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VENEZIANO MODEL FOR THE OFF-MASS-SHELL $A_1 \pi \to \pi \pi$ AMPLITUDE

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1. Introduction

The process $A_1 \pi \to \pi \pi$ is one of the rare exceptional processes involving a spinning particle, the description of which is particularly successful in the Veneziano model[1,2]. In view of this success an extension to off-mass-shell values of the momenta may also be attempted. The amplitude for three (on-shell) pions and the axial-vector current has been studied in a number of papers[3,4,5]. We want to mention in particular Suura's work[5] where coupling of the axial-vector current to all the appropriate daughters of the $\pi - A_1$ trajectory is taken into account. As emphasized by Nath et al.[6], the further extension of the amplitude to off-shell momenta of one of the pions is not an easy task. In fact, the solution they present has a pole at zero axial-vector current momentum squared $p^2 = 0$. This pole then invalidates the soft pion determination of the σ term, too.

The soft pion result for the σ term may be preserved imposing the condition, that the $p^2=0$ residue should vanish at $s=u=m_\pi^2$ x/. However, the unphysical $p^2=0$ pole can be eliminated only for $s=u=m_\pi^2$ and, necessarily remains for other values of s and u. The origin of this pole is that the amplitude has to satisfy the PCAC equation for arbitrary values of the variables.

In this paper we obtain a model for the $<\pi|A_\mu\partial^\lambda A_\lambda|\pi>$ amplitude, which has all the good properties of off-shell Veneziano-type amplitudes - e.g. it has only the appropriate poles in all the variables - and satisfies the PCAC equation exactly. Due to the proper pole structure of the amplitude the soft pion determination of the σ term remains valid. The basis for this off-shell extension in two mass variables is the successful off-shell extension of scalar amplitudes[7,8]. Therefore the $<\pi|\partial^\mu A_\mu\partial^\lambda A_\lambda|\pi>$ amplitude as well as the σ term we take from ref.[7]. Our approach is different from that of ref.[9], where only the lowest poles in the mass variables are taken into account.

In Sect. 2 for convenience we review at first kinematics, crossing relations, the off-shell $\pi\pi\to\pi\pi$ amplitude and write down the PCAC equation. Then we show that the expression obtainable from the $<\pi|A_\mu\partial^\lambda A_\lambda|\pi>$ amplitude in the soft pion limit is necessarily transversal, provided the PCAC equation is satisfied at the soft π kinematics, too. Thus the current algebra - soft π determination of the pion matrix element of the isospin current model independently yields a transversal expression. In Sect. 3 we list the properties and outline the derivation of our model for the $<\pi|A_\mu\partial^\lambda A_\lambda|\pi>$ amplitude. Sect. 4 contains a discussion of the π electromagnetic form factor obtained by the soft π method. The expression for $F_\pi(t)$ is given in terms of some unknown functions, which shows that the earlier derivations of $F_\pi(t)$ in the Veneziano model[4,5] depend on arbitrary assumptions. We also give a proof that the Adler-Weisberger relation for the $\pi\pi\to\pi\pi$ amplitude ensures through the PCAC eq. model independently the correct normalization of $F_\pi(t)$. Finally in Sect. 5 we discuss the amplitude $A_{1\pi\to\pi\pi}$ off-shell in two pion masses, (keeping the A_1 momentum on the mass-shell). The final result of Sect. 3 is given in the Appendix.

x/ For notations see Sect. 2.

2. Preliminaries

We define the off-shell amplitudes as

$$T\begin{pmatrix} i & \ell & j & n \\ q & p & q' & k\end{pmatrix} = \int d^4x\, e^{-ipx} <\pi q' j|T(\partial^\mu A_\mu^\ell(x)\partial^\lambda A_\lambda^n(0))|\pi q i> \qquad (1)$$

$$T_\mu\begin{pmatrix} i & \ell & j & n \\ q & p & q' & k\end{pmatrix} = i\int d^4x\, e^{-ipx} <\pi q' j|T(A_\mu^\ell(x)\partial^\lambda A_\lambda^n(0))|\pi q i>, \qquad (2)$$

where i, j, n, ℓ are isospin indices, q, p, q', k denote the momenta, A_μ^ℓ is the axial-vector current.

The invariant decomposition of T_μ is as follows

$$T_\mu = p_\mu T_1 + k_\mu T_2 + (q+q')_\mu T_3 . \qquad (3)$$

The isospin decomposition is

$$T\begin{pmatrix} i & \ell & j & n \\ q & p & q' & k\end{pmatrix} = \delta_{i\ell}\delta_{jn} A + \delta_{ij}\delta_{\ell n} B + \delta_{in}\delta_{\ell j} C \qquad (4)$$

$$T_\mu\begin{pmatrix} i & \ell & j & n \\ q & p & q' & k\end{pmatrix} = \delta_{i\ell}\delta_{jn} A_\mu + \delta_{ij}\delta_{\ell n} B_\mu + \delta_{in}\delta_{\ell j} C_\mu , \qquad (5)$$

where $A_\mu = p_\mu A_1 + k_\mu A_2 + (q+q')_\mu A_3$ and similar eqs. hold for B_μ and C_μ. All the invariants are of course functions of $s=(p+q)^2$, $t=(p-k)^2$, $u=(p-q')^2$, p^2 and k^2.

The crossing relations for the amplitude T ($i \to j$ as well as $\ell \to n$ crossing) are:

$$
\begin{aligned}
&A(s,t,u,p^2,k^2) = C(u,t,s,p^2,k^2)\\
&B(s,t,u,p^2,k^2) = B(u,t,s,p^2,k^2)\\
&A(s,t,u,p^2,k^2) = C(u,t,s,k^2,p^2)\\
&B(s,t,u,p^2,k^2) = B(u,t,s,k^2,p^2).
\end{aligned}
\tag{6}
$$

If k^2 is on the mass shell $(k^2=m_\pi^2)$ also have ($i \to n$ crossing):

$$C(s,t,u,p^2,m_\pi^2) = B(s,u,t,p^2,m_\pi^2). \tag{6'}$$

In case of T_μ only $i \to j$ crossing yields restrictions for the invariants, which are (suppressing the mass variables) as follows:

$$
\begin{aligned}
&A_{1,2}(s,t,u) = C_{1,2}(u,t,s)\\
&A_3(s,t,u) = -C_3(u,t,s)\\
&B_{1,2}(s,t,u) = B_{1,2}(u,t,s)\\
&B_3(s,t,u) = -B_3(u,t,s).
\end{aligned}
\tag{7}
$$

On the k^2 mass shell $(k^2=m_\pi^2)$ the following relations ($i \to n$ crossing) are also valid for the residue of the invariants:

$$
\begin{aligned}
&C_1(s,t,u) = B_1(t,u,s) + \frac{1}{2}[B_2(t,u,s) + B_3(t,u,s)]\\
&C_2(s,t,u) = -\frac{1}{2}[B_2(t,u,s) + 3B_3(t,u,s)]\\
&C_3(s,t,u) = \frac{1}{2}[B_2(t,u,s) - B_3(t,u,s)].
\end{aligned}
\tag{7'}
$$

Thus there are two independent amplitudes for T, and six for T_μ (we choose B_1 and C_1).

