experimental one is of the order of 100 keV, and beginning with $I = 23/2$ the predicted order of levels is wrong). The structure of the rotational states is shown in Table 1 which contains K-mixing amplitudes, the values of the decoupling parameter $a(I)$ (for definition see ref.[/5/]) and calculated energies $\mathcal{E}(I)$. The most noticeable feature is the appreciable values of $a(I)$ causing the doublet structure of the rotational spectrum. More compact doublets, observed in experimental spectrum, can be obtained in the theory by increasing the role of single-particle states $1/2^+$ [660] and $1/2^+$ [400]. Centrifugal and spin polarization renormalizes strongly the Coriolis force, the calculated polarization factor $R_j^\tau = 0.7$ [xx/] in this case (see Table 2).

x/ Notice that for $\delta = 0.302$ the states $1/2^+$ [400] and $1/2^+$ [660] are strongly mixed (as well as the states $3/2^+$ [402] and $3/2^+$ [651]). As a result their single-particle decoupling parameters a_{sp} are equal to 1.54 and 5.37, respectively. For $\delta = 0.275$ N-mixing of these states is rather small.

xx/ We should emphasize that the polarization renormalized all the matrix elements $\langle j_\pm \rangle$ including the decoupling parameter a_{sp}. Renormalization of a_{sp} was not taken into account in calculations presented in refs./1,4/.

TABLE 1

Ground state rotational band in ^{161}Dy.
The one-particle energies $\mathcal{E}_\nu - \lambda$ are given in the upper row. Columns 2-8 present the K-mixing amplitudes C_K^I. Decoupling term values $a(I)$ and rotational energies are displayed in columns 9 and 10. Calculations were performed with $\delta = 0.302$, $1/2\mathcal{J} = 12.0$ keV, $\Delta = 0.58$ meV and $\varkappa = 0.3$ meV.

$\mathcal{E}_\nu - \lambda$ [Kev]	-1427	-1451	-990	-1060	-139	1111	2777	$a(I)$	$\mathcal{E}(I)$ [Kev]
Spin I	1/2 [660]	1/2 [400]	3/2 [551]	3/2 [402]	5/2 [642]	7/2 [633]	9/2 [624]		
	MIXING AMPLITUDES C_K^I								
5/2	0.028	0.013	0.171	0.047	0.984	0	0	1.10	0
7/2	0.037	0.017	0.243	0.068	0.954	0.154	0	-2.40	42
9/2	0.088	0.041	0.307	0.086	0.919	0.210	0.013	3.51	99
11/2	0.067	0.031	0.337	0.095	0.899	0.252	0.023	-3.91	180
13/2	0.157	0.073	0.392	0.111	0.854	0.270	0.031	4.87	266
15/2	0.091	0.043	0.393	0.112	0.854	0.303	0.041	-4.69	397
17/2	0.224	0.105	0.448	0.129	0.795	0.295	0.045	5.76	511
19/2	0.109	0.051	0.428	0.123	0.820	0.334	0.056	-5.14	701
21/2	0.284	0.133	0.486	0.141	0.742	0.302	0.055	6.37	838
23/2	0.123	0.058	0.452	0.131	0.793	0.351	0.070	-5.42	1096
25/2	0.335	0.157	0.511	0.149	0.697	0.300	0.064	6.80	1251
27/2	0.135	0.064	0.469	0.136	0.772	0.369	0.082	-5.60	1583
29/2	0.376	0.177	0.528	0.155	0.658	0.295	0.067	7.10	1752
31/2	0.143	0.068	0.481	0.141	0.755	0.380	0.093	-5.73	2163
33/2	0.409	0.193	0.539	0.159	0.626	0.288	0.070	7.32	2344

TABLE 2

Rotational parameter $1/2\mathcal{J}$ and the energy gap values Δ extracted from the fitting of calculations to experimental energies of positive parity rotational bands. The deformation parameter δ and calculated values of R_j^σ, $\mathcal{J}_c$ and $\mathcal{J}_o = \mathcal{J} - \mathcal{J}_c$ are given as well. Calculation were performed with κ=0.3 meV.

Nuclei	δ	$1/2\mathcal{J}$ [keV]	Δ [meV]	R_j^σ	$\mathcal{J}_c$ [meV^{-1}]	$\mathcal{J}_o$ [meV^{-1}]
^{155}Gd	0.302	13.2	0.64	0.71	9.8	28.1
^{157}Gd	0.302	12.6	0.49	0.62	13.7	26.0
^{159}Gd	0.302	13.0	0.54	0.66	11.9	26.6
^{159}Dy	0.302	11.5	0.64	0.74	9.7	33.8
^{161}Dy	0.302	12.0	0.58	0.70	11.0	30.7
^{163}Dy	0.302	13.0	0.60	0.67	11.6	26.9
^{157}Er	0.275	17.5	0.60	0.81	4.8	23.8
^{159}Er	0.275	15.0	0.85	0.85	4.3	29.0
^{161}Er	0.302	16.0	1.10	0.85	4.0	27.3
^{163}Er	0.302	13.6	0.81	0.78	7.0	29.7
^{165}Er	0.302	11.8	0.69	0.73	9.8	32.5
^{167}Er	0.275	13.2	0.54	0.69	10.4	27.5
^{169}Yb	0.275	12.8	0.62	0.73	9.2	29.9
^{171}Hf	0.275	13.5	0.72	0.76	7,8	29.2
^{173}Hf	0.240	13.3	0.66	0.75	8.3	29.3
^{175}Hf	0.190	13.0	0.52	0.67	11.7	26.8
^{177}Hf	0.240	14.0	0.65	0.76	7.7	28.0
^{179}Hf	0.240	14.8	0.52	0.73	8.3	25.5

2. Application of the theory developed in ref.[5] to the description of observed rotational spectra is shown in Figs. 1-3. In all the cases, but ^{167}Er, a satisfactory description of the observed bands cannot be achieved with the help of the phenomenological formulae. Nonadiabatic effects are especially strong in $^{157-161}Er$ nuclei where the normal sequence of levels in rotational band is infringed. The structure of wave functions in these nuclei is very complicated, it is impossible practically to indicate the leading component (see Table 3).

