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N.I. Pyatov, M.I. Chernej, M.I. Baznat

POLARIZATION EFFECTS IN THE ROTATIONAL MOTION OF ODD-MASS NUCLEI. 1. THEORY

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1. Introduction

The simple Bohr rotational model[1] is found to be extremely useful in analysing experimental rotational spectra in odd-mass deformed nuclei especially after Kerman[2] has developed a procedure of taking into account the Coriolis force. It is known that in order to bring into agreement the calculations in the Kerman scheme and the experimental data it is necessary to decrease essentially the values of the single-particle matrix elements $\langle j_{\pm} \rangle$ between the states belonging to the same spherical subshell[3]. Phenomenologically this renormalization of $\langle j_{\pm} \rangle$ is accounted for by the decrease by 20-30% of the rotational parameter $1/2\mathcal{J}$ entering the Coriolis term in the Hamiltonian[4].

The Coriolis coupling of particle-hole states can be weakened by the decreasing the energy gap Δ, appearing due to the pairing interaction[5]. However, from the physical point of view it is difficult to justify the strong weakening of the pairing force in odd-mass nuclei. Earlier[6] we indicated that the regular renormalization of $\langle j_{\pm} \rangle$ may result from the static centrifugal interaction of an odd nucleon with the particles of an even-even core and gave the quasiclassical estimate of the appropriate polarization factor[x].

The present paper is devoted to the investigation of static polarization effects in rotational motion arising from the centrifugal and spin-spin interactions of an odd nucleon with the core. From the physical point of view these effects are completely analogous to the well-known effects of the spin polarization in magnetic moments[7,8]. In the subsequent sections of the paper the Hamiltonian of the intrinsic motion including pairing and static centrifugal interactions is diagonalized. The intrinsic wave functions are then used in the diagonalization of the Coriolis force matrix. Finally, the effect of the spin-spin interaction of an odd nucleon with the core is taken into account and its contribution to the polarization factor is estimated.

The application of the theory to concrete nuclei and the discussion of numerical results will be presented in the second part of this paper.

2. Rotational Model

We assume that in a deformed nucleus the intrinsic motion (for a fixed orientation of the nucleus) which depends upon the nucleon coordinates in the body-fixed system, and the collective rotational motion may be approximately separated. According to this the total angular momentum $\vec{I}$ consists of two parts, the collective rotational momentum $\vec{R}$ and the intrinsic one $\vec{j}$ formed by a single or a few nucleons moving in orbitals close to the Fermi surface.

[x] One of the authors (N.P.) expresses his gratitude to Prof. A. Bohr who drew his attention to the fact that this effect was overestimated. In the present paper necessary corrections have been introduced.

$$\vec{I} = \vec{R} + \vec{j} . \tag{1}$$

The total Hamiltonian of an axially-symmetric nucleus consists of two parts

$$H = H_{intr} + H_{rot} , \tag{2}$$

where H_{intr} describes the intrinsic motion of nucleons in a certain axially-symmetric average field and H_{rot} has the form ($\hbar = 1$)

$$H_{rot} = \frac{1}{2\mathcal{J}} (R_1^2 + R_2^2) . \tag{3}$$

Here R_i are the components of the angular momentum $\vec{R}$ and $\mathcal{J}$ is the effective moment of inertia. Thus, H_{rot} describes the rotation of the axially-symmetric rotor about the axis perpendicular to the nuclear symmetry axis ($R_3 = 0$) . With the help of (1) H_{rot} can be written in the form

$$H_{rot} = H^0_{rot} + H_j + H_c , \tag{4}$$

$$H^0_{rot} = \frac{1}{2\mathcal{J}}(I^2 - I_3^2) , \tag{4a}$$

$$H_j = \frac{1}{2\mathcal{J}}(j_1^2 + j_2^2) = \frac{1}{2\mathcal{J}} \frac{1}{4} [(j_+ + j_-)^2 - (j_+ - j_-)^2] , \tag{4b}$$

$$H_c = -\frac{1}{2\mathcal{J}}\, 2\,[I_1 j_1 + I_2 j_2] = -\frac{1}{2\mathcal{J}}\frac{1}{2}[(I_+ + I_-)(j_+ + j_-) - \\ -(I_+ - I_-)(j_+ - j_-)] \tag{4c}$$

with the usual notations

$$I_\pm = I_1 \pm iI_2 \,, \quad j_\pm = j_1 \pm i j_2 \quad . \tag{5}$$

In eq. (4) the term H^0_{rot} depends only on the total angular momentum I and its projection on the nuclear symmetry axis $I_3 = K$. In the general case, the term H_j represents the centrifugal interaction of particles induced by rotation. This interaction and related polarization effects are considered in detail later on. Finally, H_c is the Coriolis force which acts on particles in the rotated coordinate frame.

The term H_{intr} is assumed to be explicitly independent of the total angular momentum I. Since H_j, is independent of the angular momentum I it can be included in the intrinsic part of the Hamiltonian.