The amplitude T we taken from ref.[7], thus

$$B = i f_\pi^2 m_\pi^4 2(2\pi)^3 \alpha'^2 \beta_0 \Gamma(\tfrac{1}{2} - \alpha(p^2)) P(p^2) \Gamma(\tfrac{1}{2} - \alpha(k^2)) P(k^2) \times$$

$$\times \frac{1}{2}[V(t,u) + V(t,s) - V(u,s)]$$

$$C = i f_\pi^2 m_\pi^4 2(2\pi)^3 \alpha'^2 \beta_0 \Gamma(\tfrac{1}{2} - \alpha(p^2)) P(p^2) \Gamma(\tfrac{1}{2} - \alpha(k^2)) P(k^2). \times \tag{8}$$

$$\times \frac{1}{2}[V(t,u) + V(u,s) - V(t,s)],$$

where $\beta_0 = 2\frac{g^2_{\rho\pi\pi}}{4(2\pi)^6}$, $P(p^2)$ is an arbitrary non-singular function, $(P(m_\pi^2)=1)$,

$$V(s,t) = \frac{\Gamma(1-\alpha(s))\,\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(s)-\alpha(t))}$$

and $\alpha(s) = \frac{1}{2} + \alpha'(s-m_\pi^2)$ is the $\rho - f$ trajectory[10], with a slope $\alpha' = \frac{1}{2}(m_\rho^2 - m_\pi^2)^{-1}$. We have assumed that the $\pi - A_1$

trajectory is given by $\alpha(s) - \frac{1}{2}$. The eq. (6') is valid even for $k^2 \neq m_\pi^2$. T_μ and T are connected by the PCAC eq.

$$T\begin{pmatrix} i\ell\, j\, n \\ q\, p\, q'k \end{pmatrix} = p^\mu T_\mu \begin{pmatrix} i\ell\, j\, n \\ q\, p\, q'k \end{pmatrix} - i\, \Sigma^{\ell n}(q'^2, q^2, t), \qquad (9)$$

where

$$\Sigma^{\ell n}(q'^2, q^2, t) = -i \int d^4x\, \delta(x_0)\, e^{-ipx} \langle \pi q' j | [A_0^\ell(x), \partial^\lambda A_\lambda^n(0)] | \pi q i \rangle \qquad (9')$$

is the σ term $(q'^2 = q^2 = m_\pi^2)$

Expressed in terms of the invariant amplitudes eq. (9) yields the following eqs.

$$B_1 p^2 + B_2 \frac{s+u-2m_\pi^2}{2} + B_3 \frac{s-u}{2} = B(s,t,u,p^2,k^2) - B(m_\pi^2, t, m_\pi^2, 0, t) \qquad (10a)$$

$$C_1 p^2 + C_2 \frac{s+u-2m_\pi^2}{2} + C_3 \frac{s-u}{2} = C(s,t,u,p^2,k^2) \qquad (10b)$$

(On the LHS we have suppressed the variables). Here we have assumed that the σ term can be calculated from eq. (9) in the $p_\mu \to 0$ limit, setting $\lim_{p_\mu \to 0} p^\mu T_\mu \begin{pmatrix} i\ell\, j\, n \\ q\, p\, q'k \end{pmatrix} = 0$.

By a similar procedure one can get the pion electromagnetic form factor the amplitude T_μ

$$T_\mu \begin{pmatrix} i\ell\, j\, n \\ q\, p\, q'0 \end{pmatrix} = -\epsilon_{n\ell m} \langle \pi q' j | V_\mu^m(0) | \pi q i \rangle, \qquad (11)$$

where V^μ is the isospin current (we have used the usual current commutation rules) and the pion form factor $F_\pi(t)$ is defined as

$$\langle \pi q' i | V_\mu^k(0) | \pi q j \rangle = -i\epsilon_{ijk} (q+q')_\mu \frac{F_\pi(t)}{2(2\pi)^3}. \qquad (12)$$

We want to point out that eq. (11) automatically yields a transversal expression (i.e. eqs. (11) and (12) are consistent) provided eq. (9) is satisfied for $k_\mu = 0$, too. In fact, $T_\mu \begin{pmatrix} i\ell\, j\, n \\ q\, p\, q'0 \end{pmatrix}$ is transversal, and has the isospin structure implied by eqs. (11) and (12), if at $k^2 = 0$, $t = p^2$

$$C_1(m_\pi^2, t, m_\pi^2) = B_1(m_\pi^2, t, m_\pi^2) = B_3(m_\pi^2, t, m_\pi^2) = 0. \qquad (13)$$

The thrid eq. follows from crossing symmetry (eq. (7)), while $B_1 = C_1 = 0$ follows from eqs. (10a), (10b), as the RHS of these eqs. are zero in this limit and $B_{2,3}$, $C_{2,3}$ are not singular. Thus the difficulty of ref./5/ – $\langle \pi | V^\mu | \pi \rangle$ turns out to have a longitudinal component, too – appears, because the expression for T_μ given in this ref. is valid only on the mass-shell, satisfies the PCAC eq. only at $k^2 = m_\pi^2$.

We also want to emphasize that eq. (10a) obviously forces B_1 to have non Veneziano-type terms, too, as the second term on the RHS is clearly not Veneziano.

3. Model for the Amplitude with the Axial-Vector Current and an Off-Shell Pion

At first we enumerate the properties we expect for the amplitude T_μ.

i) The invariant functions of T_μ should have poles at the appropriate values of the Mandelstam variables and mass variables p^2, k^2.

ii) The residues of the poles in s, (t,u) should be polynomials in t, u $(s,u$; $s,t)$.

iii) The crossing relations of eqs.(7) should be satisfied.

iv) The residues of the simultaneous $k^2 =$ (π mass or higher excited π mass)^2, $p^2 =$ (A_1 mass or higher excited A_1 mass)^2 poles should have the Veneziano form [2].

v) The PCAC equation (eq. (9)) should be valid.

vi) The residue of the $k^2 = m_\pi^2$ pole should satisfy eq. (7').

vii) The residue of the $k^2 = m_\pi^2$ pole should have the general form of ref.[5].

viii) The $s(t,u)$ channel $I=2$ amplitudes should not have poles in $s(t,u)$ (absence of exotic resonances off the mass-shell).

ix) The leading trajectory should factorize (off-shell factorization).