It seems rather remarkable that the structure of rotational wave functions occured to be relatively weakly dependent on the choice of the single-particle level scheme and the method of diagonalizing the Coriolis force matrix. For example, in refs[1,4] the calculations for $^{161-165}Er$ and ^{155}Gd nuclei were performed in the Nilsson model with the detailed fitting of band-head energies and of a number of other parameters. Nevertheless, the largest mixing amplitudes, calculated in these papers, as a rule, agree well in value with those calculated by us (up to phases).

Usually, nonadiabatic effects decrease with increasing the quantum number of a band-head state. However, the admixtures of other states are rather large even in rotational bands based on $7/2^+[633]$ and $9/2^+[624]$ states (see Table 4).

3. The basic characteristics of positive parity rotational bands in a number of rare-earth nuclei are listed in Table 2. Values of Δ and $1/2\mathcal{J}$ are extracted from the fit of calculations to experimental data. The calculated values of R_j^σ given in table 2 show that the centrifugal and spin polarization reduces strongly (by 15 to 38%) the Coriolis coupling in all the nuclei considered. The polarization factor R_j^σ follows usually the change

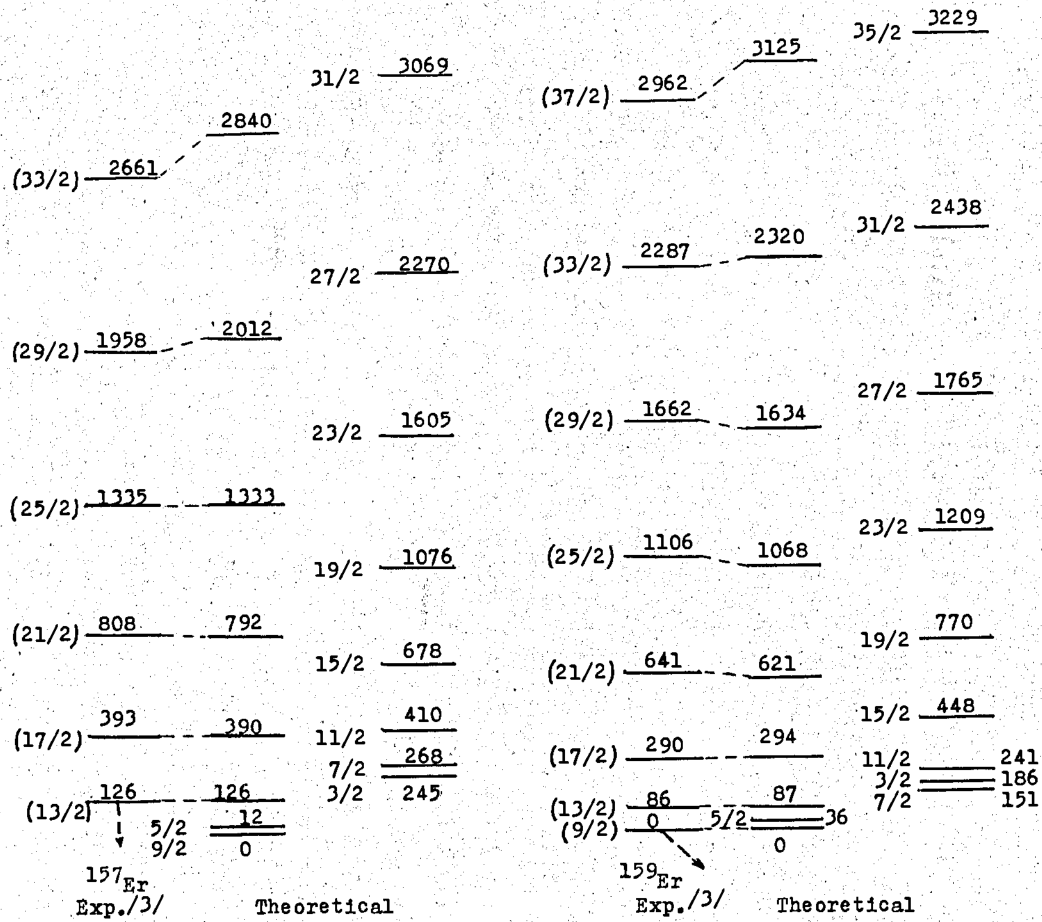

Fig. 2. Positive parity rotational bands in ^{157}Er and ^{159}Er. The parameters used in calculations are given in table 2.