The eigenfunctions and the eigenvalues of the intrinsic motion are characterized by the quantum number K ($I_3 = j_3 = K$)

$$(H_{intr} + H_j)\,\phi_K = \mathcal{E}_K \phi_K \,. \tag{6}$$

From the invariance of the intrinsic Hamiltonian under time reversal it follows that the eigenvalues $\mathcal{E}_K$ are degenerated with respect to the sign of K. Let us define the time-reversal operator T by[x/]

$$T\,\phi_K \equiv \phi_{\tilde{\tilde{K}}} = (-1)^{K+1/2}\,\phi_{-K} \tag{7}$$

where ϕ_{-K} is defined as in ref.[/10/]

$$\hat{R}_1\,\phi_K = (-1)^{\ell+K+1/2}\,\phi_{-K}\,. \tag{8}$$

Here $\hat{R}_1$ is the operator of rotation through an angle π about the 1-axis, the phase $(-1)^{\ell}$ characterizes the parity of the state. The complete symmetrized wave function can be written in the form

$$|IM\rangle = \sum_K C_K^I\,|IMK\rangle\,, \tag{9}$$

where

$$|IMK\rangle = \sqrt{\frac{2I+1}{16\pi^2}}\,\{\phi_K D^I_{MK} + (-1)^{I+\ell-1/2}\,\phi_{-K} D^I_{M,-K}\} =$$

$$= \sqrt{\frac{2I+1}{16\pi^2}}\,\{\phi_K D^I_{MK} + (-1)^{I+\ell+K}\,\phi_{\tilde{\tilde{K}}} D^I_{M,-K}\} \tag{9a}$$

---

x/ The phase of spherical harmonics $\mathcal{Y}_{\ell m}$ is assumed to be the same as that used by Condon and Shortley(see, e.g. ref./9/.)

C^I_K are the amplitudes of Coriolis mixing of states with different K. The sum in eq. (9) runs all the states with $K \leq I$. The normalization condition for the amplitudes

$$\sum_K (C^I_K)^2 = 1\,. \tag{10}$$

3. Intrinsic Motion

We consider the intrinsic motion of nucleons in an axially symmetric average field (described with, e.g. the Nilsson or Saxon-Woods potential). For the sake of simplicity we assume at first that the nucleons interact only by means of pairing residual forces. The Hamiltonian of the system in the second quantization representation is written in the form

$$H_{intr} = H_{sp} + H_{pair} \tag{11}$$

$$H_{sp} = \sum_\nu (\epsilon_\nu - \lambda)\,(a^+_\nu a_\nu + a^+_{\tilde{\nu}} a_{\tilde{\nu}}) \tag{11a}$$

$$H_{pair} = -G\,\Gamma^+\Gamma\,, \quad \Gamma = \sum_\nu a_{\tilde{\nu}} a_\nu\,. \tag{11b}$$

Here, $\epsilon_\nu - \lambda$ are the single-particle energies reckoned from the chemical potential of the system, λ, which may be found from the condition of the particle number conservation; a^+_ν and a_ν are the particle creation and annihilation operators in the $|\nu\rangle$ state respectively (the state $|\tilde{\tilde{\nu}}\rangle \equiv T|\nu\rangle$). The pairing force strength G, is assumed to be different for protons and neutrons. The centrifugal interaction, H_j, is also taken into account in the intrinsic motion. In the second quantization representation this operator reads

$$H_j = \frac{1}{2\mathcal{J}} \frac{1}{4} \sum_{r=\pm 1} r \left(F^{(r)} \right)^2 , \tag{12}$$

with definition

$$F^{(r)} \equiv \sum_{\nu\nu'} \langle \nu | j_+ + r\, j_- | \nu' \rangle\, a_\nu^+ \, a_{\nu'} \, . \tag{13}$$

With the conventional phasing of states defined by eqs. (7) and (8) the matrix elements in (13) possess the following symmetry properties:

$$\begin{aligned} \langle \nu | j_+ + r\, j_- | \nu' \rangle &\equiv j^{(r)}_{\nu\nu'} = -r\, j^{(r)}_{\tilde{\tilde{\nu}},\tilde{\tilde{\nu}}'} = r\, j^{(r)}_{\nu'\nu} \\ \langle \nu | j_+ + r\, j_- | \tilde{\tilde{\nu}}' \rangle &\equiv j^{(r)}_{\nu\tilde{\tilde{\nu}}'} = r\, j^{(r)}_{\tilde{\tilde{\nu}}\nu'} = j^{(r)}_{\nu'\tilde{\tilde{\nu}}} \, . \end{aligned} \tag{14}$$

The last matrix elements in (14) exist only between the states with $K = 1/2$. The matrix elements for $r = \pm 1$ differ only by phases i.e. $(j^{(+)}_{\lambda\lambda'})^2 = (j^{(-)}_{\lambda\lambda'})^2$.

