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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

A GRAPHICAL METHOD FOR THE DETERMINATION OF ALL RESONANCE PARAMETERS IN NUCLEAR REACTIONS

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Береги П.

Графический метод определения всех резонансных параметров в ядерных реакциях

Представляется метод для графического определения всех параметров, появляющихся в обобщенной формуле Брейта-Вигнера для изолированного ядерного резонанса. Численные примеры даются для трех различных резонансов несимметричной формы в случае непостоянного фона.

Сообщения Объединенного института ядерных исследований

Дубна, 1970

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Beregi P.

A Graphical Method for the Determination of all Resonance Parameters in Nuclear Reactions

A method for the graphical determination of all parameters appearing in the generalized Breit-Wigner approximation for an isolated nuclear resonance is presented. Numerical examples are given for three different kinds of resonances of asymmetric form in the case of non-constant background.

Communications of the Joint Institute for Nuclear Research.

Dubna, 1970

E4 - 5456

P. Beregi

A GRAPHICAL METHOD FOR THE DETERMINATION OF ALL RESONANCE PARAMETERS IN NUCLEAR REACTIONS

The proposed method is based on the resonance circle method described in /1/. We follow McVoy's discussion but we repeat only those statements which are necessary for us.

In /1/ it is discussed the necessity of using for many nuclear reactions, e.g. for isobaric analogue resonances, not the Breit-Wigner formula /2/:

$$S_{ba}(E) = e^{i(\phi_a+\phi_b)} \left[\delta_{ba} - i \frac{\Gamma_b^{1/2} \; \Gamma_a^{1/2}}{E-E_0+\frac{i}{2}\Gamma} \right] , \tag{1}$$

to which in most cases the data of the isolated resonances are fitted, but another approximation. If we consider an isolated resonance far from threshold then near the pole it is reasonable to approximate S as

$$S(E) = B - i \frac{T}{E-E_0+\frac{i}{2}\,\Gamma} . \tag{2}$$

Assuming that $B(E)$ is identically unitary across the resonance one can obtain the following expression for the matrix elements of S :

$$S_{ba}(E) = B_{ba} - i \frac{e^{i\phi_b}\,\Gamma_b^{1/2} \quad \Gamma_a^{1/2} \quad e^{i\phi_a}}{E-E_0+\frac{i}{2}\,\Gamma} . \tag{3}$$

Formula (3) is the generalized Breit-Wigner approximation in which it is assumed that the total and partial widths do not depend on E .

In(3) B can be non-diagonal. If $|B_{ba}| = 1$ then the background is elastic, for the case $|B_{ba}| < 1$ the background is non-elastic.

If we rewrite (3) in the form:

$$S_{ba}(E) = [B_{ba} - e^{i(\phi_b+\phi_a)} \rho_{ba}] + e^{i(\phi_b+\phi_a)} \rho_{ba} \frac{E - E_0 - \frac{i}{2}\Gamma}{E - E_0 + \frac{i}{2}\Gamma},$$

where $\rho_{ba} = \frac{(\Gamma_b \Gamma_a)}{\Gamma}$ and assume that the energy dependence of $S_{ba}(E)$ in the neighbourhood of the resonance is only due to the resonance term (the second term in (4)) then it is easy to see that the trajectory of each $S_{ba}(E)$ is simply a circle as the energy passes over the resonance. This fact can be used in the problem of fitting the data with eq. (3).

Taking into account the above assumption we can rewrite (3) in the following forms /1/:

$$S_{ba}(E) = e^{2i\bar{\phi}_{ba}} [|B_{ba}| - ie^{i\alpha_{ba}} \frac{\Gamma_b^{1/2} \Gamma_a^{1/2}}{E - E_0 + \frac{i}{2}\Gamma}] \tag{5a}$$

$$= e^{2i\bar{\phi}_{ba}} [|B_{ba}| - ie^{i\alpha_{ba}} \frac{\rho_{ba}\Gamma}{E - E_0 + \frac{i}{2}\Gamma}], \tag{5b}$$

where $\alpha_{ba} = \phi_b + \phi_a - 2\bar{\phi}_{ba}$.