In order to construct an amplitude with the above properties generalizing the amplitude of ref.[5], in the spirit of ref.[7,8] we start with the following expressions

$$B_\mu = i f_\pi^2 m_\pi^4 2(2\pi)^3 \alpha'^2 \beta_0 \Gamma(\frac{1}{2} - \alpha(k^2))P(k^2) \times$$

$$\times\{ p_\mu \frac{\Gamma(\frac{1}{2}-\alpha(p^2))P(p^2) - \Gamma(\frac{1}{2}-\alpha(0))P(0)}{p^2} \frac{1}{2} \times \qquad (13')$$

$$\times [B(s,t)(1-\alpha(t)-\alpha(s)) + (s\leftrightarrow u) - B(u,s)(1-\alpha(u)-\alpha(s))] +$$

$$+[-g_{\mu\lambda}\mu(p^2,k^2) + \frac{p_\mu p_\lambda}{p^2}(\mu(p^2,k^2)-\mu(0,k^2))]\frac{1}{2} \times$$

$$\times[k^\lambda (B(s,t)(2\alpha(s)-\alpha(t)-1) + (s\leftrightarrow u) - B(u,s)(2-\alpha(u)-\alpha(s)))$$

$$+(q+q')^\lambda (B(s,t)(1-\alpha(t)) - (s\leftrightarrow u) + B(u,s)(\alpha(u)-\alpha(s)))] +$$

$$+[-g_{\mu\lambda}\mu'(p^2,k^2) + \frac{p_\mu p_\lambda}{p^2}(\mu'(p^2,k^2)-\mu'(0,k^2))] \times$$

$$\times [k^\lambda(-B(s,t) - B(u,t) - 2B(u,s)) +$$

$$+(q+q')^\lambda (B(s,t)-B(u,t))]\} + \qquad (14)$$

$$+ i f_\pi^2 m_\pi^4 2(2\pi)^3 \alpha'^2 \beta_0 \Gamma(\frac{1}{2}-\alpha(0))P(0)\{p_\mu \bar{B}_1 + k_\mu \bar{B}_2 + (q+q')_\mu \bar{B}_3\}.$$

$$C_\mu = i f_\pi^2 m_\pi^4 2(2\pi)^3 \alpha'^2 \beta_0 \Gamma(\frac{1}{2} - \alpha(k^2))P(k^2) \times$$

$$\times\{ p_\mu \frac{\Gamma(\frac{1}{2}-\alpha(p^2))P(p^2) - \Gamma(\frac{1}{2}-\alpha(0))P(0)}{p^2} \frac{1}{2} \times$$

$$\times [B(s,t)(\alpha(s)+\alpha(t)-1)-(s\leftrightarrow u)+B(u,s)(1-\alpha(u)-\alpha(s))]$$

$$+[-g_{\mu\lambda}\,\mu(p^2,k^2)+\frac{p_\mu p_\lambda}{p^2}(\mu(p^2,k^2)-\mu(0,k^2))]\times\frac{1}{2}\times$$

$$\times[k^\lambda(B(s,t)(1+\alpha(t)-2\alpha(s))-(s\leftrightarrow u)+B(u,s)(2-\alpha(u)-\alpha(s)))+$$

$$+(q+q')^\lambda(B(s,t)(\alpha(t)-1)+(s\leftrightarrow u)+B(u,s)(\alpha(s)-\alpha(u)))]+ \qquad (14)$$

$$+[-g_{\mu\lambda}\,\mu'(p^2,k^2)+\frac{p_\mu p_\lambda}{p^2}(\mu'(p^2,k^2)-\mu'(0,k^2))]\times$$

$$\times[k^\lambda(B(s,t)-B(u,t)+2B(u,s))+$$

$$+(q+q')^\lambda(-B(s,t)-B(u,t))]\}+$$

$$+if_\pi^2\, m_\pi^4\, 2(2\pi)^3\alpha'^2\beta_0\Gamma(\frac{1}{2}-\alpha(0))P(0)\{p_\mu\bar{C}_1+k_\mu\bar{C}_2+(q+q')_\mu\bar{C}_3\},$$

where

$$B(s,t)=\frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))}, \qquad (15)$$

$\alpha(s)=\frac{1}{2}+\alpha'(s-m_\pi^2)$, $\alpha'=\frac{1}{2}(m_\rho^2-m_\pi^2)^{-1}$ as in ref.[10].

$\mu(p^2,k^2)$ and $\mu'(p^2,k^2)$ are unknown functions, the pole structure of which is however known;

$$\mu(p^2,k^2)=\bar{\bar{\mu}}(p^2,k^2)\Gamma(\frac{3}{2}-\alpha(p^2))$$

$$\mu'(p^2,k^2)=\bar{\bar{\mu}}'(p^2,k^2)\Gamma(\frac{3}{2}-\alpha(p^2))$$

and $\bar{\bar{\mu}}(p^2,k^2)P(k^2)$, $\bar{\bar{\mu}}'(p^2,k^2)P(k^2)$ have no poles at all.

Later we shall use the notation

$$\bar{\mu}(p^2,k^2)P(p^2)=\bar{\bar{\mu}}(p^2,k^2)(\frac{1}{2}-\alpha(k^2));\ \bar{\mu}'(p^2,k^2)P(p^2)=\bar{\bar{\mu}}'(p^2,k^2)(\frac{1}{2}-\alpha(k^2))$$

too.

It can be easily seen that for a pole contribution to $\mu(p^2,k^2)$ or $\mu'(p^2,k^2)$, $(\frac{a}{p^2-M_A^2})$ the expression

$$-g_{\mu\lambda}\,\mu(p^2,k^2)+\frac{p_\mu p_\lambda}{p^2}(\mu(p^2,k^2)-\mu(0,k^2))$$

reduces to

$$(-g_{\mu\lambda}+\frac{p_\mu p_\lambda}{M_A^2})\frac{a}{p^2-M_A^2}$$

μ and μ' correspond to the two parameters of ref.[2]. The ratio of the s and d wave coupling constants of $A_1\to\rho\pi$ decay is

$$\frac{g_s}{g_d\, m^2_\rho} = -2\,\frac{m^2_\rho - m^2_\pi}{m^2_\rho}\,\frac{\mu'(p^2,k^2)}{\mu(p^2,k^2)}\Big|_{p^2=m^2_A,\;k^2=m^2_\pi},$$

where the $A_1\rho\pi$ vertex is defined as

$$V_{A_1\rho\pi} = g_s\,\epsilon_\rho\cdot\epsilon_A + g_d\,\epsilon_\rho\cdot k\,\epsilon_A\, p\,,$$

with the polarization vectors denoted as ϵ_ρ, ϵ_A and the momenta (ρ and A_1 respectively) denoted as p, k. As will be seen later, the ratio of the s and d wave coupling constants of the decays of higher excited A_1 mesons is given by the same formula, (the only difference being that we must set equal p^2 to the mass squared of the excited A_1). $\bar{B}_1$ and $\bar{C}_1$ are necessary in eqs. (13) and (14) in order to satisfy the PCAC eqs. (10a,b).