We pass to the quasiparticle representation by means of the Bogolubov canonical transformation

$$
\begin{aligned}
a_{\nu} &= u_{\nu}\, \alpha_{\tilde{\nu}} + v_{\nu}\, \alpha^{+}_{\nu} \\
a_{\tilde{\nu}} &= u_{\nu}\, \alpha_{\nu} - v_{\nu}\, \alpha^{+}_{\tilde{\nu}} ,
\end{aligned}
\qquad (15)
$$

where u_ν and v_ν are the transformation parameters. Introducing the quasiboson operators

$$
\begin{aligned}
A^{(\tau)}_{\nu\nu'} &\equiv \alpha_{\nu}\, \alpha_{\tilde{\tilde{\nu}}'} + \tau\, \alpha_{\tilde{\tilde{\nu}}}\, \alpha_{\nu'} \\
B^{(\tau)}_{\nu\nu'} &\equiv \alpha^{+}_{\nu}\, \alpha_{\nu'} - \tau\, \alpha^{+}_{\tilde{\tilde{\nu}}}\, \alpha_{\tilde{\tilde{\nu}}'} \\
\bar{A}^{(\tau)}_{\nu\nu'} &\equiv \alpha_{\nu}\, \alpha_{\nu'} - \tau\, \alpha_{\tilde{\tilde{\nu}}}\, \alpha_{\tilde{\tilde{\nu}}'} \\
\bar{B}^{(\tau)}_{\nu\nu'} &\equiv \alpha^{+}_{\nu}\, \alpha_{\tilde{\tilde{\nu}}'} + \tau\, \alpha^{+}_{\tilde{\tilde{\nu}}}\, \alpha_{\nu'}
\end{aligned}
\qquad (16)
$$

and using eqs. (14)–(16), one can write the operator $F^{(\tau)}$ in the form (the phase is arbitrary)

$$
F^{(\tau)} = F^{(\tau)}_{B} + F^{(\tau)}_{A} \qquad (17)
$$

$$
F^{(\tau)}_{B} \equiv \sum_{\nu,\nu'>0} M_{\nu\nu'} \left[i^{(\tau)}_{\nu\nu'}\, B^{(\tau)}_{\nu\nu'} - i^{(\tau)}_{\nu\tilde{\tilde{\nu}}'}\, \bar{B}^{(\tau)}_{\nu\nu'} \right] \qquad (17a)
$$

$$
\begin{aligned}
F^{(\tau)}_{A} \equiv \frac{1}{2} \sum_{\nu,\nu'>0} L_{\nu\nu'} \Big[& i^{(\tau)}_{\nu\nu'} \left(A^{(\tau)+}_{\nu\nu'} + \tau A^{(\tau)}_{\nu\nu'} \right) + \\
& + i^{(\tau)}_{\nu\tilde{\tilde{\nu}}'} \left(\bar{A}^{(\tau)+}_{\nu\nu'} + \tau \bar{A}^{(\tau)}_{\nu\nu'} \right) \Big] ,
\end{aligned}
\qquad (17b)
$$

where

$$M_{\nu\nu'} \equiv u_\nu u_{\nu'} + v_\nu v_{\nu'} \tag{17c}$$

$$L_{\nu\nu'} \equiv u_\nu v_{\nu'} - u_{\nu'} v_\nu .$$

In the independent quasiparticles approximation the Hamiltonian (11) has the form[x/]

$$H^0_{intr} = U_0 + H_{sqp} \tag{18}$$

$$U_0 = 2 \sum_\nu (\epsilon_\nu - \lambda) v^2_\nu - \frac{\Delta^2}{G} \tag{18a}$$

$$H_{sqp} = \sum_\nu E_\nu B^{(-)}_{\nu\nu} \tag{18b}$$

$$\Delta = G \sum_\nu u_\nu v_\nu \quad ; \quad E_\nu = \sqrt{\Delta^2 + (\epsilon_\nu - \lambda)^2} . \tag{18c}$$

We look now for the eigenfunctions and the eigenvalues of the total intrinsic Hamiltonian

$$\mathcal{H}_{intr} \equiv H^0_{intr} + H_j . \tag{19}$$

[x/] We neglect here the pairing vibrations and the coupling between quasiparticles and pairing vibrations.

The centrifugal interaction (12) generates the excitations with $K^{\pi} = 1^{+}$ in even-even nuclei and leads to the three-quasi-particle admixtures in low-energy states of odd-mass nuclei. The trial wave function for an odd-mass nucleus may be chosen in the form ($K > 0$)

$$\phi_K = \{ N_K a^+_K + \frac{1}{2} \sum_{\tau} \sum_{K'} \sum_{\lambda\lambda' \neq K'} [R^{KK'}_{\lambda\lambda'}(\tau) a^+_{K'} A^{(\tau)+}_{\lambda\lambda'} + \tag{20}$$

$$+ \delta_{K,1/2} \delta_{K',1/2} R^{K\tilde{\tilde{K}}'}_{\lambda\lambda'}(\tau) a^+_{\tilde{K}'} A^{(\tau)+}_{\lambda\lambda'} +$$

$$+ (\text{analogous terms with operators } \bar{A}^{(\tau)}_{\lambda\lambda'})]\}| 0>).$$

Here N_K and $R^{KK'}_{\lambda\lambda'}$ are variational amplitudes, $|0>$ the quasiparticle vacuum. The blocking effect is taken into account in all the sums in eq. (20) ($\lambda\lambda' \neq K'$). The bosons described by the operators $A^{(\tau)}_{\lambda\lambda'}$ are assumed to have the total projection of angular momenta and parity $K^{\pi} = 1^{+}$. The three-quasi-particle admixtures have the same total K quantum number as the one-quasiparticle has in the wave function ϕ_K (i.e. $K' = K \pm 1$).

