5401.

ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E4 - 5401

ЛАБОРАТОРИЯ ВЫЧИСЛИТЕЛЬНОЙ ТЕХНИКИ
И АВТОМАТИЗАЦИИ

N.A. Toropkov

COMPLETE EXPERIMENT IN HOLOGRAPHY

1970

Торопков Н.А. E4-5401

Полный опыт в голографии

В работе показана возможность полного опыта в голографии. Получены формулы для зависимости $< I(x) >$ от поляризации и других параметров, характеризующих рассеянный и опорный фотоны. Обсуждаются эксперименты, позволяющие регистрировать полный набор параметров, определяющих рассеянный фотон.

Препринт Объединенного института ядерных исследований.
Дубна, 1970

Toropkov N.A. E4-5401

Complete Experiment in Holography

The possibility of complete experiment in holography is shown. The expression for $< I(x) >$ dependence on polarization and other parameters, characterizing the scattered and reference photons, are obtained. The experiments, allowing detection of full set of parameters, defining the scattered photon, are discussed.

Preprint. Joint Institute for Nuclear Research.
Dubna, 1970

E4 - 5401

N.A. Toropkov

COMPLETE EXPERIMENT IN HOLOGRAPHY

Submitted to International Conference on Instrumentation for High Energy Physics, Dubna, 1970.

It is known, that in the classical approach a coherent light wave is described in any plane of an optical tract by a complex amplitude or a real amplitude and phase. In holography it can also be described by a real function plus a known coherent reference wave /1-6/.

However, one should know the intensity, wave vector $\vec{k}$, frequency ω and the Stokes parameters, characterizing a polarization state, to describe a light wave completely. And the quantum mechanical approach is required for the description of interaction process. A brief review of theory of the complete experiment in holography is given below with quantum electrodynamical formalism as far as photon interaction (registration) is connected with the change of the number of particles.

In complete experiment formalism the extraction of maximum information from experimental data depends on the measuring of full set of parameters, defining the system before and after interaction. If measured parameters are remaining values then operators commutate with the Hamiltonian of the process, so the unique interpretation of these data is possible. In general case operators do not commutate with the Hamiltonian, it, therefore, appears that the amplitude and the phase of the interaction process are not reconstructed simultaneously.

In fact, let us consider the detection of a scattered photon in a coherent state $|u_{k,\lambda}>$ /7/ appropriate to a pure one-mode state. Let an operator

$$A(x) = \sum_{k,\lambda} \frac{1}{\sqrt{2\omega}} [\vec{e}_{k,\lambda} \; c_{k,\lambda} e^{ikx} + \vec{e}^{\,*}_{k,\lambda} c^{+}_{k,\lambda} e^{-ikx}] \quad , \tag{1}$$

correspond to a selected state of photon. Here $\vec{e}_{k,\lambda}$ is the polarization vector, $c_{k,\lambda}$ ($c^{+}_{k,\lambda}$) is the annihilation (creation) operator of the photon with $k(\vec{k}, i\omega)$ momentum, λ (=1.2) polarization, ω is the photon frequency, normalization volume is taken as one. Write down the expression for the expectation value of the intensity at the space-time point (x) making use of the expansion of photon final state on the complete orthonormal set of initial states:

$$|u_{k,\lambda'}>_f = \sum_i a_{if} \, |u_{k,\lambda}>_i \, , \tag{2}$$

where expantion coefficients are transition amplitudes with phase $e^{i\phi}$:

$$\begin{aligned} I(x) &= \frac{\vec{e}_{k,\lambda'} \vec{e}^{\,*}_{k,\lambda'}}{2\omega'} <u_{k,\lambda'}|_f \; c^{+}_{k,\lambda'} c_{k,\lambda} \; |u_{k,\lambda'}>_f \quad , \\ &= \frac{|a_{if}|^2}{2\omega'} \; |u_{k,\lambda'}|^2 <u_{k,\lambda} \; |_i u_{k,\lambda}>_i \; . \end{aligned} \tag{3}$$

This expression shows that the expectation value of $I(x)$ is determined by amplitude transition square and the information about its phase is lost in the process of measuring the scattering photon.