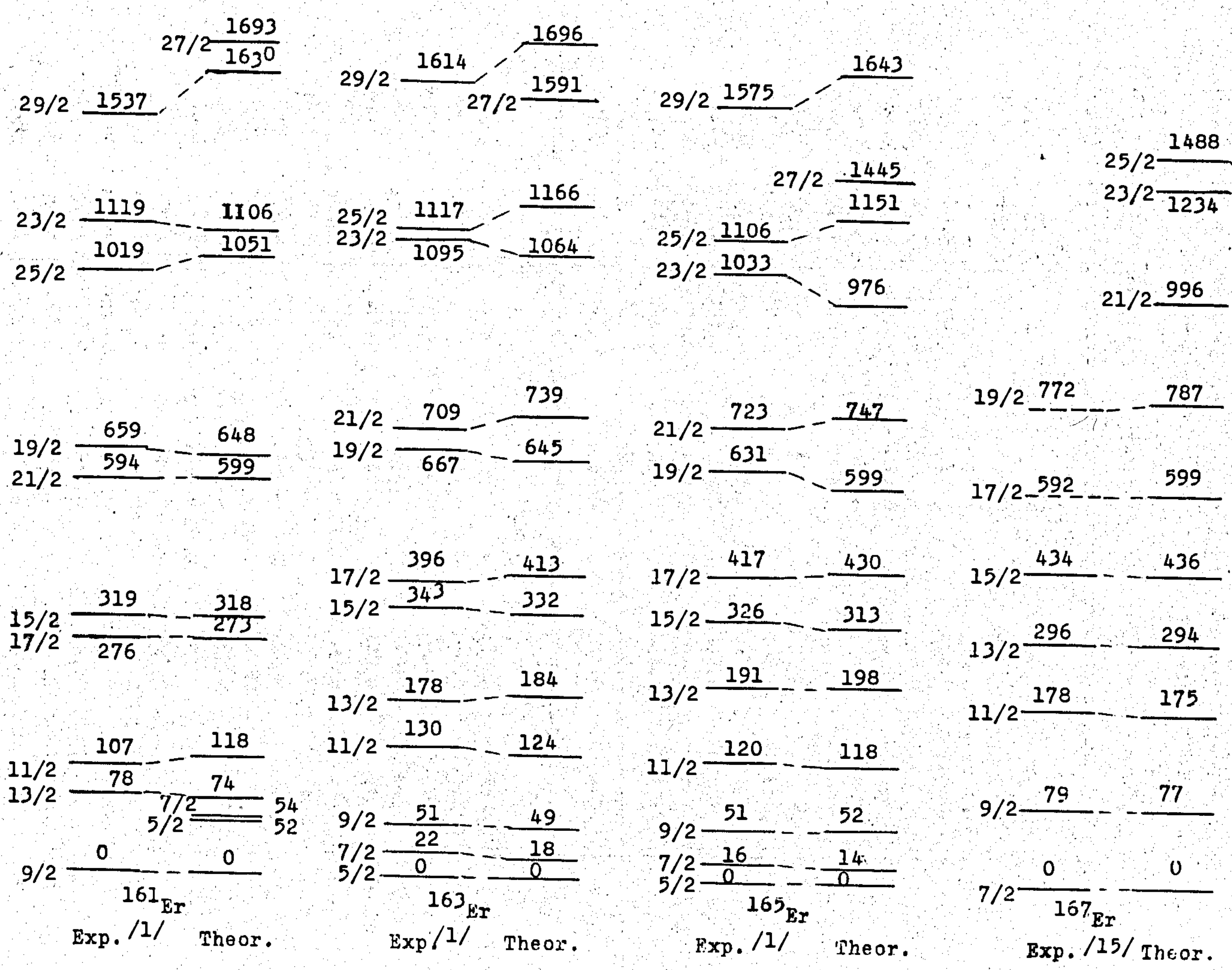

TABLE 3

Positive parity rotational bands in ^{157}Er and ^{159}Er (see caption to table 1). Calculations are performed with parameters as given in table 2. Only the largest amplitudes C_K^I are presented.

I	1/2 [660]	3/2 [651]	5/2 [642]	7/2 [633]	$a(I)$	$\mathcal{E}(I)$ [keV]
			^{157}Er			
3/2	0.420	0.908	–	–	-1.71	245
5/2	0.854	0.508	0.114	–	7.59	12
7/2	0.365	0.853	0.369	0.050	-4.47	268
9/2	0.822	0.535	0.192	0.031	8.02	0
11/2	0.326	0.808	0.478	0.114	-5.51	410
13/2	0.802	0.546	0.238	0.055	8.22	126
15/2	0.299	0.771	0.537	0.167	-6.08	678
17/2	0.786	0.551	0.269	0.075	8.34	390
19/2	0.279	0.740	0.573	0.211	-6.45	1076
21/2	0.773	0.555	0.293	0.093	8.43	792
23/2	0.263	0.715	0.596	0.248	-6.71	1605
25/2	0.762	0.557	0.311	0.108	8.49	1333
27/2	0.250	0.693	0.612	0.278	-6.91	2270
29/2	0.753	0.558	0.326	0.122	8.54	2012
31/2	0.240	0.674	0.623	0.304	-7.06	3069
33/2	0.744	0.558	0.339	0.135	8.58	2840
			^{159}Er			
3/2	0.297	0.955	–	–	-1.63	186
5/2	0.711	0.658	0.248	–	7.82	36
7/2	0.273	0.806	0.519	0.080	-5.28	151
9/2	0.710	0.624	0.322	0.062	8.52	0
11/2	0.257	0.751	0.588	0.157	-6.20	241
13/2	0.708	0.607	0.349	0.092	8.76	87
15/2	0.246	0.715	0.618	0.211	-6.66	448
17/2	0.705	0.597	0.365	0.115	8.89	294
19/2	0.236	0.689	0.635	0.253	-6.96	770
21/2	0.701	0.590	0.376	0.133	8.96	621
23/2	0.228	0.668	0.645	0.286	-7.16	1209
25/2	0.698	0.585	0.385	0.147	9.01	1068
27/2	0.221	0.650	0.651	0.313	-7.31	1765
29/2	0.694	0.581	0.391	0.160	9.05	1634
31/2	0.215	0.635	0.654	0.336	-7.43	2438
33/2	0.691	0.577	0.397	0.171	9.08	2320
35/2	0.210	0.622	0.657	0.355	-7.53	3229
37/2	0.688	0.575	0.402	0.180	9.10	3125