Later on we employ the following approximate commutation relations

$$[A^{(\tau)}_{\nu\nu'}, A^{(\tau')+}_{\lambda\lambda'}] \cong 2\delta_{\tau\tau'} (\delta_{\nu\lambda} \delta_{\nu'\lambda'} - \tau \delta_{\nu\lambda'} \delta_{\nu'\lambda}) \tag{21}$$

$$[a_K, A^{(\tau)+}_{\lambda\lambda'}] \cong 0$$

and exact ones

$$[A^{(\tau)}_{\nu\nu'}, B^{(\tau')}_{\lambda\lambda'}] = \delta_{\tau\tau'} [\delta_{\lambda\nu} A^{(-)}_{\lambda'\nu'} - \tau \delta_{\nu'\lambda} A^{(-)}_{\lambda'\nu}] +$$

$$+\delta_{\tau,-\tau'} [\delta_{\lambda\nu} A^{(+)}_{\lambda'\nu'} - \tau \delta_{\nu\lambda} A^{(+)}_{\lambda'\nu}] . \tag{22}$$

Similar relations can be easily obtained for the operators $\bar{A}$ and $\bar{B}$.

The normalization condition for the wave function (20) reads

$$<\phi^{+}_{K} \phi_{K}> = N^{2}_{K} + \sum_{\tau} \sum_{K'} \sum_{\lambda\lambda' \neq K'} \{ (R^{KK'}_{\lambda\lambda'}(\tau))^{2} +$$

$$+ (R^{K\tilde{\tilde{K}}'}_{\lambda\lambda'}(\tau))^{2} \delta_{K,1/2} \delta_{K',1/2} \} = 1 . \tag{23}$$

Here and below the terms arising from the coupling of quasiparticles to bosons $\bar{A}^{(\tau)}_{\lambda\lambda'}$ are not written explicitly but it is assumed instead that summation over $(\lambda\lambda')$ includes the states with $K = 1/2$.

Employing relations (21) and (22) the following expressions for a number of matrix elements are obtained

$$<\phi^{+}_{K} H_{sqp} \phi_{K}> \cong N^{2}_{K} E_{K} + \sum_{\tau} \sum_{K'} \sum_{\lambda\lambda' \neq K} (E_{\lambda} + E_{\lambda'} + E_{K'}) \times \tag{24}$$

$$\times [(R^{KK'}_{\lambda\lambda'}(\tau))^{2} + \delta_{K,1/2} \delta_{K',1/2} (R^{K\tilde{\tilde{K}}'}_{\lambda\lambda'}(\tau))^{2}]$$

$$<\phi_K^+ F_B^{(\tau)} F_A^{(\tau)} \phi_K> \cong 2\tau N_K \sum_{K'} M_{KK'} [j_{KK'}^{(\tau)} \Phi_{KK'}^{(\tau)} -$$

$$- \delta_{K,1/2} \delta_{K',1/2} j_{K\tilde{\tilde{K}}'}^{(\tau)} \Phi_{K\tilde{\tilde{K}}'}^{(\tau)}] , \qquad (25)$$

where

$$\Phi_{KK'}^{(\tau)} \equiv \sum_{\lambda\lambda' \neq K'} R_{\lambda\lambda'}^{KK'}(\tau) \, j_{\lambda\lambda'}^{(\tau)} L_{\lambda\lambda'}$$

$$\Phi_{K\tilde{\tilde{K}}'}^{(\tau)} \equiv \sum_{\lambda\lambda' \neq K'} R_{\lambda\lambda'}^{K\tilde{\tilde{K}}'}(\tau) \, j_{\lambda\lambda'}^{(\tau)} L_{\lambda\lambda'} .$$

$$<\phi_K^+ (F_A^{(\tau)})^2 \phi_K > \cong 2\tau \sum_{K'} [\Phi_{KK'}^{(\tau)\,2} + \delta_{K,1/2} \delta_{K',1/2} \Phi_{K\tilde{\tilde{K}}'}^{(\tau)\,2}] + \qquad (26)$$

+ (terms not including the variational amplitudes).

Introducing the Lagrange multiplier W_K to ensure the requirement (23) we derive the variational equations for the amplitudes N_K and $R_{\lambda\lambda'}^{KK'}(\tau)$ x/

---

x/ The contribution from terms $(F_B^{(\tau)})^2$ in H_j is not taken into account in eqs. (27). Approximately the same energy shift of all one-quasi-particle levels results from these terms which, however, are assumed to be included in the average field.

