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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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GENERAL METHOD FOR SETTING UP A MICROSCOPIC THEORY OF NUCLEAR COLLECTIVE MOTION

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GENERAL METHOD FOR SETTING UP A MICROSCOPIC THEORY OF NUCLEAR COLLECTIVE MOTION

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In the previous papers [1,2] the authors developed a theory of nuclear rotation, taking into account the three-dimensional kinematics. The similarity of this theory with RPA (the random phase approximation) for vibrations [3] suggests the possibility of solving the old problem of a microscopic description of coupling between different modes in nuclear spectra. Here the authors present their philosophy on this topic, which generalizes the RPA for vibrations and their previous description of nuclear rotation [1,2,4]. Possible applications of this philosophy will be given in the following papers.

We write the Hamiltonian of an even-even nucleus in the second quantized form

$$H = \sum_{ij} \left[e_i \delta_{ij} - \sum_k \bar{v}_{ik,jk} \right] a_i a_j + \frac{1}{4} \sum_{ij,k\ell} \bar{v}_{ij,k\ell}\, a_i^{+} a_j^{+} a_\ell a_k \qquad (1)$$

in its Hartree-Fock basis, e being the H.F. energies, $\underline{k}$ - hole states and $\bar{v}_{ij,k\ell} = v_{ij,k\ell} - v_{ij,\ell k}$ the interaction. We denote the true (spherically symmetric) ground state by $|0\rangle$ and the H.F. ground state (eventually axially deformed) by $|\ \ \rangle$.

Let us consider the linear space N built on a set of low-lying eigenstates of different nature, say, up to a definite energy. The states may belong to the same nucleus whose H.F. ground state is given by $|\ \ \rangle$ or to its odd-even or odd-odd neighbour. Another linear space S of linear operators R^+ (transition operators) is defined so that the space of all $R^+|0\rangle$ coincides with N . We prescribe some commutation, anticommutation, or multiplication relations between the R^+ operators and proceed to find a few operators $r^+ \in s \subset S$ such that any R^+ is a polynomial of r^+ .

Suppose now that we find a subspace $M \subset N$ and a corresponding subspace $P\,(s \subset P \subset S)$, such that $R^+|-\rangle$ form a space coinciding with N and $R|-\rangle \in N$ if $|-\rangle \in M$ and $R^+ \in P$. For instance, S may include all products of r^+ with powers up to η and $|-\rangle = R^+|0\rangle$ with R^+ containing powers of r^+ up to λ . In this example the operators $R^+ \in P$ contain powers of r^+ up to $\mu = \eta - \lambda$.

Suppose further that a simple model Hamiltonian $h = h\,(p, R, R^+, I)$ is known which is easy to diagonalize in the basis $R^+|0\rangle$. Here p are free parameters, I –some well-known operators such as the total angular-momentum and $R^+ \in S$ the transition operators whose relation to the quantities included in H is an object of the theory. We impose on h the condition, according to which the parameters p may be chosen so that

$$H\,|\,-\,\rangle = h\,|\,-\,\rangle \tag{2}$$

for any $|-\rangle \in N$. If the space N is limited and the number of parameters is large enough this can be easily achieved. The best choice of h corresponds to the smallest number of parameters. In fact, one prefers to work with as few parameters as possible, thus leaving only a few eigenstates in M and N. But then one chooses h so that (2) and the following equations for R^+ which are exact for N and M are valid approximately for much larger spaces $M' \supset M$ and $N' \supset N$. This is expected to improve the accuracy of practical solutions of these equations.

Now, let us take an arbitrary state $|-\rangle \in M$. Since $r^+ \in s \subset P$ (in particular, $P = s$ is also possible) it follows that $r^+|-\rangle \in N$, $r|-\rangle \in N$ and therefore

$$[H, r^+]|-\rangle - [h, r^+]|-\rangle = \langle -|[H, r^+] - \langle -|[h, r^+] = 0 . \quad (3)$$

From this it follows

$$\langle -|[Q, [H, r^+] - [h, r^+]]|-\rangle = 0 \quad (4)$$

for any linear operator Q. Note, that only the diagonal matrix element of (3) is missing in eq. (4). In some special cases equations (3), (4) are known: from the author's theory of rotation [4] (eqs. (3) and from Rowe's formulation [3] of RPA (eq. (4)).