TABLE 4

Positive parity rotational bands in ^{169}Yb, ^{177}Hf and ^{179}Hf (see caption to table 1). Calculations are performed with parameters as given in table 2. Only the largest amplitudes C_K^I are presented.

I	3/2 [651]	5/2 [642]	7/2 [633]	9/2 [624]	$a(I)$	$\mathcal{E}(I)$ [keV]	$\mathcal{E}_{exp}$ [13,14] [keV]
		^{169}Yb					
7/2	0.025	0.174	0.984	–	-0.94	0	0
9/2	0.046	0.248	0.968	0.138	2.12	69	70.9
11/2	0.067	0.302	0.931	0.191	-2.97	157	161.1
13/2	0.090	0.346	0.906	0.225	3.63	265	269.5
15/2	0.108	0.380	0.883	0.251	-4.11	396	404.7
17/2	0.135	0.410	0.859	0.269	4.56	547	546.6
19/2	0.147	0.432	0.842	0.285	-4.82	724	735.6
21/2	0.178	0.455	0.817	0.293	5.21	918	902.3
23/2	0.182	0.470	0.806	0.304	-5.30	1144	1155.7
25/2	0.220	0.488	0.780	0.304	5.61	1382	1334.4
27/2	0.212	0.497	0.776	0.315	-5.65	1660	1664.4
29/2	0.259	0.513	0.743	0.308	6.11	1940	1843.1

I	5/2 [642]	7/2 [633]	9/2 [624]	11/2 [615]	$a(I)$	$\mathcal{E}(I)$ [keV]	$\mathcal{E}_{exp.}$ [15] [keV]
		^{177}Hf					
9/2	0.028	0.188	0.982	–	0.76	0	0
11/2	0.050	0.264	0.957	0.112	-1.71	106	105.3
13/2	0.073	0.320	0.932	0.155	2.44	236	233.8
15/2	0.095	0.363	0.908	0.185	-3.01	388	387.1
17/2	0.117	0.399	0.885	0.208	3.48	565	561.5
19/2	0.138	0.428	0.863	0.226	-3.85	767	765.6
21/2	0.160	0.452	0.842	0.240	4.19	994	980.0
23/2	0.180	0.472	0.823	0.251	-4.45	1246	–
		^{179}Hf					
9/2	0.015	0.135	0.991	–	0.62	0	0
11/2	0.028	0.194	0.971	0.140	-1.42	121	122.7
13/2	0.042	0.239	0.951	0.192	2.04	266	268.9
15/2	0.056	0.277	0.932	0.227	-2.53	437	438.7
17/2	0.071	0.310	0.914	0.252	2.94	634	631.5
19/2	0.086	0.338	0.896	0.271	-3.28	857	848.5
21/2	0.102	0.364	0.880	0.285	3.58	1106	1085.2
23/2	0.117	0.387	0.865	0.295	-3.83	1383	–

of Δ from nucleus to nucleus. The moment of inertia $\mathcal{J}_c$, arising from the centrifugal interaction, depends strongly on Δ and on the degree of particle population of the levels coupled by the Coriolis interaction (the latter dependence is characterized by the variation of $\mathcal{J}_c$ value in different nuclei with approximately the same Δ value).

To characterize the "inert" core that does not take part in centrifugal interaction the values of $\mathcal{J}_o = \mathcal{J} - \mathcal{J}_c$ are calculated and listed in table 2. It is necessary to note that the fluctuation of $\mathcal{J}_o$ in different nuclei with the same number of neutrons does not exceed usually 10% and is essentially smaller than the fluctuation of $\mathcal{J}_c$ (compare, e.g. ^{155}Gd and ^{159}Er, ^{157}Gd and ^{161}Er and so on).

At last, we should point out that comparatively small number of levels near the Fermi surface, coupled by the Coriolis force, gives an essential contribution to the effective moment of inertia (usually $\mathcal{J}_c \approx 1/4\,\mathcal{J}$).

4. Comparative behaviour of the energy gap and the moment of inertia in odd-mass and even-even nuclei is presented in Fig.4. As a rule, Δ_{odd} obtained from the rotational band analysis, follows the trend of Δ_{even}, obtained from the analysis of pair energies in neighbouring even-even nuclei. Their difference can result from the blocking effect, e.g.