At first we discuss the determination of $\bar{B}_1$. We want to choose these functions so as to satisfy the PCAC eq. (10a) which yields using eq. (13') the following equation :

$$\bar{B}_1 p^2 + \bar{B}_2\,\frac{s+u-2m^2_\pi}{2} + \bar{B}_3\,\frac{s-u}{2} = \Gamma\left(\frac{1}{2}-\alpha(k^2)\right)P(k^2)\frac{1}{2}.$$

$$\times\,[B(s,t)(1-\alpha(t)-\alpha(s)) + (s\leftrightarrow u) - B(u,s)(1-\alpha(u)-\alpha(s))] -$$

$$-\Gamma\left(\frac{1}{2}\right)\Gamma(1-\alpha(t))P(t) + \tag{16}$$

$$+\Gamma\left(\frac{1}{2}-\alpha(k^2)\right)P(k^2)\{\bar{\mu}'(0,k^2)\,\frac{s+u-2m^2_\pi}{2}\,\frac{1}{2}\,\times$$

$$\times[B(s,t)(2\alpha(s)-\alpha(t)-1) + (s\leftrightarrow u) - B(u,s)(2-\alpha(u)-\alpha(s))] +$$

$$+\bar{\mu}(0,k^2)\,\frac{s-u}{2}\,\frac{1}{2}\,[B(s,t)(1-\alpha(t)) - (s\leftrightarrow u) + B(u,s)(\alpha(u)-\alpha(s))] + \tag{16}$$

$$+\bar{\mu}'(0,k^2)\,\frac{s+u-2m^2_\pi}{2}\cdot[-B(s,t) - B(u,t) - 2B(u,s)] +$$

$$+\bar{\mu}'(0,k^2)\,\frac{s-u}{2}\,[B(s,t) - B(u,t)]\}.$$

The RHS of this eq. has no poles in p^2, thus it is possible to choose $\bar{B}_1$ so that they have no poles in p^2, which ensures that our amplitudes B_1 satisfy property iv. For $\bar{B}_1$ we assume

$$\bar{B}_1 = \beta^1_1 B(u,t) + \beta^2_1 B(s,t) + \beta^3_1 B(u,s) + b'_1, \tag{17}$$

where β^j_1 are at most first order polynomials in s and u and the role of b'_1 is to compensate for the non Veneziano term of the RHS of eq. (16). The $B(u,t)$ ($B(s,t)$ and $B(u,s)$) terms shoul[d] cancel in eq. (16). Here we meet a difficulty, as the coefficients of the $B(u,t)$ (and $B(s,t)$) terms do not cancel. In fact assuming the proper pole structure - it is easy to see that for $s=u=m^2_\pi$, $p^2=0$ (i.e. $t=k^2$) the coefficients of $B(u,t)(B(s,t))$ cancel only on the k^2 mass shell ($k^2=m_\pi^2$) . However, this is not an indication that we must give up the proper pole structure, as done in ref. /6/, it means only that the eq. for b'_1 should contain some Veneziano-like terms, too. In this way we get uniquely the eq. for b'_1 :

$$b'_1 p^2 + b'_2 \frac{s+u-2m_\pi^2}{2} + b'_3 \frac{s-u}{2} = \tag{18}$$

$$= \Gamma(\frac{1}{2} - \alpha(k^2)) P(k^2) \frac{1}{2} [B(t,s) + B(t,u)] (\frac{1}{2} - \alpha(k^2)) - \Gamma(\frac{1}{2}) \Gamma(1 - \alpha(t)) P(t).$$

The solution of the eqs. for β_1^i is now straightforward we have determined the general solution of these eqs.

We turn now to the solution of eq. (18). Again b'_1 may have only the right position poles. The RHS of the eq. vanishes if simultaneously $t = k^2$, $s = u = m_\pi^2$ (i.e. $p^2 = 0$). Thus a simple assumption like $b'_2 = b'_3 = 0$ leads to a singularity in b'_1 at $p^2 = 0$ (as in ref./6/). Generalizing the experience obtained from the pole dominance solution of the eq., we write the RHS as

$$\begin{aligned} RHS = & \frac{1}{2} \Gamma(\frac{3}{2} - \alpha(k^2)) P(k^2) [B(s,t) - B(m_\pi^2, t)] + \\ & + \frac{1}{2} \Gamma(\frac{3}{2} - \alpha(k^2)) P(k^2) [B(u,t) - B(m_\pi^2, t)] \\ & + B(m_\pi^2, t) [\Gamma(\frac{3}{2} - \alpha(k^2)) P(k^2) - \Gamma(\frac{3}{2} - \alpha(t)) P(t)]. \end{aligned} \tag{19}$$

Now, the first term vanishes if $s = m_\pi^2$ for arbitrary u , t , k^2 ; the second for $u = m_\pi^2$ (arbitrary s , t , k^2) the third for $k^2 = t$ (arbitrary s , u). Thus a solution of the eqs.

$$\bar{b}_1 p^2 + (\bar{b}_2 + \bar{b}_3) \frac{s - m_\pi^2}{2} + (\bar{b}_2 - \bar{b}_3) \frac{u - m_\pi^2}{2} = \Gamma(\frac{3}{2} - \alpha(k^2)) P(k^2) [B(t,s) - B(t, m_\pi^2)] \times$$

$$\bar{\bar{b}}_1 p^2 + (\bar{\bar{b}}_2 + \bar{\bar{b}}_3) \frac{s - m_\pi^2}{2} + (\bar{\bar{b}}_2 - \bar{\bar{b}}_3) \frac{u - m_\pi^2}{2} = \Gamma(\frac{3}{2} - \alpha(k^2)) P(k^2) [B(t,u) - B(t, m_\pi^2)] \tag{20}$$

$$(\bar{\bar{\bar{b}}}_1 + \frac{\bar{\bar{\bar{b}}}_2}{2}) p^2 + \bar{\bar{\bar{b}}}_2 \frac{k^2 - t}{2} + \bar{\bar{\bar{b}}}_3 \frac{s - u}{2} = B(m_\pi^2, t) [\Gamma(\frac{3}{2} - \alpha(k^2)) P(k^2) - \Gamma(\frac{3}{2} - \alpha(t)) P(t)]$$

may be easily obtained, writing

$$\bar{b}_1 = \bar{b}_2 - \bar{b}_3 = \bar{\bar{b}}_1 = \bar{\bar{b}}_2 + \bar{\bar{b}}_3 = \bar{\bar{\bar{b}}}_1 + \frac{\bar{\bar{\bar{b}}}_2}{2} = \bar{\bar{\bar{b}}}_3 = 0. \tag{21}$$