Eqs. (3), (4) together with the relations between I and R^+ (e.g. commutation relations of the angular momentum with R^+) can be used to determine r^+ and p. In general, they do not define r^+ and R^+ completely, but only the effect of $R^+ \in P$ on the states $|-\rangle \in M$. Eqs. (3), (4) are exact, but their applicability rests upon the knowledge of $|-\rangle$. Although, it is not clear how to choose $|-\rangle$ the H.F. ground state $|\ \rangle$ might be well known. Inserting into (3) and (4) $|\ \rangle$ instead of $|-\rangle$ one comes to approximate, but plausible equations:

$$[H, r^+] | > - [h, r^+] | > = < | [H, r^+] - < | [h, r^+] = 0 , \tag{5}$$

$$< | [Q , [H, r^+] - [h, r^+]] | > = 0 . \tag{6}$$

To make these equations practical a second approximation is needed: one treats r^+ as simple operators, e.g. as linear combinations of particle-hole excitations. In this way one obtains linear equations for the coefficients of r^+ giving their microscopic structure.

How to go beyond the second approximation is known at least formally. In the case of vibrations one resorts to the higher orders of RPA /5/. The effect of the first one, i.e. of the substitution of $| - >$ by $| >$ is less evident and not easily estimated. Our main point here is that when increasing the space M not only more states are included in the description, but its accuracy is also improved. Indeed, let us choose $| - >$ to be the closest state to $| >$ in some sense, e.g. with respect to their overlap. Increasing the space M one can increase the overlap and therefore improve the approximation, obtaining the full overlap and thus eliminating all errors due to the substitution $| - > \to | >$ if M coincides with the space of all states.

This points to the possibility of using successive approximations as follows. Consider the sequence M_ν, h_ν, p_ν, r_ν^+, $| - > = | \nu >$ ($\nu = 1,2,3...$) in which the space M_ν and the number of parameters p_ν in h_ν increase with ν. If the quantities r_ν^+ and p_ν are known one may try to use them in order to find $p_{\nu+1}$, $r_{\nu+1}^+$ in a simplified way. Now , since $< \nu | > \to 1$ for $\nu \to \infty$ we obtain from (4)

$$<\nu|[Q,[H,r_\nu^+]-[h_\nu,r_\nu^+]]|\nu> = <|[Q,[H,r_\nu^+]-[h_\nu,r_\nu^+]]|> + 0(\nu) = 0 \tag{7}$$

and from (6)

$$<|[Q,[H,r_\nu'^+]-[h_\nu,r_\nu'^+]]|> = 0 . \tag{8}$$

Here the primed quantities are solution of approximate eq. (8) while r_ν^+ are the solution of the exact equation (7). With suitable mathematical requirements one will have $0(\nu)\to 0$ $(\nu\to\infty)$ and thus $R_\nu'^+ - R_\nu^+ \to 0$ $(\nu\to\infty)$ for $R_\nu^+ \subset P_\nu$.

These arguments work when one uses higher orders of RPA describing the anharmonicity of vibration, since then one increases M and includes more parameters p in h . Another example is the inclusion of higher powers of the angular momentum squared in h to describe corrections to the simple rotational formula/4/. However, it may not be always the best choice to include states belonging to one vibrational (or rotational) band in M , because this does not allow to improve the overlap $<-|>$ indefinitely. It might be preferable to describe phenomenologically the coupling of different modes by defining appropriate operators R^+ and including sufficient number of parameters in h which together with R^+ should be determined from the microscopic equations (5), (6).

Let us illustrate the scheme outlined above by two well known examples. In the example of vibrations in deformed nuclei $r^+ = R^+ = B^+ \subset S = P = s$, $h = h\omega B^+B$ and $p = h\omega$, B^+ being the phonon creation operator, such that $[B, B^+] = 1$. In this case M includes only the ground state $|0,>$, N contains $|0>$ and $B^+|0>$. However, in N' several approximately harmonic vibrational states are involved.

Eq. (6) coincides with the RPA equation[3]. As it has been pointed out, one can introduce additional parameters accounting for the anharmonicity of vibrations.