In Fig. 1a it is presented a resonance trajectory, where the "rapid" counterclockwise resonance circle interrupts the "slow" motion of the inelastic background term. This circle would have started and stopped at the point B_{ba} if the background had been completely constant across the resonance. From (4) and (5) it can be seen that the centre of the circle is at $B_{ba} - \rho_{ba} e^{i(\phi_b+\phi_a)} = e^{2i\bar{\phi}_{ba}}[|B_{ba}| - \rho_{ba} e^{i\alpha_{ba}}]$, the radius of the circle is ρ_{ba} .

Formula (5b) contains 6 parameters: $|B_{ba}|$, ρ_{ba}, $\bar{\phi}_{ba}$, α_{ba}, E_0 and Γ. It is similar to the simple Breit-Wigner expression (1) but it contains two extra paremeters

$|B_{ba}|$ and α_{ba} to allow the background to be inelastic and non-diagonal and the resonant term to have a different phase from that of the background. From Fig. 1a one can easily obtain $|B_{ba}|$, ρ_{ba}, $\bar{\phi}_{ba}$ and α_{ba}. McVoy has stated for that case that "these parameters being known, only E_0 and Γ remain to be found by fitting (5) to the data".

It is easy to see that in the case when the energy dependence of S_{ba} is known, one can obtain E_0 and Γ by graphical method without fitting, too. To this end one has to put in formula (5b) for the energy $E = E_0 - \frac{\Gamma}{2}$, $E = E_0$ and $E = E_0 + \frac{\Gamma}{2}$, respectively:

$$S^{(-)}_{ba} = S_{ba}(E_0 - \frac{\Gamma}{2}) = |B_{ba}|\, e^{2i\bar{\phi}_{ba}} - (1-i)\rho_{ba}\, e^{i(2\bar{\phi}_{ba}+\alpha_{ba})}, \tag{6a}$$

$$S^{(0)}_{ba} = S_{ba}(E_0) = |B_{ba}|\, e^{2i\bar{\phi}_{ba}} - 2\rho_{ba}\, e^{i(2\bar{\phi}_{ba}+\alpha_{ba})}, \tag{6b}$$

$$S^{(+)}_{ba} = S_{ba}(E_0 + \frac{\Gamma}{2}) = |B_{ba}|\, e^{2i\bar{\phi}_{ba}} - (1+i)\rho_{ba}\, e^{i(2\bar{\phi}_{ba}+\alpha_{ba})}. \tag{6c}$$

In Fig. 1b a non-closed resonance circle is to be seen. From (6a-c) we can conclude that the values $S^{(-)}_{ba}$, $S^{(0)}_{ba}$ and $S^{(+)}_{ba}$ correspond to $\delta = \frac{\pi}{4}$, $\delta = \frac{\pi}{2}$ and $\delta = \frac{3\pi}{4}$, respectively. After determining the real and imaginary parts of $S^{(-)}_{ba}$, $S^{(0)}_{ba}$, and $S^{(+)}_{ba}$ (either by means of formulae (6a-c) if the parameters $|B_{ba}|$, ρ_{ba}, $\bar{\phi}_{ba}$ and α_{ba} are known or graphically if we are interested only in E_0 and Γ) one can find the energy values $E_- = E_0 - \frac{\Gamma}{2}$ and $E_+ = E_0 + \frac{\Gamma}{2}$ on the curves $\mathrm{Re}\, S(E)$ and $\mathrm{Im}\, S(E)$. Of course, in the ideal case the equation $E_+ - E_0 = E_0 - E_-$ must be fulfilled.

