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СООБЩЕНИЯ  
ОБЪЕДИНЕННОГО  
ИНСТИТУТА  
ЯДЕРНЫХ  
ИССЛЕДОВАНИЙ

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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

S.I. Gabrakov, A.A. Kuliev, N.I. Pyatov

**1<sup>+</sup> STATES IN DEFORMED NUCLEI.  
DIPOLE SUM RULE  
AND STRENGTH FUNCTION  
FOR M1 TRANSITIONS**

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ОРЕНБУРГ

## 1. Introduction

It is well known that the spin-spin component of residual nuclear interaction leads to the polarization phenomena in odd-mass nuclei. This interaction strongly affects the magnitudes of magnetic moments, decoupling parameters, etc./1,2/. It is possible to understand the polarization effects by assuming a coupling of the odd particle to the  $1^+$  excitations of the even core, these excitations being generated by the spin-spin force. In ref./3/ we have investigated some properties of the  $1^+$  excitations in even deformed nuclei in the framework of the Tamm-Dankoff approximation (TDA). The model Hamiltonian investigated, the dispersion equation for the energy and expressions for the wave functions of  $1^+$  states were given in detail also in ref./3/.

In the present communication we treat in more detail the dipole sum rule and M1 gamma-ray strength function and we also discuss the position of the giant M1 resonance in the region of high energies<sup>x/</sup>.

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<sup>x/</sup> Similar investigation of the strength function for M1 transitions in the region of energy 2-9 MeV have recently been made/4/.

## 2. Characteristics of $1^+$ States

To describe the  $1^+$  states in even-even nuclei we make use of the Hamiltonian of the type<sup>/3/</sup>

$$H = H_{sp} + H_{pair} + H_{\sigma} \quad (1)$$

where  $H_{sp}$  describes the single-particle motion,  $H_{pair}$  is the pairing interaction and  $H_{\sigma}$  is the spin-spin interaction which is specified by the following strength parameters:  $\kappa_{nn} = \kappa_{pp} = \kappa$  and  $\kappa_{np} = q\kappa$ . The wave functions are built as linear combinations of two quasi-particle states, whose amplitudes are found by variational method. The energies  $\omega_i$  of  $1^+$  states are the solutions of the dispersion equation of TDA.

In order to characterize the  $1^+$  states we make use of the following quantities:

(i) The reduced probability of M1 excitation from the ground state  $B(M1, 0 \rightarrow 1)$  (for the expression cf. ref.<sup>/3/</sup>).

(ii) The dipole sum rule

$$\langle \Psi_0 | [D_{\mu}, [HD_{\mu}]] | \Psi_0 \rangle = \frac{8\pi}{3} \sum_i \omega_i B_1(M1), \quad (2)$$

where  $|\Psi_0\rangle$  is the wave function of the ground state and  $D_{\mu}$  is the magnetic dipole operator. We consider the r.h.s. of (2) as a function  $X_n$  of the number of states involved in the summation. This part of the sum rule depends on the strength of the spin-spin interaction.

(iii) The strength functions  $S_{\mu}(\mu=0,1)$  for M1 transitions, defined as

$$S_{\mu}(E_{\gamma}) \Delta E = \sum_{(\Delta E)} |\langle 1^+ K = \mu | \mathfrak{M}(M1, \mu) | \Psi_0 \rangle|^2, \quad (3)$$

where  $\mathfrak{M}(M1, \mu)$  is the operator for M1 transition, and the summation runs over a certain energy interval  $\Delta E$ .

The strength functions are related to the average reduced width of M1 transition  $\bar{k}_{M1}$  by the following expression<sup>/5/</sup>

$$\bar{k}_{M1} = 2.76 \times 10^{-3} S(E_\gamma) = \bar{\Gamma}_\gamma D^{-1} E_\gamma^{-3}, \quad (4)$$

where  $\bar{\Gamma}_\gamma$  is the average partial width for M1 transition in eV,  $D$  is the average spacing between the levels of the same spin and parity (in MeV) and  $E_\gamma$  is the gamma-ray energy (in MeV). The strength function  $S(E_\gamma)$  is the statistical sum of  $S_\mu$  for  $\mu=0$  and 1

$$S(E_\gamma) = S_1(E_\gamma) + \frac{1}{3} [S_0(E_\gamma) - S_1(E_\gamma)]. \quad (5)$$

Having in mind these characteristics we have investigated the properties of the  $1^+$  states in the following three energy regions:

- (i) Spectroscopic  $1^+$  states in the region of energy  $\omega \approx 2-4$  MeV.
- (ii)  $1^+$  states in the region of the neutron binding energy (region of the resonance capture of slow neutrons)  $\omega \approx 5-8$  MeV.
- (iii)  $1^+$  states in the region of the giant magnetic dipole resonance ( $\omega > 9$  MeV).