Then we shall demonstrate that "reference photon" in the registration of scattered photon enables us to detect the amplitude and phase of the interaction process. All the parameters characterizing reference photon are suggested to be known. We shall analyze the measurement of two photons at the space-time point (x). The expectation value of the intensity, summed over all possible modes is then:

$$
\begin{aligned}
I(x) = & \sum_{\substack{k\lambda \\ k'\lambda'}} \left| < u_{k,\lambda}\, u_{k',\lambda'} \left| A^{+}_{k,\lambda}(x) + A^{+}_{k',\lambda'}(x) \right| u_{k',\lambda'}\, u_{k,\lambda} > \right|^2 \\
& = \left[\frac{|u_{k\lambda}|^2}{2\omega} + \frac{|u_{k',\lambda'}|^2}{2\omega'} \,|a_{if}|^2 \right] < u_{k,\lambda} | u_{k,\lambda} > + \\
& + \frac{< u_{k,\lambda} | u_{k,\lambda} >}{2\sqrt{\omega\omega'}} \mathrm{Re}\{ a^{*}_{if}\, u^{*}_{k\lambda'} u_{k,\lambda}\, \vec{e}_{k,\lambda}\, \vec{e}^{*}_{k\lambda} \exp i[(k-k')X + (\phi-\phi')] + \\
& + a_{if}\, u_{k',\lambda'}\, u^{*}_{k,\lambda}\, \vec{e}^{*}_{k,\lambda}\, \vec{e}_{k',\lambda'} \exp i[(k'-k)x + (\phi'-\phi)]\} .
\end{aligned}
\tag{4}
$$

In the relation (4) the first two terms are the expected intensity sum of reference and scattered photons. The last two terms correspond to the interference between scattered and reference photons in the case they are coherent, provided that phase difference is not accidental. The intensity modulated by the periodic factor is recorded by a detector. Mention that interference picture will remain steady for time intervals short compared with the frequency spread $\approx \frac{1}{\omega'-\omega}$ and its effective size is $\approx \frac{1}{|\vec{k}'-\vec{k}|}$. From expression (4) one may see that the complete experiment is realized within scalar theory since an amplitude and phase of interaction may be reduced from the experimentally detected intensity.

Further note that while receiving relation (4), we restrict ourselves to the examination of the pure state of photons, for which

$$|u_{k,\lambda}\ ; u_{k',\lambda'}> = |u_{k,\lambda}>|u_{k',\lambda'}>. \quad (5)$$

In addition, the photon and scattering system state was supposed to factorize.

If two photons are of the same polarization, then $\vec{e}_{k,\lambda}\cdot\vec{e}^{\,*}_{k',\lambda'} = \delta_{kk'}\cdot\delta_{\lambda\lambda'}$ in expression (4). In general, as scattered photon polarization is not fixed, it can be defined as follows:

$$\vec{e}_{k'\lambda'} = d^{(1)}_{k'}\,\vec{e}^{\,(1)}_{k'} + d^{(2)}_{k'}\,\vec{e}^{\,(2)}_{k'}\ , \quad (6)$$

where $d^{(1)}_{k}$, $d^{(2)}_{k}$ are coefficients dependent on the interaction parameters and are determined by density matrix:

$$\rho_{ij} = <|d_i d^*_j|>. \quad (7)$$

The expectation is made on scattering system parameters. The density matrix of the polarization state of photon ρ depends on the three real measured Stokes paremeters.

$$\rho = 1/2\ (1 + \vec{\xi}\,\vec{\sigma}\ ),$$

where $\vec{\sigma}$ –are Pauli's matrices. The measurement of ξ_1, ξ_2 and ξ_3 paremeters allows us to determine the density matrix and to reconstruct the polarization state of scattered photon. Really:

$$\begin{aligned} \xi_1 &= \rho_{11} + \rho_{22} + \rho_{12} - \rho_{21} - 1 \\ \xi_2 &= \rho_{11} + \rho_{22} + i\,(\rho_{12} - \rho_{21}) - 1 \\ \xi_3 &= 2\rho_{11} - 1. \end{aligned} \quad (9)$$

The ξ_1, ξ_2 and ξ_3 parameter measurements may be performed using polarization filters. The ξ_1 and ξ_3 parameters determine the probability of the linear polarization photon along the axis, the angle between them is equal to $\pi/4$, and ξ_2 determines the circle polarization probability.