In Figs. 2a, 3a, 4a we present some resonance circle for partial wave matrix elements with $L = 0$ obtained for resonances discussed in /3/. Outside and inside the circles we indicate some energy values in given units. The corresponding resonances in the cross-sections

$$\frac{d\sigma^{0}_{ba}(E_a)}{d\Omega} = \frac{1}{4E_a} \mid \delta_{ba} - S_{ba}(E_a) \mid^2 , \tag{7}$$

(where E_a is the energy of the bombarding particle) are shown in Figs. 2b, 3b, 4b. In Fig. 2a a resonance circle for a potential resonance is shown. Fig. 3a presents a compound resonance in a three-body system. For both cases only one channel is open. In Fig. 4a it is presented a resonance circle for a quasi-compound resonance appearing in an elastic scattering for case of three open channels.

The determination of E_0 and Γ is illustrated for the potential resonance. Fig. 5 shows the energy dependence of $\mathrm{Re}\,S$ and $\mathrm{Im}\,S$ in the neighbourhood of E_0 and we indicate the energy values E_- , E_0 and E_+ obtained from S_- , S_0 and S_+ of Fig. 2a.

In Table one can find the obtained resonance energies (E_0) and resonance widths (Γ) for resonance circles discussed in our paper. The accuracy of the method depends on the gap in the resonance circle which is due to non-constancy of B_{ba} . For the potential resonance we can compare our result with the "exact" one because we know the situation of the pole of the S -matrix in the E plane, namely $E_0 = 0.102145$ and $\Gamma = 6,954 \times 10^{-3}$. One can see that for a broad gap (which is slightly more than $\pi/3$) the resonance energy is obtained with very high accuracy and the error in the determination of Γ is only 3,5%. We think that the errors for E_0 and Γ of the quasi-compound resonance are of the same order and the resonance energy and width of the compound resonance where we have a narrow gap, are obtained with high accuracy.

Finally, we want to discuss the possibility of improving and using our method. We can check our result by putting $E_n = E_0 + \frac{n\Gamma}{2}$

(where n is not necessarily an integer number) and determining the value $\frac{n\Gamma}{2}$ e.g. from $S^{(0)}_{ba}$ and

$$S^{(n)}_{ba} = S_{ba}\ (E_0 + \frac{n\Gamma}{2}) = |B_{ba}|\ e^{2i\bar{\phi}_{ba}} - \frac{2-2ni}{n^2+1}\rho_{ba}\ e^{i(2\bar{\phi}_{ba}+\alpha_{ba})} \quad (7)$$

with the help of the described method. The method can be improved by taking into account the non-constancy of B_{ba} . This is important for resonances sitting in a very steep background.

The proposed method can be used both in the case of evaluation of experimental data when there is performed a phase analysis and also in the case of theoretical calculations when the resonances are obtained not from the Breit-Wigner expression but e.g. from the solution of integral equation for the transition matrices /3,4/. Of course, if the values $S^{(-)}_{ba}$ (and/or $S^{(+)}_{ba}$) and $S^{(0)}_{ba}$ are known it is possible to obtain the values E_0 and Γ from the cross-section $\frac{d\sigma_{ba}}{d\Omega}$ (or in the case of broad resonances from the cross section multiplied by the energy of the incident particle) which is proportional to $|\delta_{ba} - S_{ba}|^2$ but in that case one must be careful because the same value of the cross-section can belong to different energies.

After this work has been performed Dr. Z. Kunszt pointed to the similarity of our method and the method based on the Argand - diagrames and used in elementary particle physics /5/. We think that our discussion may be useful because it shows that even for energy -dependent background we can obtain the resonance energy and resonance width with high accuracy, in most cases the resonance circle is not distorted even for the case of many open channels, when the trajectory of S_{ba} is not required to lie on the circle. (Of course in the case of one open channel the trajectory can not leave the unit circle.) Prof. V.V. Babikov have drawn the author's attention to another (and very elegant) graphical method /6/ which uses the differential cross section for a given angle rather than the resonance circle. The method proposed in /6/ is obtained for the case of constant background and for the case discussed in/6/ one can obtain only 5 parameters of the generalized Breit-Wigner approximation. As we have shown, in the resonance circle method it is easy to determine all of the appearing parameters.