It is remarkable also that the effective moments of inertia in odd-mass nuclei are close to experimental ones in neighbouring even-even nuclei and follow the general trend of the latter. Deviations from a smooth behaviour of $\mathcal{J}_{odd}$ are connected, as a rule with the change of the band -head level (roughly speaking, it can be associated with the dependence of $\mathcal{J}_{odd}$ on the approximate K

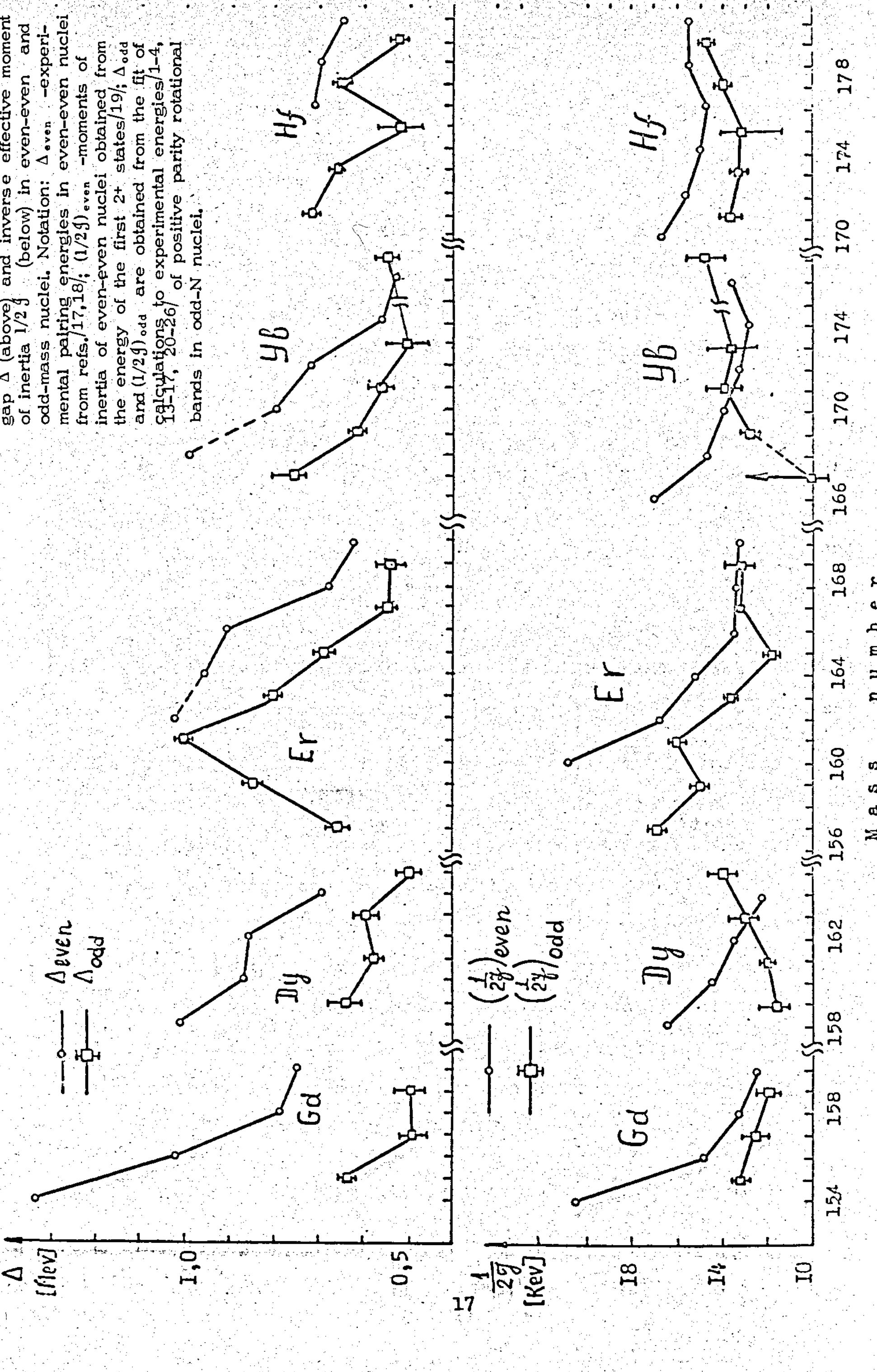

Fig. 4. Comparitive behaviour of the energy gap Δ (above) and inverse effective moment of inertia $1/2\mathcal{J}$ (below) in even-even and odd-mass nuclei. Notation: Δ_{even} -experimental pairing energies in even-even nuclei from refs./17,18/; $(1/2\mathcal{J})_{even}$ -moments of inertia of even-even nuclei obtained from the energy of the first 2+ states/19/; Δ_{odd} and $(1/2\mathcal{J})_{odd}$ are obtained from the fit of calculations to experimental energies/1-4, 13-17, 20-26/ of positive parity rotational bands in odd-N nuclei.

quantum number). That is especially noticeable in the Er isotopes: rotational bands in ^{157}Er and ^{159}Er contain large components of $1/2^+$ [660] and $3/2^+$ [651] states, in $^{161-165}Er$ $5/2^+$ [642] state and in ^{167}Er $7/2^+$ [633] state. In a number of nuclei only a few lowest rotational levels are known, therefore it is difficult to obtain reliable values of Δ and $1/2\mathcal{J}$ for them (e.g., in ^{167}Yb).

5. Characteristic deviations from the Alaga rule for the intraband branching ratio connected with the nonadiabaticity are shown in Fig.5. The Alaga rule is violated essentially only in the case of a very strong K-mixing (e.g., in ^{157}Er, ^{161}Er, ^{161}Dy nuclei). This result is quite natural, since the Coriolis interaction, concerning a small number of single-particle states, cannot perturb strongly the nuclear quadrupole moment.