$$N_K (E_K - W_K) + \frac{1}{2\mathcal{G}} \frac{1}{2} \sum_{K',\tau} M_{K,K'} [j^{(\tau)}_{KK'} \Phi^{(\tau)}_{KK'} -$$

$$-\delta_{K,1/2} \delta_{K',1/2} j^{(\tau)}_{K\tilde{\tilde{K}}'} \Phi^{(\tau)}_{K\tilde{\tilde{K}}'}] = 0$$

$$R^{KK'}_{\lambda\lambda'}(\tau) [E_\lambda + E_{\lambda'} + E_{K'} - W_K] + \frac{1}{2\mathcal{G}} \frac{1}{2} j^{(\tau)}_{\lambda\lambda'} L_{\lambda\lambda'} [\Phi^{(\tau)}_{KK'} +$$

$$+ N_K j^{(\tau)}_{KK'} M_{KK'}] = 0 \tag{27}$$

$$\{ R^{K\tilde{\tilde{K}}'}_{\lambda\lambda'}(\tau) [E_\lambda + E_{\lambda'} + E_{K'} - W_K] + \frac{1}{2\mathcal{G}} \frac{1}{2} j^{(\tau)}_{\lambda\lambda'} L_{\lambda\lambda'} \times$$

$$\times [\Phi^{(\tau)}_{K\tilde{\tilde{K}}'} - N_K j^{(\tau)}_{K\tilde{\tilde{K}}'} M_{KK'}] \} \delta_{K,1/2} \delta_{K',1/2} = 0 .$$

From (27) the equation for the energy shift $E_K - W_K$ of one-particle levels associated with the three-quasi-particle admixtures and the expressions for the amplitudes $R^{KK'}_{\lambda\lambda'}(\tau)$ can be derived

$$E_K - W_K = \frac{1}{2\mathcal{G}} \sum_{K'} M^2_{KK'} (j^2_{KK'} + \delta_{K,1/2} \delta_{K',1/2} j^2_{K\tilde{\tilde{K}}'}) \times \frac{\mathcal{G}_{KK'}}{2\mathcal{G} + \mathcal{G}_{KK'}} \tag{28}$$

$$R^{KK'}_{\lambda\lambda'}(\tau) = -\frac{1}{2} N_K \frac{j^{(\tau)}_{KK'} M_{KK'}}{2\mathcal{J} + \mathcal{J}_{KK'}} \quad \frac{j^{(\tau)}_{\lambda\lambda'} \quad L_{\lambda\lambda'}}{E_\lambda + E_{\lambda'} + E_{K'} - W_K} \tag{29}$$

$$R^{K\tilde{\tilde{K}}'}_{\lambda\lambda'}(\tau) = -\frac{1}{2} N_K \frac{j^{(\tau)}_{K\tilde{\tilde{K}}'} M_{KK'}}{2\mathcal{J} + \mathcal{J}_{KK'}} \quad \frac{j^{(\tau)}_{\lambda\lambda'} \quad L_{\lambda\lambda'}}{E_\lambda + E_{\lambda'} + E_{K'} - W_K} \tag{30}$$

the function $\mathcal{J}_{KK'}$ being defined by

$$\mathcal{J}_{KK'} \equiv \frac{1}{2} \sum_{\lambda\lambda' \neq K'} \frac{j^2_{\lambda\lambda'} \quad L^2_{\lambda\lambda'}}{E_\lambda + E_{\lambda'} + E_{K'} - W_K} . \tag{31}$$

Now the eigenvalue for the intrinsic Hamiltonian (19) reads

$$\mathcal{E}_K = U_0 + W_K . \tag{32}$$

4. Renormalization of Single-Particle Matrix Elements

The wave functions (20) can be used for the calculation of the matrix elements of operators $j_+ + \tau j_-$ entering the Coriolis coupling term (4c). By means of eqs. (21), (22), (29) and (30) one obtains

$$< \phi^+_K \; F^{(\tau)} \phi_{K'} > = j^{(\tau)}_{KK'} M_{KK'} R_j (K, K') \tag{33}$$

$$< \phi^{+}_{\tilde{\tilde{K}}} F^{(\tau)} \phi_{K'} > = j^{(\tau)}_{K\tilde{\tilde{K}}'} M_{KK'} R_{j} (K,K') \delta_{K,1/2} \delta_{K',1/2} \qquad (34)$$

where the matrix elements of the polarization matrix R_j are defined by

$$R_j (K,K') = N_K N_{K'} \{ 1 - \frac{\mathcal{J}_{KK'}}{2\mathcal{J} + \mathcal{J}_{KK'}} - \frac{\mathcal{J}_{K'K}}{2\mathcal{J} + \mathcal{J}_{K'K}} \} \qquad (35)$$

$\mathcal{J}_{KK'}$ and $\mathcal{J}_{K'K}$ being given by (31).

Thus the centrifugal interactions lead to the reduction of the one-particle matrix elements $j^{(\tau)}_{KK'}$ and the decoupling parameters $j^{(\tau)}_{K\tilde{\tilde{K}}}$ since $R_j (K,K')<1$ for the states with $W_K<E_\lambda+E_{\lambda'}+E_{K'}$ (the lowest solution of eq. (28), which corresponds to the state with small three-quasiparticle admixtures).