Then the solution of eq. (18) is

$$b'_1 = \bar{b}_1 + \bar{\bar{b}}_1 + \bar{\bar{\bar{b}}}_1 . \tag{22}$$

Thus we have obtained

$$\begin{aligned} b'_1 = & B(m_\pi^2, t) \frac{\Gamma(\frac{3}{2} - \alpha(k^2)) P(k^2) - \Gamma(\frac{3}{2} - \alpha(t)) P(t)}{t - k^2} \\ b'_2 = & \Gamma(\frac{3}{2} - \alpha(k^2)) P(k^2) \frac{1}{2} \{ \frac{1}{s - m_\pi^2} [B(s,t) - B(m_\pi^2, t)] + (s \leftrightarrow u) \} + \\ & + B(m_\pi^2, t) 2 \frac{\Gamma(\frac{3}{2} - \alpha(k^2)) P(k^2) - \Gamma(\frac{3}{2} - \alpha(t)) P(t)}{k^2 - t} \\ b'_3 = & \Gamma(\frac{3}{2} - \alpha(k^2)) P(k^2) \frac{1}{2} \{ \frac{1}{s - m_\pi^2} [B(s,t) - B(m_\pi^2, t)] - (s \leftrightarrow u) \}. \end{aligned} \tag{23}$$

This solution then ensures (or does not disturb) that B_t satisfies the requirements i – v .

We note that if the "subtractions" in the RHS of eq. (18) are carried out in different order, we get solutions which do not fulfil requirement ii, i.e. the residues of the poles in the Mandelstam variables are not polynomials. Of course, we may always add a solution of the homogeneous eq. (if otherwise this fulfils our requirements) to b'_t , however, if this solution contains resonances it must have Veneziano form, so it is already included in our solution, as we have determined the general solution for the β_t^i -s .

The solution of the eq. for $\bar{C}_t$ can be obtained in a similar way. Again

$$\bar{C}_t = \gamma_t^1 B(u,t) + \gamma_t^2 B(s,t) + \gamma_t^3 B(u,s) + c'_t . \tag{24}$$

There are no crossing restrictions for γ_t^i thus the general solution contains even more arbitrary functions than the expression of the β_t^i-s . Although in the eq. for $\bar{C}_t$ non Veneziano terms do not appear, the eq. of the c'_t -s is not homogeneous. The inhomogeneous term appears for reasons similar to those which led to the first term in eq. (18). In this way we get the eq.

$$c'_1 p^2 + c'_2 \frac{s+u-2m_\pi^2}{2} + c'_3 \frac{s-u}{2} = \Gamma(\tfrac{3}{2} - \alpha(k^2)) P(k^2) \frac{1}{2} \times$$
$$\times [B(u,t) - B(s,t)] \tag{25}$$

The (essentially unique) solution of which is

$$c' = 0$$

$$c'_2 = \Gamma(\tfrac{3}{2} - \alpha(k^2)) P(k^2) \frac{1}{2} \{ \frac{1}{s-m^2} [B(m_\pi^2, t) - B(s,t)] - (s \leftrightarrow u) \} \tag{26}$$

$$c'_3 = \Gamma(\tfrac{3}{2} - \alpha(k^2)) P(k^2) \frac{1}{2} \{ \frac{1}{s-m_\pi^2} [B(m_\pi^2, t) - B(s,t)] + (s \leftrightarrow u) \} .$$

Again properties i-v of the invariants C_t are ensured (or not disturbed) by the solution eq. (26).

Eqs. (23) and (26) show non Regge behaviour. It is very interesting to note that eq. (23), which is part of the solution of an originally inhomogeneous equation, behaves as ~ s^0, while eq. (26) which is part of the solution of an originally homogeneous equation, behaves as $\sim s^{-1}$ for large s and fixed t .

Having ensured properties i-v, the further requirements must also be fulfilled. These eliminate most of the arbitrary functions appearing in β_t^i and γ_t^i , (only two of them remain), as well as force $\mu(p^2,k^2)$, $\mu'(p^2,k^2)$ to be independent of k^2 . The full solution we give in the Appendix.

Actually our solution reduces to the general solution of ref.[5] (property vii) only if we set $\eta_1(p^2)=0$. We do not see any reason for this special choice, thus our solution is more general than ref.[5].

We want to emphasize the importance of requirement viii. In fact absence of exotic resonances in the s and in the t channel yields independent conditions, in particular, the latter forces out symmetry properties for the Veneziano-like terms in C_t .

We note another favourable feature of our model. On the pion mass-shell ($k^2 = m_\pi^2$) the leading trajectory contributions to the residues of the $\alpha(s)=\ell$ poles correspond to only $I=0$ particles for ℓ = even and $I=1$ for ℓ = odd. Off the mass-shell this isospin selection rule does not follow from Bose statistics, however, it is true in our model. Thus our off-shell extension does not introduce new leading trajectories.

The daughter structure of our amplitude is rather complicated. Imposing off-shell factorization of the daughter trajectories, we must introduce new particles (some of which are ghosts). A similar situation has been found also in case of the scalar particle amplitude[7].