Comparison of the calculated branching ratio $B(E2,I\to I-2)/B(E2,I\to I-1)$ with the experimental data was made for $9/2^+$ [624] rotational bands in ^{177}Hf and ^{179}Hf (Fig.6). Deviations from the Alaga rule are within the experimental measurement errors. In both the nuclei however, nonadiabatic and polarization effects are quite noticeable in the rotational energies and wave functions (see Tables 2 and 4). The reliable experimental data for branching ratio in other nuclei are not yet available.

6. In order to test the validity of the quasi-classical approximation in calculating R_j, equation (28)[x/] for the energy shift E_K-W_K of one-quasi-particle levels arising from the three-quasi-particle admixtures has been solved. It turned out that these shifts are small ($\lesssim$ 50-100 keV) and slightly varied

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x/ Here and below we refer to the equations and formulae from paper/5/.

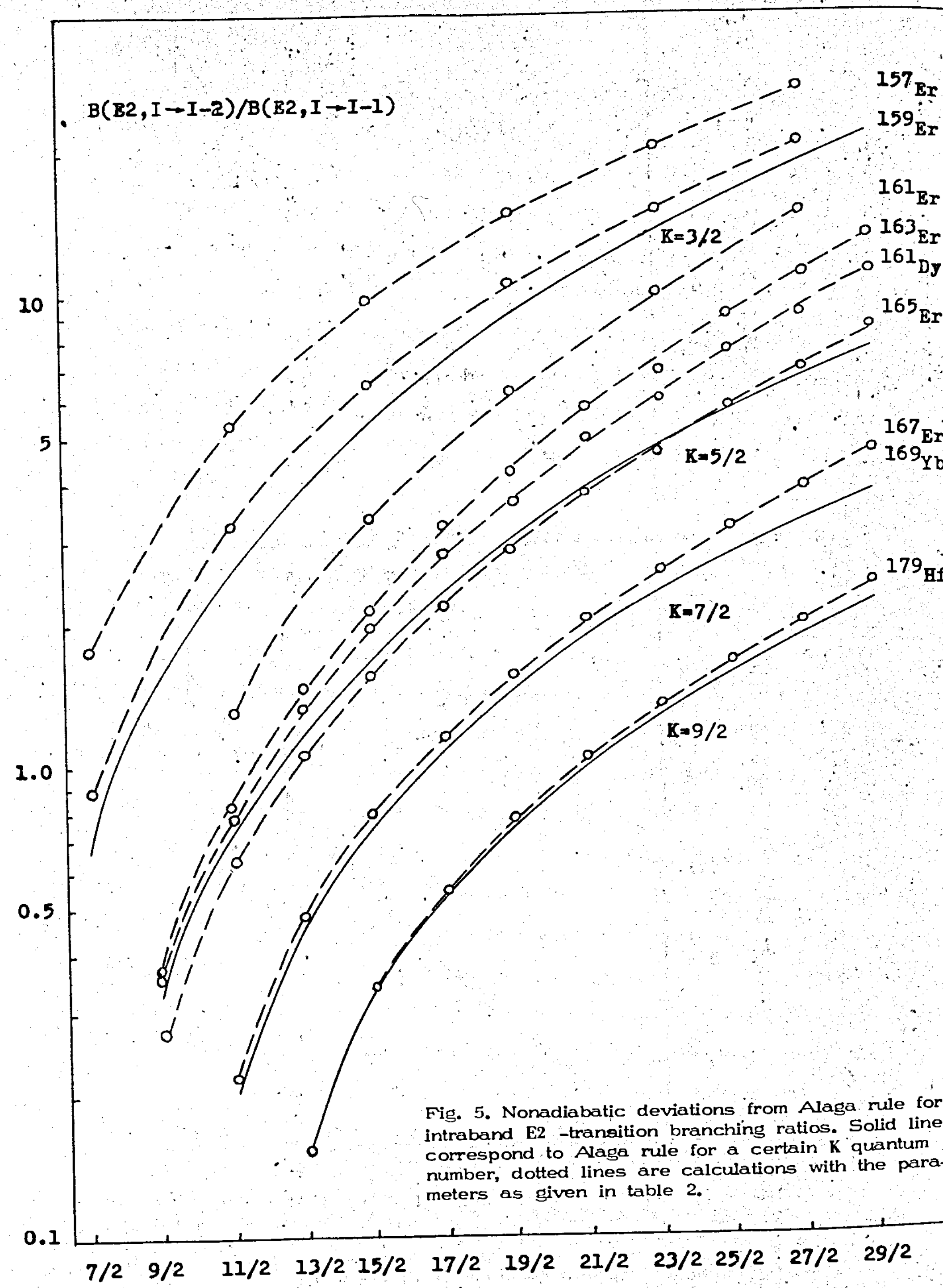

Fig. 5. Nonadiabatic deviations from Alaga rule for intraband E2 -transition branching ratios. Solid lines correspond to Alaga rule for a certain K quantum number, dotted lines are calculations with the parameters as given in table 2.