Now examine the case when reference photon polarization state is known and scattered one is determined by expression(6). Define unit vector as following:

$$\vec{e}_{k}^{(1)} = \frac{[\vec{k},\vec{k}']}{|[\vec{k},\vec{k}']|}, \qquad \vec{e}_{k}^{(2)} = \frac{[\vec{k},\vec{e}_{k'}]}{\omega}$$

$$e_{k'}^{(1)} = \frac{[\vec{k},\vec{k}']}{|[\vec{k},\vec{k}']|}, \qquad \vec{e}_{k'}^{(2)} = \frac{[\vec{k}',\vec{e}_{k'}^{(1)}]}{\omega'}. \tag{10}$$

Let $|d_{k}^{(1)}|$ be the probability of reference photon polarization along the axis defined by $\vec{e}_{k}^{(1)}$, and $|d_{k}^{(2)}|^2$ that along unit vector $\vec{e}_{k}^{(2)}$, $|d_{k}^{(1)}|^2+|d_{k}^{(2)}|^2=1$. We shall find the scalar product $\vec{e}_{k,\lambda}\cdot\vec{e}_{k',\lambda'}^{*}$ entering expression (4). Taking into account the definition of the unit vectors (10) we shall obtain:

$$\begin{aligned}\vec{e}_{k\lambda}\cdot\vec{e}_{k'\lambda'}^{*} &= d_{k}^{(1)}\cdot d_{k'}^{(1)*} + d_{k}^{(2)}\cdot d_{k'}^{(2)*}\,\frac{[\vec{k},\vec{e}_{k'}]}{\omega}\cdot\frac{[\vec{k}'\vec{e}_{k'}]}{\omega'}, \\ &= d_{k}^{(1)}\cdot d_{k'}^{(1)*} + d_{k}^{(2)}\cdot d_{k'}^{(2)*}\cos\theta\end{aligned} \tag{11}$$

where θ is the angle between $\vec{k}$ and $\vec{k}'$ vectors, $d_{k}^{(1)}$ and $d_{k}^{(2)}$ are known, but $d_{k'}^{(1)}$ and $d_{k'}^{(2)}$ are found from relations (6) ÷ (9).

The result proves that the scalar approximation is insufficient for the unique determination of the amplitude and phase of the interaction process as the expectation value depends on polarization states of scattered and reference photons and $\vec{e}_{k\lambda}\cdot\vec{e}_{k'\lambda'}=1$ just only for the entirely coherent scattering with $\theta=0$.

It should be noted, that photon state in the momentum representation is defined entirely by 4-vector k and the Stokes parameters $\vec{\xi}$ and 1. The measurement of these parameters allows the complete experiment /9/ to be realized. Besides, the direct computation of the operator commutators of these values with the Hamiltonian shows the conservation of these values in holography. That makes possible to interprete uniquely the experimental data, the phase resolution being determined by wave length λ and the effective apperture R ($\Delta\phi \approx \frac{\lambda}{R}$) . The frequency ω may be obtained using Michelson interferometer, wave vector $\vec{k}'$ is defined from the law of conservation of energy-momentum and from the scattering angle measurement. The Stokes parameters are measured using the polarization filters.

References

1. D. Gabor. Proc. Roy. Soc. A197, 454 (1949).
2. Ю.Н. Денисюк. ДАН СССР 7, 543(1962).
3. E.N. Leith and J. Upathieks. J. Opt. Am. Soc. 52, 1123 (1962).
4. G.W. Stroke. Introduction to Coherent Optics and Holography. Acad. Press. New York. London 1968.
5. Л.М. Сороко. УФН, 90 (1966).
6. D. Gabor. Reports on Progress in Physics. 32, 395, 1969.
7. R.J. Glauber. Phys. Rev. 131, 2766 (1963).
8. А.И. Ахиезер и В.Б. Берестецкий. Квантовая электродинамика. Наука, Москва, 1969.
9. Н.А. Торопков. Препринт ОИЯИ, 13-4131, 1968.

Received by Publishing Department
on October 14, 1970.