4. Pion Electromagnetic Form Factor

As pointed out at the end of Sect. 2 the pion electromagnetic form factor may be obtained without any difficulty from T_μ by the soft π method, our expression for T_μ being transversal in this limit. Thus we get

$$F_\pi(t)=-i\,2(2\pi)^3\, C_3(m_\pi^2,t,m_\pi^2)\big|_{p^2=t,\,k^2=0} \tag{27}$$

which yields

$$\begin{aligned} F_\pi(t) &= 4(2\pi)^6 f_\pi^2 m_\pi^4 \alpha'^2 \beta_0 \Gamma\left(\tfrac{1}{2}-\alpha(0)\right) P(0)\, B(m_\pi^2,t)\,. \\ &\Big\{ \Gamma\left(\tfrac{3}{2}-\alpha(t)\right)\left[\bar{\bar{\mu}}(t)\left(\tfrac{1}{2}-\alpha'(t-m_\pi^2)\right)+2\bar{\bar{\mu}}'(t)\right] - \\ &-\Gamma\left(\tfrac{3}{2}-\alpha(0)\right)\left[\bar{\bar{\mu}}(0)\left(\tfrac{1}{2}-\alpha'(t-m_\pi^2)\right)+2\bar{\bar{\mu}}'(0)\right] + \\ &+P(0)\,\Gamma\left(\tfrac{1}{2}-\alpha(0)\right)\left[-4t\left(\eta_1(t)-\bar{\eta}_1(t,0)\,m_\pi^2\right)+\alpha'\right] + \\ &+P(0)\,\Gamma\left(\tfrac{3}{2}-\alpha(0)\right)\alpha'\left[\psi\left(\tfrac{1}{2}\right)-\psi\left(\tfrac{3}{2}-\alpha(t)\right)\right]\Big\}, \end{aligned} \tag{28}$$

where $\bar{\bar{\mu}}(t)$, $\bar{\bar{\mu}}'(t)$, $\eta_1(t)$, $\bar{\eta}_1(t,k^2)$ are arbitrary nonsingular functions ($\eta_1,\bar{\eta}_1$) come from the arbitrariness of γ_3^j), $\psi(x)=\Gamma'(x)/\Gamma(x)$. It is clear, that apart from the position of the poles, which is as expected, we cannot say anything about $F_\pi(t)$ without further assumptions. Thus, e.g. it is not possible to predict the absence of the $\rho'(\alpha(t)=2)$ pole, and the other predictions of ref.[5] do not follow either.

The condition $F_\pi(0)=1$ is automatically fulfilled, if the off-shell $\pi\pi\to\pi\pi$ amplitude satisfies the Adler-Weisberger relation[11]. This has been proved by Geffen[9], assuming single π and A_1 pole dominance for the mass dependence of the amplitude. The proof can be carried out along similar lines without any specific assumption on the mass dependence, so the above statement is really model independent.

In fact in our notations the Adler-Weisberger theorem states

$$\frac{\partial A}{\partial s}=-\frac{\partial C}{\partial s}=-\frac{i}{2|2\pi|^3}\,;\quad \frac{\partial B}{\partial s}=0, \tag{29}$$

where the arguments are $t=p^2=k^2=0$, $(u=-s+m_\pi^2)$ and we must take the value of the derivative at $s=m_\pi^2$, which is the required kinematics for $F_\pi(0)$. Using eq. (10b) we write

$$C_3=\frac{2}{s-u}\left(C-C_1p^2-C_2\frac{s+u-2m_\pi^2}{2}\right).$$

From eq. (10b) it follows that $C=0$ at $s=m_\pi^2$, $t=p^2=k^2=0$, as C_1, C_2, C_3 are not singular in this limit. So taking $p^2=k^2=t=0$ $(u=-s+2m_\pi^2)$ in the limit $s\to m_\pi^2$ we get from the above eq.

$$C_3=\frac{\partial C}{\partial s}=i\,\frac{1}{2(2\pi)^3}\,.$$

which is the required result.

We want to emphasize that in the framework of our model it is not consistent to compare the Weinberg low energy formula [12] for $\pi\pi\to\pi\pi$ scattering with our expressions. Thus the KSFR relation (with the usual Veneziano model modification) does not follows. Instead we may use eq. (29) to fix the normalization, which yields

$$g^2_{\rho\pi\pi} = \frac{1}{2f^2_\pi} \frac{1}{\sqrt{\pi}\, m^2_\pi\, \alpha'^2 P^2(0)\, \Gamma(\frac{1}{2}-\alpha(0))\Gamma(1-\alpha(0))} \times \tag{30}$$

$$\times \frac{1}{1+\alpha' m^2_\pi [\psi(\frac{1}{2}) - \psi(\frac{3}{2}-\alpha(0))]} .$$

Using eq. (30) we get for the σ term (defined in eq. (9)) the following expression $(i = j,\ q'^2 = q^2 = m^2_\pi)$

$$\Sigma^{\ell n}(0) = -\delta_{\ell n} \frac{m^2_\pi}{2(2\pi)^3} \frac{1}{1+\alpha' m^2_\pi [\psi(\frac{1}{2}) - \psi(\frac{3}{2}-\alpha(0))]} . \tag{31}$$

So

$$\Sigma^{\ell n}(0) \approx -\delta_{\ell n} \frac{m^2_\pi}{2(2\pi)^3} .$$

Thus the Gell-Mann, Oakes, Renner [13] result for the σ term (which is exact for $q^2 = q'^2 = 0$) follows independently of the KSFR relation. Of course assuming $P(0) \approx P(m^2_\pi) = 1$ (which is a reasonable assumption) the modified KSFR relation may be also obtained.

5. Model for the Amplitude with Two Pions Off-Shell

In this section for comletness we discuss briefly the amplitude off-shell in two pion masses (keeping at the same time the A_1 on the mass-shell).

The definition of the off-shell amplitude is

$$\bar{M}\begin{pmatrix} i\ \ell\ j\ n \\ q\ p\ q'\ k \end{pmatrix} = \int d^4x\, e^{-iqx} \langle \pi j q' | T\, \partial^\lambda A^i_\lambda(x) \partial^\mu A^k_\mu(0) | A \ell p \epsilon \rangle , \tag{32}$$

where ϵ denotes the polarization of the A_1. Following the recipe of ref. [7], we write

$$\bar{M}\begin{pmatrix} i\ \ell\ j\ n \\ q\ p\ q'\ k \end{pmatrix} = i f^2_\pi\, m^4_\pi 2(2\pi)^3 \alpha'^2 \Gamma(\frac{1}{2}-\alpha(q^2)) P(q^2) \Gamma(\frac{1}{2}-\alpha(k^2)) P(k^2) M(s,t,u), \tag{33}$$

where we have used the definition

$$\langle \pi j q', \pi n k | \pi i q, A \ell p \epsilon \rangle_{\substack{\\ out\ \ in}} = -i(2\pi)^4 \delta(p+q-k-q') M(s,t,u). \tag{34}$$

In eq. (33) M denotes the Veneziano amplitude for the process, which is, of course, given by the appropriate residue of T_μ.