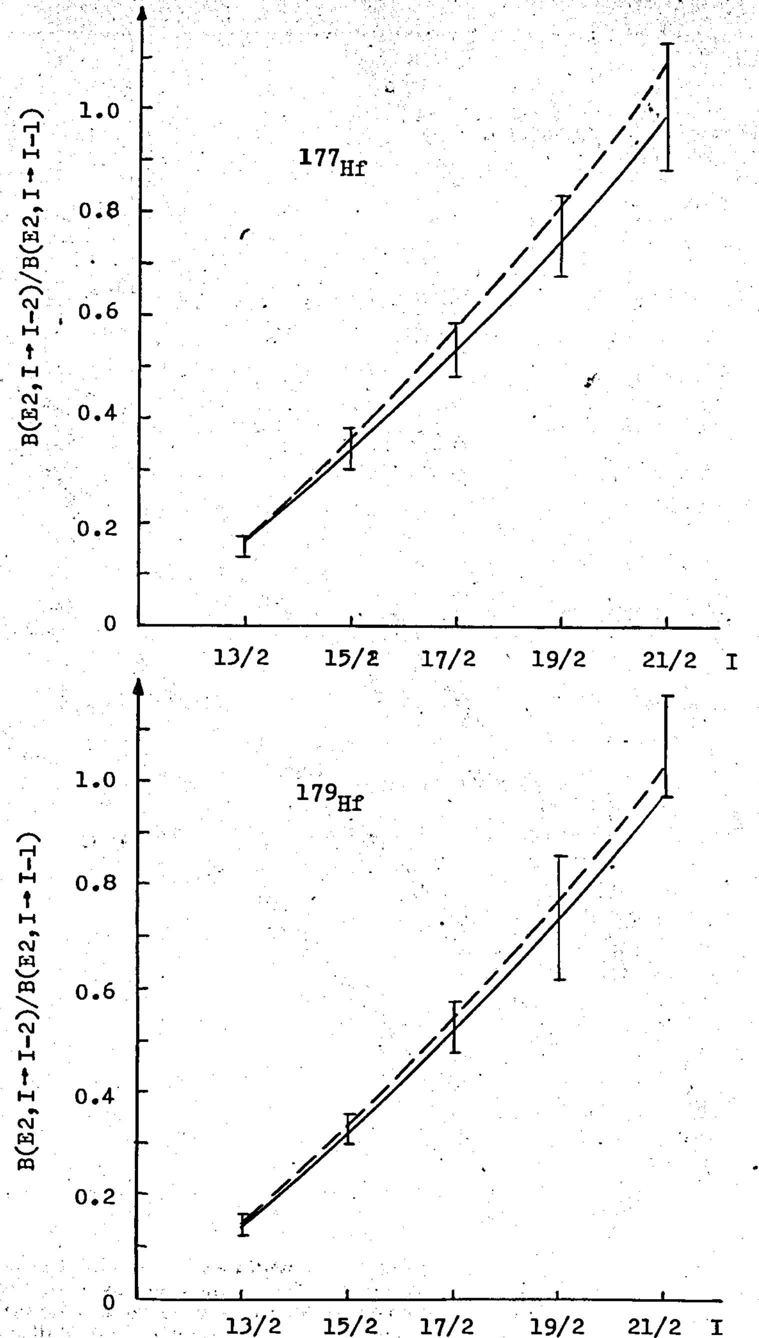

Fig. 6. Experimental E2 -transition branching ratios within $9/2^+[624]$ rotational band in ^{177}Hf and ^{179}Hf (vertical lines, corresponding to the error of measurements)/15/ are compared with the predictions of Alaga rule (solid lines) and nonadiabatic calculations with the parameters as given in table 2 (dotted lines).

from level to level. The total admixture of three-quasi-particle states does not exceed 1-2% of the norm. Thus, approximation (36) seems to be justified in the case when $|E_K - E_{K'}| \lesssim 2\Delta$. In the opposite case, the quantity $\mathcal{J}_{KK'}$ determined by eq.(29) may become negative. In that event the polarization factor $R_j(K;K') \gtrsim 1$ (see, eq. (35)). However, the effects of the Coriolis coupling of widely spaced levels $(|E_K - E_{K'}| > 2\Delta)$ are usually small (see, e.g. the contribution of $9/2^+$ [624] state, table 1). An example of calculation of the polarization matrix $R_j(K,K')$ as given by eqs. (29) and (35) is presented in Fig.7. The tendency of decreasing of $R_j(K,K')$ with increasing the spacing between coupled levels is seen. This tendency is valid only for the coupling of levels with $|E_K - E_{K'}| < 2\Delta$. However, because of both the use of other approximations (static approximation for Δ and $\mathcal{J}$, neglect of the residual quadrupole force, etc.) and the complication of numerical calculations, the employment of such a detailed polarization matrix seems to be unjustified.

At first sight it seems surprising the applicability of the static approximation $\Delta = const.$ and $\mathcal{J} = const.$ to the description of high spin rotational states in odd-mass nuclei. Indeed, the theory predicts for even-even nuclei a quick change of the energy gap (especially the neutron gap) and the moment of inertia with the increase of rotational frequency (see, e.g. ref./10/). It have been pointed out in /11/ that in odd-mass nuclei the mixing of single-particle states stabilizes effectively the energy gap value, due to the strong deviation of the rotational energy from the $I(I+1)$ law. Justification of the static approximation can be found, however, by means of solving the dynamical equations for the energy gap and moment of inertia in odd-mass nuclei. This