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R e f e r e n c e s

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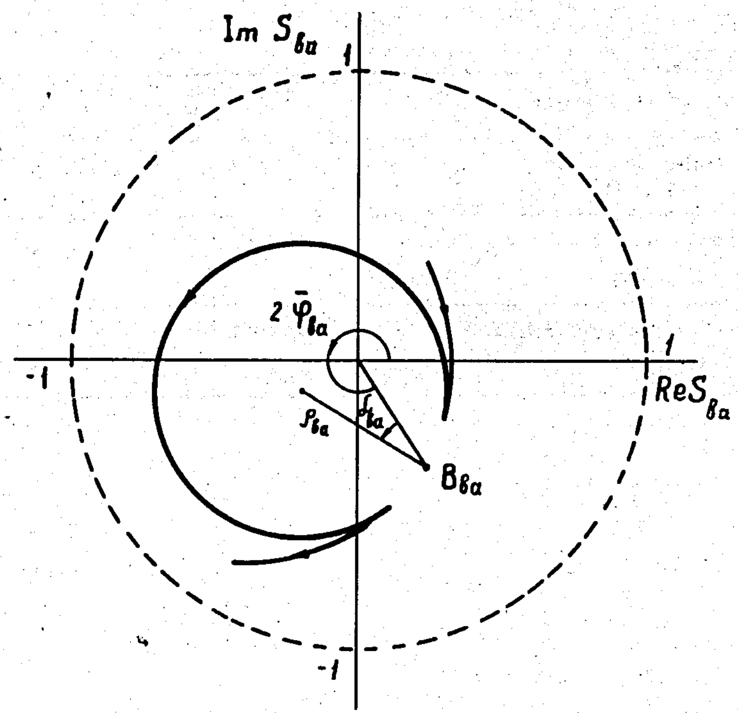
Table

Type of resonance	E_0	Γ
Potential resonance (Fig. 2)	0.1022	7.2×10^{-3}
Compound resonance (Fig. 3)	0.070878	2.0×10^{-5}
Quasi-compound resonance (Fig. 4)	0.5285	1.3×10^{-2}

a



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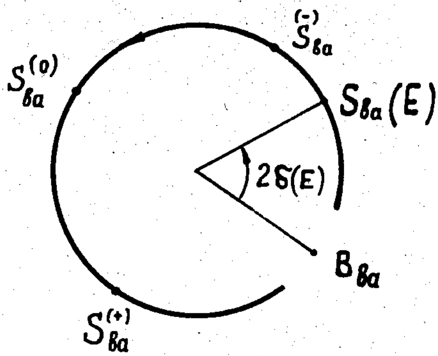


Fig. 1. A typical resonance trajectory (a) and a non-closed resonance circle (b).