We discuss now the Adler condition for $\bar{M}$, as it is easily seen that eq. (33) satisfies all the other usual requirements. The Adler conditions have the following form

$$\bar{M}\begin{pmatrix} i\ \ell\ j\ n \\ 0\ p\ q'\ k \end{pmatrix} = -\int d^4x \langle \pi j q' | \delta(x_0) [A^i_0(x), \partial^\mu A^n_\mu(0)] | A \ell p \epsilon \rangle = -i \bar{\Sigma}^{in}(u) \tag{35}$$

$$\bar{M}\begin{pmatrix} i\ \ell\ j\ n \\ q\ p\ q'\ 0 \end{pmatrix} = -\int d^4x \langle \pi j q' | \delta(x_0) [A^n_0(x), \partial^\mu A^i_\mu(0)] | A \ell p \epsilon \rangle = -i \bar{\Sigma}^{ni}(u), \tag{36}$$

The σ terms in eqs. (35) and (36) should be equal and also $\bar{\Sigma}^{ln} \approx \delta_{ln}\delta_{jl}$. This conditions are not satisfied automatically off the mass-shell ($u = q^2 \neq m_\pi^2$). In fact we get using eq. (35)

$$\bar{\Sigma}^{ln}(u) = i2(2\pi)^3 \alpha'^2 \frac{G_A}{m_A^2} f_\pi^3 m_\pi^6 \beta_0 \Gamma(\frac{1}{2} - \alpha(0)) P(0) \times$$
$$\times \{ \delta_{ln} \delta_{lj} P(u) \Gamma(\frac{1}{2}) \Gamma(1-\alpha(u)) \epsilon \cdot q' 4\mu' -$$
$$-(\delta_{il}\delta_{jn} - \delta_{ij}\delta_{\ell n}) P(u) \Gamma(\frac{1}{2}) \Gamma(1-\alpha(u)) \epsilon \cdot q' 4 (\frac{\mu}{2} - \mu') \}. \quad (37)$$

From eq. (36) we get a similar formula for $\bar{\Sigma}^{nl}$ the only difference being that the sign of the $(\delta_{il}\delta_{jn} - \delta_{ij}\delta_{\ell n})$ term is opposite. (We denote $\mathrm{res}_{p^2=m_A^2}\, \mu(p^2) = \frac{\mu}{\alpha}$, and similarly for μ'). This means that the $\delta(x_0)[A_0^l(x), \partial^\mu A_\mu^n(0)]$ commutator contains besides the usual $I=0$ term an $I=1$ term, too. ($I=2$ does not appear). At any rate, the pole structure of the σ term is appropriate.

If we take only one pion off the mass-shell, the Adler condition tells us that the amplitude should vanish at the soft π point. This condition is readily met by our expressions, as the RHS of eq. (37) has no pole at $u = m_\pi^2$. Thus there is no difficulty with the Adler condition in this case, as has been observed already in ref./2/.

One way out of the difficulty with the "off-shell" σ term is to set $\mu - 2\mu' = 0$, which gives a prediction for the ratio of the s and d wave amplitudes in $A_1 \to \rho\pi$ decay. Using the notation of ref./14/, we get $\left| \frac{a_T}{a_L} \right| = 1.62$ while experimentally /15/ it is 0.64 ± 0.25.

Another way is to modify suitable the off-shell amplitude eq. (33). To do this we write down the isospin decomposition of $\bar{M}$:

$$\bar{M} = \delta_{il}\delta_{jn} \bar{A} + \delta_{ij}\delta_{\ell n} \bar{B} + \delta_{in}\delta_{\ell j} \bar{C}. \quad (38)$$

We must then modify the expression given for $\bar{A}$, $\bar{B}$ by eq.(33). The simplest way is to add terms of the form $\epsilon k f(u) + \epsilon(q+q')f'(u)$ (we do not want to introduce Veneziano satellite terms, as we want to modify the amplitude only off the mass-shell). With this assumption we get

$$\bar{A}' = -\bar{B}' = -f_\pi^3 m_\pi^6 \frac{G_A}{m_A^2} 2(2\pi)^3 \alpha'^2 \beta_0 \Gamma(\frac{1}{2} - \alpha(0)) P(0) \times$$
$$\times \frac{1}{2}(3\epsilon k + \epsilon(q+q')) P(u) \Gamma(\frac{1}{2}) \Gamma(1-\alpha(u)) 4(\frac{\mu}{2} - \mu'), \quad (39)$$

$\bar{A}'$ and $\bar{B}'$ should be added to the expressions obtained for $\bar{A}$ and $\bar{B}$ from eq. (33). It can easily be seen that this new terms do not spoil the good properties of the original amplitude. Thus eqs. (33), (39) determine a satisfactory Veneziano type expression for our amplitude.

Finally, I want to thank Dr. Z. Kunszt for his help in preparing the manuscript.

Appendix

We give here the full expression for the amplitude T_μ. B_μ and C_μ are given by eqs. (13) and (14) respectively, where $\mu(p^2, k^2)$, $\mu'(p^2, k^2)$ should be replaced by $\mu(p^2)$, $\mu'(p^2)$ and $\bar{B}_1$, $\bar{C}_1$ are given below.

$$\bar{B}_1 = \Gamma\left(\frac{1}{2} - \alpha(k^2)\right)P(k^2)\times\{$$

$$B(s,t)\left[-\frac{\alpha'}{2} + x(\eta_1 + \bar{\eta}_1(m_\pi^2 - k^2)) - y(\eta_1 - \bar{\eta}_1(m_\pi^2 - k^2))\right] +$$

$$+(s\leftrightarrow u) + B(s,u)2x\eta_1\}+$$

$$+B(m_\pi^2,t)\frac{\Gamma(\frac{3}{2} - \alpha(k^2))P(k^2) - \Gamma(\frac{3}{2} - \alpha(t))P(t)}{t-k^2}$$

$$\bar{B}_2 = \Gamma\left(\frac{1}{2} - \alpha(k^2)\right)P(k^2)\{$$

$$B(s,t)\left[\frac{\alpha'}{2} - 2p^2(\eta_1 + \bar{\eta}_1(m_\pi^2 - k^2)) + \frac{\bar{\mu}(0)}{2}\left(-\frac{1}{2}\cdot\alpha' z\right) - \bar{\mu}'(0) + x\bar{\mu}(0)\alpha' + y\frac{\bar{\mu}(0)\alpha'}{2}\right]+$$

$$+(s\leftrightarrow u) + B(s,u)\left[\alpha' - 4p^2\eta_1 - \frac{\bar{\mu}(0)}{2} - 2\bar{\mu}'(0) + \frac{x\bar{\mu}(0)\alpha'}{2}\right]\}+$$

$$+\frac{1}{2}\Gamma\left(\frac{3}{2} - \alpha(k^2)\right)P(k^2)\left\{\frac{1}{s-m_\pi^2}(B(t,s) - B(t,m_\pi^2)) + (s\leftrightarrow u)\right\} -$$