In the quasiclassical approximation when the following conditions are fulfilled

$$\begin{aligned} & N^2_K \approx N^2_{K'} \approx 1 \\ & E_{K'} - W_K \ll E_\lambda + E_{\lambda'} \\ & E_K - W_{K'} \ll E_\lambda + E_{\lambda'} \end{aligned} \qquad (36)$$

and the blocking effect is neglected we obtain for the function $\mathcal{J}_{KK'}$

$$\mathcal{J}_{KK'} \approx \mathcal{J}_{K'K} \approx \frac{1}{2} \sum_{\lambda\lambda'} \frac{j^2_{\lambda\lambda'} \quad L^2_{\lambda\lambda'}}{E_\lambda + E_{\lambda'}} \equiv \mathcal{J}_o \quad . \tag{37}$$

Apparently $\mathcal{J}_o$ represents the contribution of particles interacting through the centrifugal force to the effective moment of inertia (the sum in (37) runs over the states contributing to the intrinsic angular momentum $\vec{j}$). The quasiclassical expression for R_j now can be written

$$R_j\,(K, K') \approx 1 - \frac{2\mathcal{J}_o}{2\mathcal{J} + \mathcal{J}_c} \equiv R_j \quad . \tag{38}$$

In that event all the matrix elements $<j_\pm>$ (including the decoupling parameters) are reduced by the same factor R_j .

5. The Coriolis Force

Using the representation (13) one can write the Coriolis term (4c) in the form

$$H_c = -\frac{1}{2\mathcal{J}} \frac{1}{2} \sum_{\tau=\pm 1} \tau \,(I_+ + \tau I_-)\, F^{(\tau)} \quad . \tag{39}$$

Now for diagonal matrix element of (39) in the state (9) it is easy to obtain

$$<IM|H_c|IM> = \sum_{KK'} C^I_K\, C^I_{K'} <IMK|H_c|IMK'> \quad , \tag{40}$$

where

$$\langle IMK | H_c | IMK' \rangle = -\frac{1}{2\mathcal{J}} \frac{1}{2} \sum_{r=\pm 1} r \{ \langle \phi^+_K F^{(r)} \phi_{K'} \rangle \times$$

$$[\sqrt{(I-K)(I+K+1)}\, \delta_{K',K+1} + r\sqrt{(I+K)(I-K+1)}\, \delta_{K',K-1}] + \tag{40a}$$

$$+(-1)^{I+\ell+1/2}\, \delta_{K,1/2}\, \delta_{K',1/2}\, (I+1/2) \langle \phi^+_{\tilde{K}} F^{(r)} \phi_{K'} \rangle \} \, .$$

Having in mind the results (33) and (34) we came to the conclusion that the centrifugal interactions weaken the Coriolis force. In the quasiclassical approximation the polarization effects can be accounted for by the use of the renormalized effective moment of inertia for the Coriolis force

$$\frac{1}{2\mathcal{J}_{eff}} = \frac{1}{2\mathcal{J}} R_j \, . \tag{41}$$

Assuming $\mathcal{J}_o \ll \mathcal{J}$ we obtain

$$\mathcal{J}_{eff} \cong \mathcal{J} + \mathcal{J}_o \, . \tag{42}$$

The latter result was obtained by A. Bohr /11/.

6. Spin-Polarization Effects

Additional renormalization of the matrix elements $< j_{\pm} >$ may arise from the spin-spin interaction of particles /8/. It occured that the most important contribution to the polarization matrix comes from the monopole part (in angular variables) of this interaction which can be taken to have the form of the model Hamiltonian ($\kappa_{nn} = \kappa_{pp} \equiv \kappa > 0$, $\kappa_{np} = 0$)

$$H_{\sigma} = \frac{1}{4} \kappa \sum_{\substack{\tau = \pm 1 \\ (n, p)}} \tau \left(T^{(\tau)} \right)^2 , \tag{43}$$

with the notation

$$T^{(\tau)} = \sum_{\nu\nu'} < \nu | \sigma_{+} + \tau \sigma_{-} | \nu' > a_{\nu}^{+} a_{\nu'} , \tag{43a}$$

where $\sigma_{\pm} = \sigma_x \pm i \sigma_y$ are the Pauli spin matrices, the matrix elements $< \nu | \sigma_{+} + \tau \sigma_{-} | \nu' > \equiv \sigma^{(\tau)}_{\nu\nu'}$, are assumed to have the symmetry properties (14), κ is the strength parameter.

Employing the wave functions (20) one can diagonalize the total intrinsic Hamiltonian $\mathcal{H}_{intr} + H_{\sigma}$. Omitting the details of calculations we give here the final results. The secular equation for W_K now takes the form

$$E_{K} - W_{K} = \sum_{K'} M^2_{KK'} \{ \frac{1}{2\mathcal{J}} [j^2_{KK'} + \delta_{K,1/2} \delta_{K',1/2} j^2_{K\tilde{K}'}] \times$$

$$\times \frac{\mathcal{J}^{(\kappa)}_{KK'}}{2\mathcal{J} + \mathcal{J}^{(\kappa)}_{KK'}} + \kappa [\sigma^2_{KK'} + \delta_{K,1/2} \delta_{K',1/2} \sigma^2_{K\tilde{\tilde{K}}'}] \times \tag{44}$$

$$\times \frac{\kappa S^{(j)}_{KK'}}{1 + \kappa S^{(j)}_{KK'}} ,$$

where

$$\mathcal{J}^{(\kappa)}_{KK'} \equiv \mathcal{J}_{KK'} - \frac{\kappa X^2_{KK'}}{1 + \kappa S_{KK'}} . \tag{44a}$$