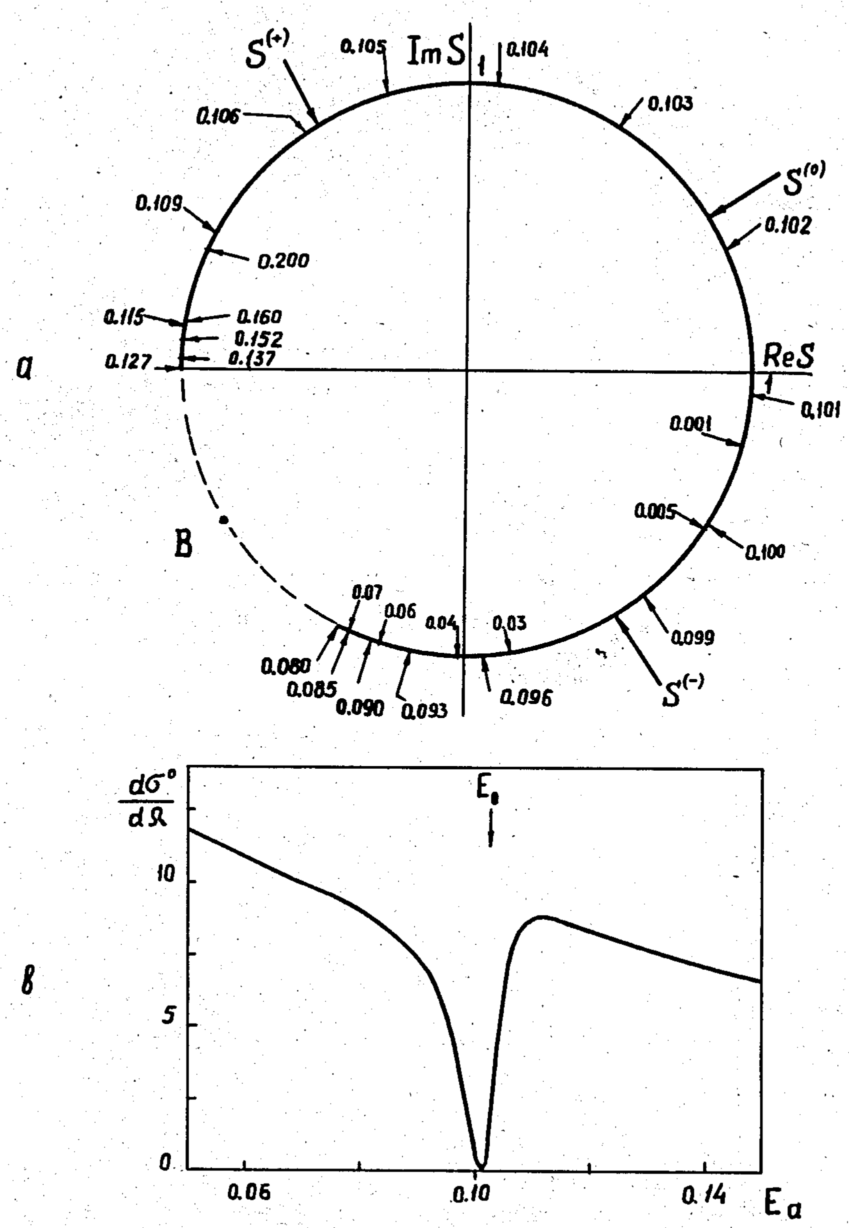
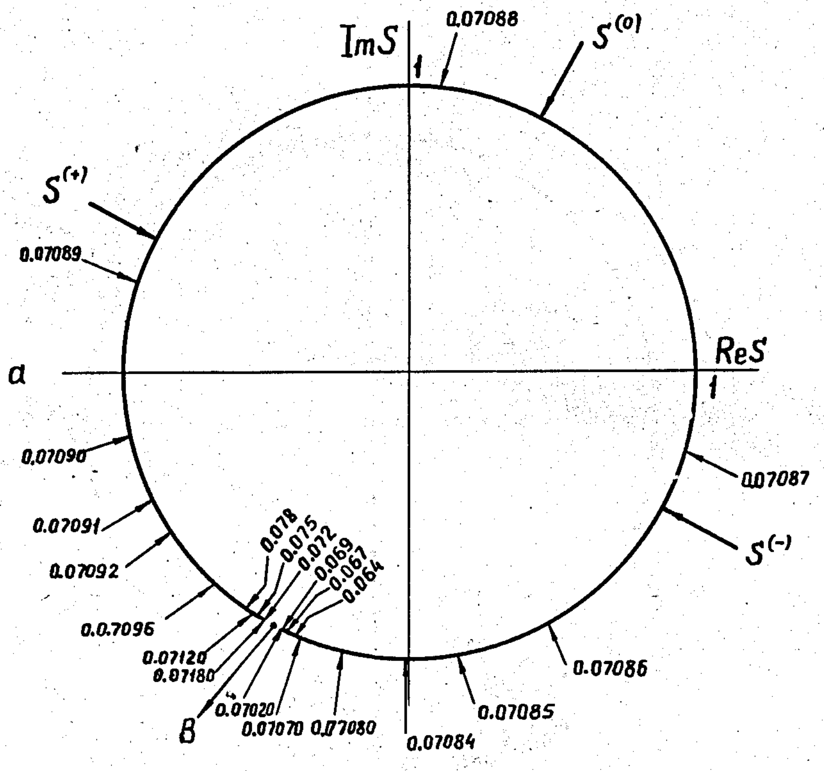


Fig. 2. The resonance trajectory (a) and the cross-section (b) in the neighbourhood of the potential resonance.