$$-B(m_\pi^2,t)\,2\,\frac{\Gamma(\frac{3}{2} - \alpha(k^2))P(k^2) - \Gamma(\frac{3}{2} - \alpha(t))P(t)}{t-k^2}$$

$$\bar{B} = \Gamma\left(\frac{1}{2} - \alpha(k^2)\right)P(k^2)\{$$

$$B(s,t)\left[-\frac{\alpha'}{2} + 2p^2(\eta_1 - \bar{\eta}_1(m_\pi^2 - k^2)) + \frac{\bar{\mu}(0)}{2}\left(\frac{1}{2} - \alpha' z\right) + \bar{\mu}'(0) + x\frac{\bar{\mu}(0)}{2}\alpha'\right]$$

$$-(s\leftrightarrow u) + B(s,u)\left[-y\frac{\bar{\mu}(0)\alpha'}{2}\right]\}+$$

$$+\frac{1}{2}\Gamma\left(\frac{3}{2} - \alpha(k^2)\right)P(k^2)\left\{\frac{1}{s-m_\pi^2}(B(t,s) - B(t,m_\pi^2)) - (s\leftrightarrow u)\right\}$$

$$\bar{C}_1 = \Gamma\left(\frac{1}{2} - \alpha(k^2)\right)P(k^2)\{$$

$$B(s,t)\left[\frac{\alpha'}{2} - x(\eta_1 + \bar{\eta}_1(m_\pi^2 - k^2)) + y(\eta_1 - \bar{\eta}_1(m_\pi^2 - k^2))\right] -$$

$$-(s\leftrightarrow u) + B(s,u)[-2x\eta_1]\}$$

$$\bar{C}_2 = \Gamma\left(\frac{1}{2} - \alpha(k^2)\right)P(k^2)\{$$

$$B(s,t)\left[-\frac{\alpha'}{2} + 2p^2(\eta_1 + \bar{\eta}_1(m_\pi^2 - k^2)) + \bar{\mu}(0)\frac{1}{2}\left(\frac{1}{2} + \alpha' z\right) + \bar{\mu}'(0) - x\bar{\mu}(0)\alpha' - y\frac{\bar{\mu}(0)\alpha'}{2}\right]$$

$$-(s\leftrightarrow u) + B(s,u)\left[-\alpha' + 4p^2\eta_1 + \frac{1}{2}\bar{\mu}(0) + 2\bar{\mu}'(0) - x\frac{\bar{\mu}(0)\alpha'}{2}\right]\}+$$

$$+\frac{1}{2}\Gamma\left(\frac{3}{2} - \alpha(k^2)\right)P(k^2)\left\{\frac{1}{s-m_\pi^2}(B(t,m_\pi^2) - B(t,s)) - (s\leftrightarrow u)\right\}$$

$$\bar{C}_3 = \Gamma\left(\frac{1}{2} - \alpha(k^2)\right) P(k^2) \{$$

$$B(s,t)\left[\frac{\alpha'}{2} - 2p^2(\eta_1 - \bar{\eta}_1(m_\pi^2 - k^2)) - \bar{\mu}(0)\frac{1}{2}\left(\frac{1}{2} - \alpha' z\right) - \bar{\mu}'(0) - x\frac{\bar{\mu}(0)\alpha'}{2}\right]$$

$$+ (s \leftrightarrow u) + B(s,u)\left[y \frac{\bar{\mu}(0)\alpha'}{2}\right]\} +$$

$$+ \frac{1}{2}\Gamma\left(\frac{3}{2} - \alpha(k^2)\right) P(k^2)\left\{\frac{1}{s - m_\pi^2}(B(t, m_\pi^2) - B(t,s)) + (s \leftrightarrow u)\right\}.$$

The notation is

$$x = s + u - 2m_\pi^2, \quad y = s - u, \quad z = p^2 + k^2 - m_\pi^2,$$

$$\mu(p^2) = \bar{\bar{\mu}}(p^2)\Gamma\left(\frac{3}{2} - \alpha(p^2)\right) = \Gamma\left(\frac{1}{2} - \alpha(p^2)\right) P(p^2)\bar{\mu}(p^2)$$

$$\mu'(p^2) = \bar{\bar{\mu}}'(p^2)\Gamma\left(\frac{3}{2} - \alpha(p^2)\right) = \Gamma\left(\frac{1}{2} - \alpha(p^2)\right) P(p^2)\bar{\mu}'(p^2)$$

η_1 is an arbitrary nonsingular function of p^2, $\bar{\eta}_1$ is an arbitrary nonsingular function of p^2 and k^2.

References

1. G. Veneziano. Nuovo Cim., 57A, 190 (1968).
2. C.J. Goebel, M.L. Blackmon, K.C. Wali. Phys.Rev., 182, 1487(1969); Fayyazuddin, Riazuddin. Phys.Lett., 28B, 561 (1960); ibid. 29B, 262 (E) (1969).
3. S.P. de Alwis, D.A. Nutbrown, P. Brooker, J.M. Kosterlitz. Phys. Lett., 29B, 362 (1969); H.J. Schnitzer. Phys.Rev.Lett., 22, 1154 (1969); R. Arnowitt, P. Nath, Y. Srivastava, M.H. Friedman. Phys.Rev.Lett., 22, 1158 (1969).
4. J.L. Rosner, H. Suura. Argonne preprint, July 1969; Y. Oyanagi. Nucl.Phys., 14B, 375 (1969).
5. H. Suura. Phys.Rev.Lett., 23, 551 (1969).
6. P. Nath, R. Arnowitt, M.H. Friedman. Boston preprint, August 1969.
7. F. Csikor. JINR preprint, E2-4865, Dubna (1969).
8. F. Csikor. Phys.Lett., 31B, 141 (1970).
9. D.A. Geffen. Phys.Rev.Lett., 23, 897 (1969).
10. C. Lovelace. Phys.Lett., 28B, 264 (1968).
11. S.L. Adler. Phys.Rev., 140B, 736 (1965); K. Kawarabayashi, W.D. Mc Glinn, W.W. Wada. Phys.Rev.Lett., 15, 897 (1965).
12. S. Weinberg. Phys.Rev.Lett., 17, 616 (1966).
13. M. Gell-Mann, R.J. Oakes, B. Renner. Phys.Rev., 175, 2195(1968).
14. F.J. Gilman, H. Harari. Phys.Rev., 165, 1803 (1968).
15. J. Ballam et al. Bull.Am.Phys.Soc., 14, 573 (1969).

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