$$S^{(j)}_{KK'} \equiv S_{KK'} - \frac{X^2_{KK'}}{2\mathcal{J} + \mathcal{J}_{KK'}} \tag{44b}$$

$$X_{KK'} \equiv \frac{1}{2} \sum_{\lambda\lambda' \neq K'} \frac{\sigma_{\lambda\lambda'} \, j_{\lambda\lambda'} \, L^2_{\lambda\lambda'}}{E_{\lambda} + E_{\lambda'} + E_{K'} - W_{K}} \tag{44c}$$

$$S_{KK'} \equiv \frac{1}{2} \sum_{\lambda\lambda' \neq K'} \frac{\sigma^2_{\lambda\lambda'} \, L^2_{\lambda\lambda'}}{E_{\lambda} + E_{\lambda'} + E_{K'} - W_{K}} . \tag{44d}$$

The summation in $\mathcal{G}_{KK'}$ and $X_{KK'}$ extends over the states involved in the centrifugal coupling, while no such restrictions are imposed on the summation in $S_{KK'}$. The mixing of the centrifugal and spin-spin interactions leads to the mutual renormalization of the corresponding polarization sums $S_{KK'}$ and $\mathcal{G}_{KK'}$ (see, eqs. (44a) and (44b)). The matrix element of the polarization matrix R^{σ}_{J} is now given by

$$R^{\sigma}_{j}(K,K') = N_K N_{K'} \{ 1 - \frac{\mathcal{G}^{(\kappa)}_{KK'}}{2\mathcal{G} + \mathcal{G}^{(\kappa)}_{KK'}} - \frac{\mathcal{G}^{(\kappa)}_{K'K}}{2\mathcal{G} + \mathcal{G}^{(\kappa)}_{K'K}} -$$

$$- \frac{\sigma_{KK'}}{i_{KK'}} [\frac{2\mathcal{G}}{2\mathcal{G} + \mathcal{G}^{(\kappa)}_{KK'}} \frac{\kappa X_{KK'}}{1 + \kappa S_{KK'}} + \tag{45}$$

$$+ \frac{2\mathcal{G}}{2\mathcal{G} + \mathcal{G}^{(\kappa)}_{K'K}} \frac{\kappa X_{K'K}}{1 + \kappa S_{K'K}}] \} .$$

The quasiclassical expression for R^{σ}_{J} (see eq. (36)) reads

$$R^{\sigma}_{j} \cong 1 - \frac{2\mathcal{G}^{(\kappa)}_{c}}{2\mathcal{G} + \mathcal{G}^{(\kappa)}_{c}} - \left(\overline{\frac{\sigma_{KK'}}{j_{KK'}}} \right) \frac{2\mathcal{G}}{2\mathcal{G} + \mathcal{G}^{(\kappa)}_{c}} \frac{2\kappa X_c}{1 + \kappa S_c} \tag{46}$$

where $\mathcal{g}_c^{(\kappa)}$, X_o and S_c denote the corresponding quasiclassical limits of (44a) - (44d). For the matrix elements between the states of a single j -shell the following estimate of the ratio $\overline{\left(\frac{\sigma_{KK'}}{j_{KK'}}\right)}$ can be obtained

$$\overline{\left(\frac{\sigma_{KK'}}{j_{KK'}}\right)} \cong \frac{1}{j} \; . \tag{47}$$

Hence

$$X_o \cong \frac{1}{j}\, \mathcal{g}_o \; . \tag{48}$$

The magnitude of κ and S_o was estimated from the calculation magnetic moments /8/

$$\kappa \approx 300 \text{ Kev}$$
$$1 + \kappa S_c \approx 2 \; . \tag{49}$$

With the estimates (47) - (49) it can be shown that the renormalization of $\mathcal{g}_o$ due to the spin-spin force is small $(\mathcal{g}_c^{(\kappa)} \approx \mathcal{g}_o)$ and the spin polarization gives the contribution to R_j^σ

$$- \frac{2\mathcal{g}_c}{2\mathcal{g} + \mathcal{g}_c} \, \frac{\kappa \mathcal{g}}{j^2} \tag{50}$$

which is approximately 20% of that from the centrifugal polarization in case of $i_{13/2}$ shell (with $\mathcal{J} \approx 30$ MeV^{-1}).

7. Equation of Motion for C^I_K

The equations of motion for the Coriolis mixing amplitudes C^I_K can be obtained by minimizing the total energy $\mathcal{E}(I)$ for each spin value

$$\mathcal{E}(I) \equiv \langle IM | \mathcal{H}_{intr} + H_\sigma + H^0_{rot} + H_c | IM \rangle = \tag{51}$$

$$= U_0 + \Sigma (C^I_K)^2 W_K + \frac{1}{2\mathcal{J}} g(I) ,$$

with $g(I)$ given by

$$g(I) = I(I+1) - \sum_K (C^I_K)^2 K^2 + (-1)^{I+1/2}(I+1/2)\, a(I) . \tag{51a}$$