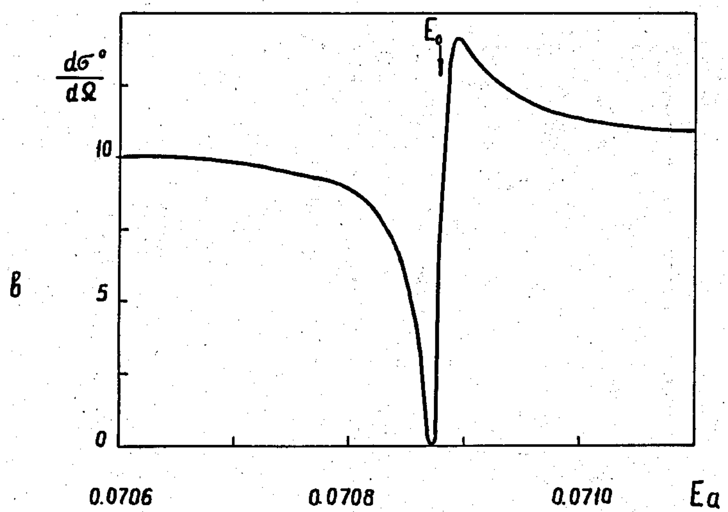


Fig. 3. The resonance trajectory (a) and the cross-section (b) in the neighbourhood of the compound resonance.