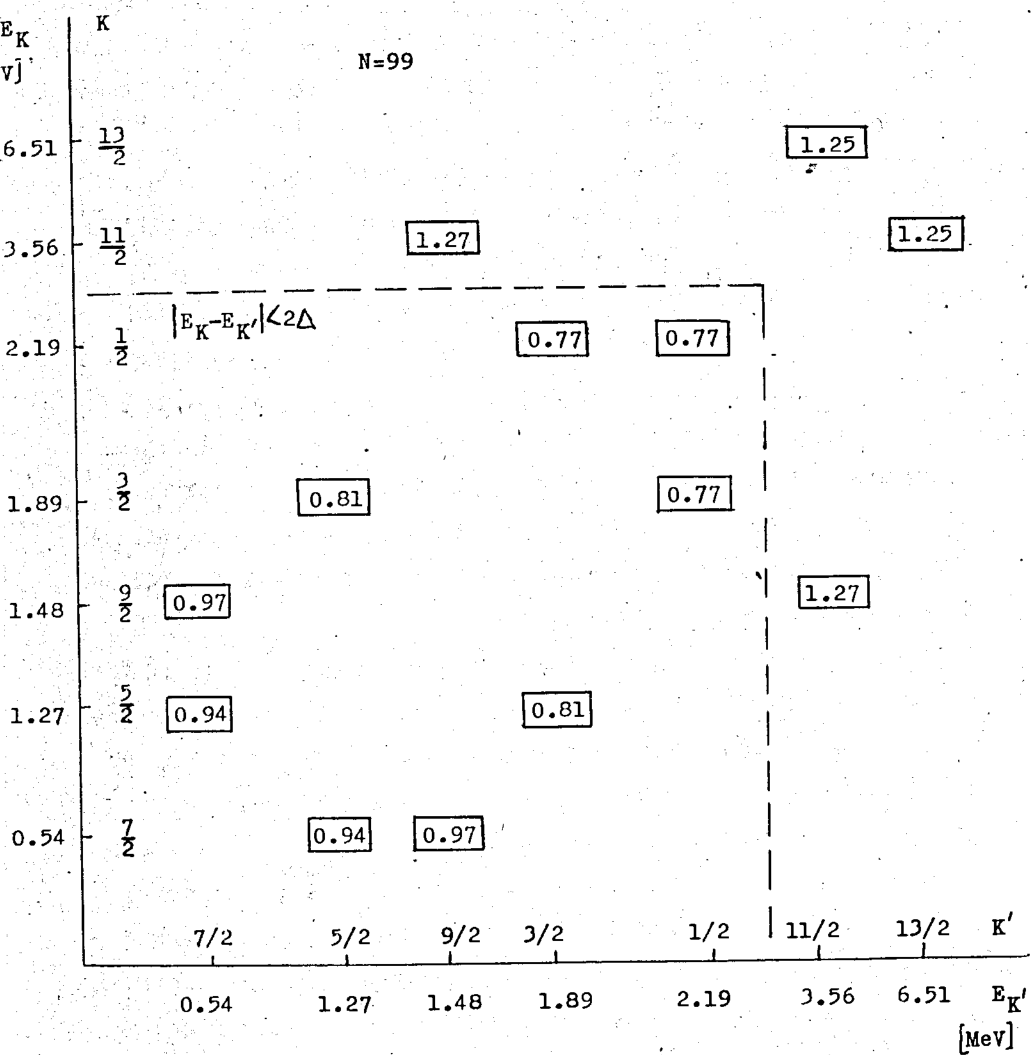

Fig. 7. Polarization matrix $R_j(K,K')$ for the renormalization of the matrix elements $\langle K|j_\pm|K'\rangle$ between the states of $i_{13/2}$ subshell in ^{169}Yb (rotational band on $7/2^+[633]$ state). Calculations are performed with $\delta = 0.275$, $\kappa = 0$, $\Delta = 0.53$ MeV and $1/2\mathcal{J} = 12.8$ keV.

problem will be considered in the third part of this series.

Conclusion

In choosing the residual interactions between nucleons we were influenced by the following considerations.

i) It is necessary to take into account those interactions which strongly affect the single-particle spectra and give the coherent contribution to the renormalization of the Coriolis force.

ii) On the other hand, it is desirable not to overcomplicate at the first stage the theory and to conserve the possibility of performing the comparatively simple numerical calculations. For that reason we do not include in the intrinsic Hamiltonian the quadrupole and octupole residual forces which complicate noticeably the structure of excited states (see, e.g., ref./12/). All the rotational bands, considered in the present paper, are built upon the ground or low-lying intrinsic excited states, which contain small admixtures from the interaction between quasiparticles and vibrational phonons.

The polarization factors R_j^σ were calculated in the quasiclassical approximation which permits us to give the simple physical interpretation of the polarization effect in terms of renormalization of the effective moment of inertia entering the Coriolis force. This approximation allows one to simplify significantly the numerical calculations.

In all the nuclei considered the polarization effects are strong and the dispersion of values of R_j^σ around the average value 0.7 is not large. The renormalization of the Coriolis force is mostly due to the centrifugal interactions. The role of the spin-spin interactions is effectively reduced to the change of the energy gap parameter. But such a conclusion can be drawn only for the rotationa bands which are built upon the single-particle states originating from the spherical subshell with large j-value.

The calculations performed have shown that the method suggested in ref.[5] gives the fairly good description of the strongly distorted by the Coriolis force rotational bands up to the large spin values using the effective parameters $\mathcal{J}$ and Δ in the static limit. It occured that the effective values of $\mathcal{J}$ and Δ in odd-mass nuclei follow the behaviour of those values in even-even nuclei. Usually the results of calculations slightly change with the small variation of deformation if the Coriolis mixing involves more than three single-particle states.

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