Rotation in deformed nuclei uncoupled from other modes is described by $r^+ = R^+_{1m} \in s$; $R^+_{\ell m}$ being the roton operators with multiplication properties of spherical harmonics[1]. In this case $h = \frac{1}{2\mathcal{J}}\vec{I}^{\,2}$, $p = \frac{1}{2\mathcal{J}}$ and M includes formally only the ground state $|0>$ while N includes $|0>$ and the states $|2m> = R^+_{2m}|0>$. However, N' contains several states of the rotational band for which the simple rotational formula is approximately valid (or even the states of different rotational bands having approximately equal moments of inertia). Including higher powers of $\vec{I}^{\,2}$ in h , i.e. writing[4] $h = f(\vec{I}^{\,2})$ one incorporates the whole band into N . Eq. (6) coincides with the authors equations in [1,2] determining the microscopic structure of the rotons.

Appendix

The following examples illustrate the variety of possible definitions of r^+ and $s \subset S$.

a. <u>The Quasi-Spin Operators</u>

Let us designate the states in N by an index r spanning all integers (or half integers) from the interval $-T \leq r \leq T$ $(2T+1=\{N\}$ - being the total number of states in N). The space S is defined as the manifold of all the operators

$$R^+_{r\,r'} = |r><r'|$$

or of their linear combinations

$$R^+_{tr} = \sum_{r_1 r_2} (T\,T\,r_1\,r_2|\,t\,r)\,(-1)^{T-r_2}\,R^+_{r_1 r_2} .$$

The space s contains the following three operators

$$\Sigma_r = \frac{(-1)^{2T}}{\sqrt{2T(T+1)}} R^{+}_{1r} \qquad (r = 0, \pm 1)$$

satisfying the commutation relations of the angular momentum operator.

b. Transition Operators in an Elliott-Type Model

The quasi-spin operators may be introduced formally for any value of $\{N\} \geq 2$. However, if

$$\{N\} = (1 + p)(1 + q)\{1 + \frac{1}{2}(p + q)\},$$

where p, q are positive integers one may label the states in N by eigenvalues defining states in the (p, q) representation of the SU(3) group, rather than of the SU(2) group considered above. In this case the 9 operators $r^{+}_{\nu} = R^{+}_{ij}$ $(i, j = 1, 2, 3)$ may be defined so that

$$\sum_i R^{+}_{ii} = 0$$

$$[R^{+}_{ij}, R^{+}_{kl}] = R^{+}_{il}\,\delta_{jk} - R^{+}_{kj}\,\delta_{il}.$$

The space so defined is the space of generators of the SU_3 group contained in the group of unitary transformations of operators operating in N.

c. Phonon and Roton Operators

If the number of states in N is infinite, then labelling the states by subsequent positive integers and defining

$$B^{+} = \sum_{n=0}^{\infty} \sqrt{n+1}\; |n+1\rangle\langle n|$$

one comes to the space s containing the phonon creation and annihilation operators B^+ and B, such that

$$[B, B^+] = e \equiv \sum_{n=0}^{\infty} |n\rangle\langle n| \quad .$$

There exists an another way of labelling states which leads to the roton operators[2]. To define rotons one splits the states from N into multiplets containing $(2\ell+1)$ states distinguished by an integer m ($|m| \leq \ell$) so that every combination of two integers (ℓ, m) ($\ell \geq 0$) appears in N only once. The roton operators are defined by the equation[1,2].

$$R^+_{\ell m} |\ell_1 m_1\rangle = \sum_{\ell_2 m_2} [(2\ell_1+1)(2\ell_2+1)]^{1/2} \begin{pmatrix} \ell & \ell_1 & \ell_2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell & \ell_1 & \ell_2 \\ m & m_1 & -m_2 \end{pmatrix} (-1)^{m_2} |\ell_2 m_2\rangle .$$

It may be shown that the operators defined in this way commute and satisfy the following multiplication relation

$$R^+_{\ell_1 m_1} R^+_{\ell_2 m_2} = \sum_{\ell m} (2\ell+1) \begin{pmatrix} \ell & \ell & \ell \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell & \ell & \ell \\ m_1 & m_2 & m \end{pmatrix} R_{\ell m} \quad .$$

In this case the space s contains the three operators $R^+_{\ell=1, m}$ ($m = 0, \pm 1$) .

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