The generalized notion of the decoupling parameter $a(I)$ is introduced here

$$(-1)^{I+1/2} (I+1/2)\, a(I) \equiv 2\mathcal{J} \langle IM | H_c | IM \rangle . \tag{51b}$$

We will take the pairing force into account in the static limit in which the energy gap Δ (and therefore, the one-quasi-particle energy E_ν) is spin-independent. Hence, we neglect the Coriolis antipairing effect, arising from the interaction between pairing and rotation, which gives in the lowest order in $I(I+1)$ the energy contribution $B_\Delta I^2 (I+1)^2$ $(B_\Delta < 1)$ /12/. As a rule in odd-mass nuclei this correction is small /13/ as compared with that from the Coriolis force, which can be characterized by the value of $a(I)$ ($|a(I)| \approx 10$ for the rotational bands based on $i_{13/2}$ shell states /6/).

Introducing the Lagrange multiplier ω_K to ensure the normalization condition (10) we obtain the variational equations for C^I_K in the form

$$C^I_K \{ U_0 + W_K + \frac{1}{2\mathcal{J}} [I(I+1) - K^2 +$$

$$+\delta_{K,1/2} (-1)^{I+1/2} (I+1/2) a_{sp} R^\sigma_j (K,K)] - \omega(I) \} + \tag{52}$$

$$+ \sum_{K' \neq K} C^I_{K'} < IMK | H_c | IMK' > = 0 ,$$

where a_{sp} denotes the unperturbed single particle decoupling parameter

$$a_{sp} \equiv -(-1)^\ell \; j^{(+)}_{\tilde{1}/2,\,1/2} \tag{52a}$$

and

$$< IMK \mid H_c \mid IMK' = K \pm 1 > = - \frac{1}{2\mathcal{J}} \sqrt{(I \mp K)(I \pm K + 1)} \times$$

$$\delta_{K',K\pm 1} \quad i^{(+)}_{K,K'} \quad R^{\sigma}_{j}(K,K') \, M_{KK'} \, . \tag{52b}$$

Equations (52), (44) and (10) now define completely the spectra of quasi-particle and rotational excitations of a given parity in odd-mass nuclei.

8. Discussion

In the proceeding sections it was shown that the centrifugal polarization (i.e. the renormalization of matrix element $< j_{\pm} >$) essentially depends on the value of $\mathcal{J}_{KK'}$ (or the value of $\mathcal{J}_c$ in quasiclassical limit), which is particularly large if the states originating from a strongly degenerated spherical subshell appear near the Fermi surface. It is known that rotational bands upon such states are strongly distorted by the Coriolis force[3,4,6]. Therefore, the correlation between the centrifugal polarization and the Coriolis coupling of single-particle states is found, i.e. the polarization effects are noticeable in rotational bands strongly distorted by the Coriolis force. This is just the main difference between the centrifugal and spin polarization which leads to the stable renormalization of the spin gyromagnetic ratio g_s in all states[7,8]. We hope, that both the polarization effects from the centrifugal interactions and the non-adiabatic effects from the Coriolis coupling will essentially depend upon a comparatively small number of single particle levels near the Fermi surface.

The effective moment of inertia $\mathcal{J}$ is considered here as a parameter the magnitude of which must be close to that in neighbouring even-even nuclei (some difference however may arise from the difference of the energy gap Δ in odd-mass and even-even nuclei, e.g).

We do not touch upon the dynamics of pairing interaction and the effective moment of inertia. The dependence of Δ and $\mathcal{J}$ on the frequency of rotation will be considered later on when solving the dynamical equations given in[6].

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Пятов Н.И., Черней М.И., Базнат М.И. E4-5468

Поляризационные эффекты во вращательном движении нечётных атомных ядер. I. Теория

Рассмотрены поляризационные эффекты во вращательном движении нечётных атомных ядер, возникающие от центробежного и спинового взаимодействий нечётного нуклона с частицами чётно-чётного остова. Эти взаимодействия приводят к перенормировке одночастичных матричных элементов $< j_{\pm} >$ и, следовательно, взаимодействия Кориолиса. В квазиклассическом приближении поляризационные эффекты сводятся к перенормировке (увеличению) эффективного момента инерции при взаимодействии Кориолиса. Показано, что эти эффекты наиболее существенны для вращательных полос на одночастичных состояниях, исходящих из сильно вырожденных сферических подоболочек (т.е. в тех случаях, когда сильны неадиабатические эффекты от взаимодействия Кориолиса).

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Pyatov N.I., Chernej M.I., Baznat M.I. E4-5468

Polarization Effects in the Rotational Motion of Odd-Mass Nuclei. I. Theory

Polarization effects in the rotational motion of odd-mass nuclei associated with centrifugal and spin-spin interactions of an odd nucleon with the particles of an even-even core are considered. These interactions lead to a renormalization of the single-particle matrix elements $< j_{\pm} >$ and, consequently, the Coriolis force. In the quasiclassical approximation the polarization effects are reduced to a renormalization (increase) of the effective moment of inertia in the Coriolis coupling term. It is shown that these effects are most essential for rotational bands based on the single-particle levels originating from strongly degenerated spherical subshells (i.e. in the cases when the non-adiabatic effects due to the Coriolis force are strong).

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