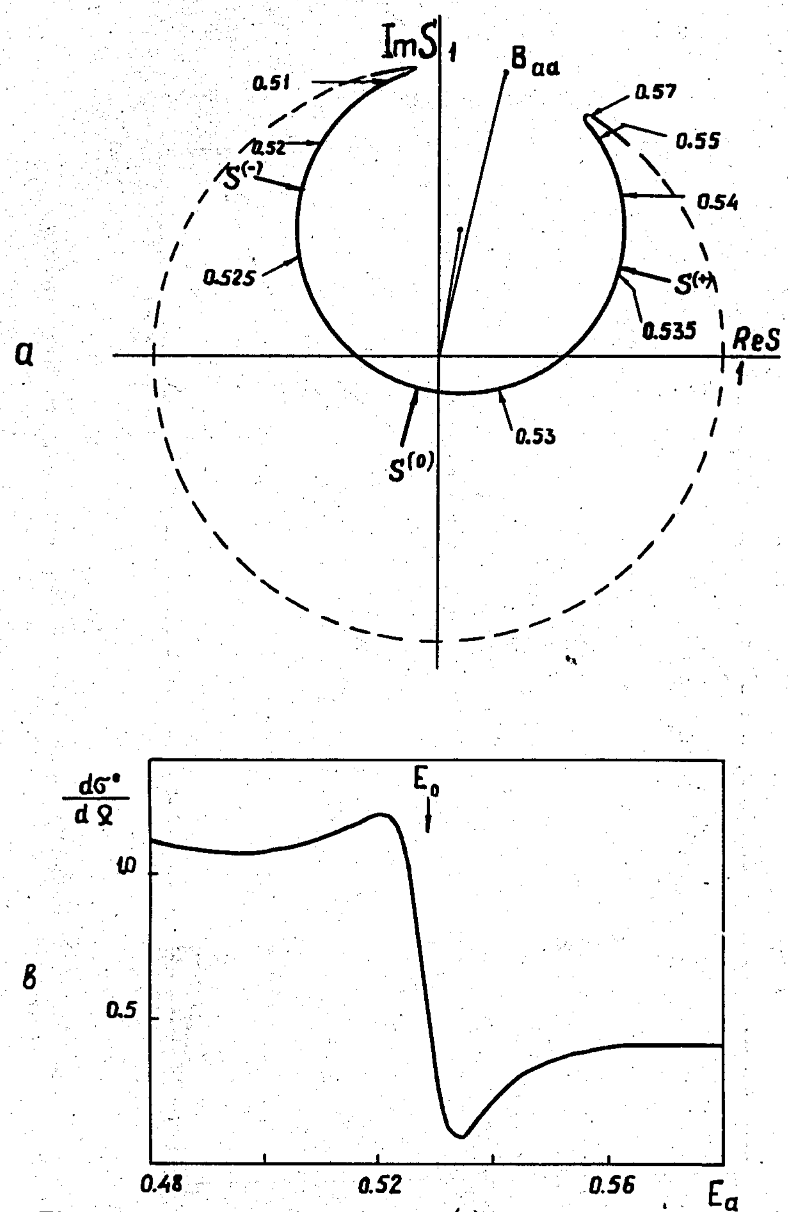


Fig. 4. The resonance trajectory (a) and the cross-section (b) for the elastic scattering in the neighbourhood of the quasi-compound resonance.

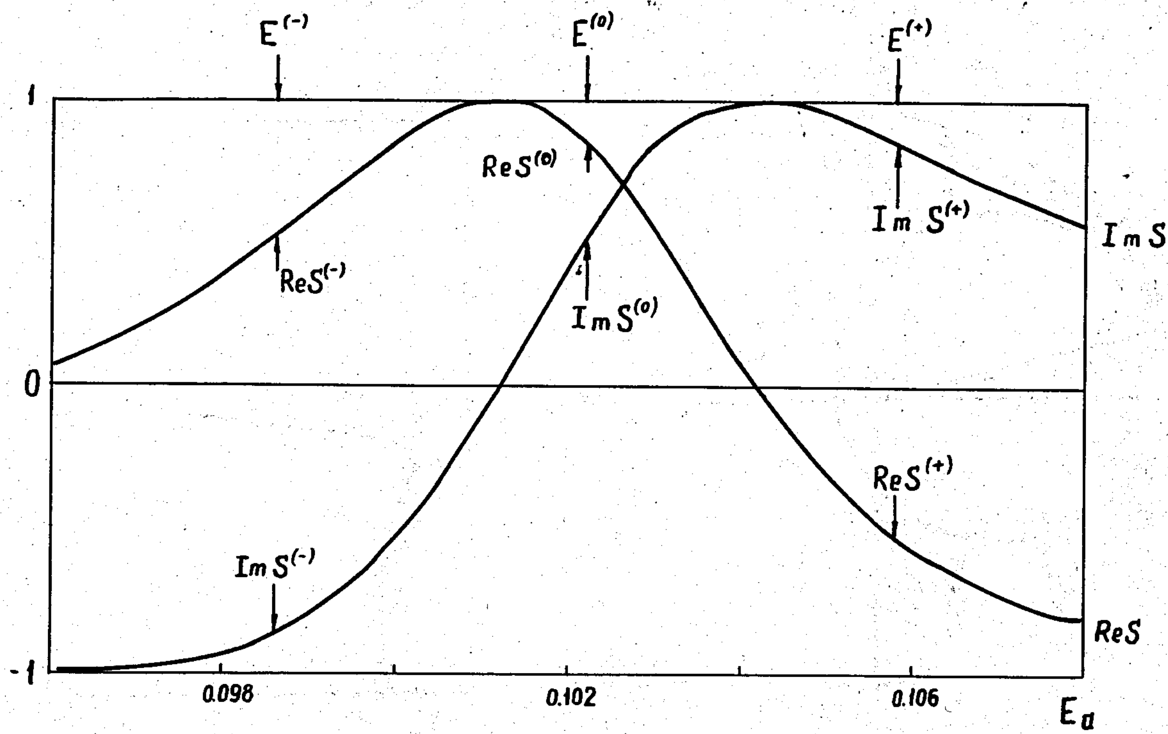


Fig. 5. The energy dependence of ReS and ImS for the potential resonance.