### 3. Discussion of the Results

The Nilsson scheme<sup>/6/</sup> of single-particle levels, improved for getting the best description of the spectra of the single-particle and collective excitations in rare-earth nuclei was used. Forty neutron and forty proton levels were taken into account. All the single-particle levels involved lie below the neutron binding energy.

From calculations of magnetic moments the strength parameters  $\kappa$  and  $q$  were determined to lie within the range  $0.04\hbar\omega_0 \leq \kappa \leq 0.05\hbar\omega_0$  and  $0 \geq q \geq -1/4$ .

Due to the repulsive character of the spin-spin interaction the collective mode of excitation associated with this interaction is expected to appear in a high-energy region; consequently, the r.h.s.

of the sum rule (2) must exceed the l.h.s. The behaviour of the function

$$\chi_n = \frac{8\pi}{3} \sum_{i=1}^n \omega_i B_i (M1) \quad (6)$$

shows the location of the collective mode and the energy region of the saturation of the r.h.s. of the sum rule.

The strength functions  $S(E_\gamma)$  were calculated for two-quasi-particle excitations<sup>xx</sup> ( $\kappa=0$ ) and for collective  $1^+$  states. The averaging was carried out over the energy interval of  $\Delta E = 1.25$  MeV, as in ref.<sup>4/</sup>.

Now we shall discuss the results obtained for each energy region of  $1^+$  excitations.

(i) The spectroscopic  $1^+$  states may be observed by means of ordinary spectroscopic methods. In tables 1,2 the calculations for the low-lying  $1^+$  states in the nuclei of  $^{160}\text{Dy}$  and  $^{176}\text{Hf}$  are presented. The weak collectivization and small values of  $B(M1)$  are characteristic of these states. The particle-hole  $1^+$  states ( $K=1$ ) with  $B(M1, 0 \rightarrow 1) \approx 1B(M1)_{s.p.}^{x/}$  which are formed by quasi-particles in the levels belonging to one spherical subshell may sometimes appear among them.

The number of the low-lying states with  $K=0$  is rather small. The spectroscopic  $1^+$  excitations give a small contribution to the sum rule (see fig.1). The spin-spin interaction decreases the magnitude of the strength function  $S_0$  in this energy region (see fig.2). The strength function  $S_1$  and the reduced width  $\bar{k}_{M1}$  have relatively small maxima at  $E_\gamma \approx 4$  MeV (fig.2,3).

All characteristics of the spectroscopic  $1^+$  states depend weakly on the strength of the spin-spin interaction.

(ii) The  $1^+$  states show more varied properties in the energy region of the resonance neutron capture (5-8 MeV). The states with

$$1B(M1)_{s.p.}^{x/} = \frac{90}{16\pi} \left( \frac{e\hbar}{2Mc} \right)^2$$

$K=0$  in this region are also weakly collectivized and give a small contribution to the sum rule. The strength function  $S_0(E_\gamma)$  is also not very large (fig.1,2). But a number of collective  $1^+$  states with  $K=1$  and relatively large values of  $B(M1, 0 \rightarrow 1) \approx 1B_{s.p.}$  does appear. In that case the main contribution to  $B(M1)$  comes from the single-particle transitions between the levels of one spherical sub-shell and sometimes between the levels of spin-orbital partners. The strength function  $S_1(E_\gamma)$  and the average reduced width  $k_{M1}$  in this energy region have second maxima, although their values are smaller than those for two-quasi-particle excitations. It is possible to find such states in  $(n, \gamma)$  reactions, if strong M1-transitions to the ground state are observed. The predicted values of  $k_{M1}$  are of the same order of magnitude as those measured experimentally for M1 transitions in  $^{188}\text{Er}$  [7]. But since these transitions are coming from resonance states with spins  $3^+$  and  $4^+$  [8] a direct comparison of theory with experiment seems to us to be difficult. In connection with this the most interesting are the investigations of  $(n, \gamma)$  reactions on nuclei, in which the resonances with spins  $1^+$  or  $0^+$  are formed.

As one can see from fig.1 the contribution to the sum rule of  $1^+$  states in the energy region of 5-8 MeV, although being dependent on  $\kappa$  but for the physically interesting values of  $\kappa > 0.03 \hbar \omega_0$  does not exceed 20% of the total sum. Therefore, the collective mode associated with the spin-spin interaction (magnetic dipole resonance) is shifted to higher energy.

(iii) The theory predicts the most strongly collectivized  $1^+$  states to appear at energies of the order of 10 MeV ( $K=0$ ), and 13 MeV ( $K=1$ ). The structure of some of such states in  $^{170}\text{Yb}$  was given in ref. [3]. These excitations are mainly connected with single-particle transitions between levels belonging to spin-orbital partners and situated in the region of discrete spectrum. Therefore, such collective excitations although lying in the energy region of the continuous spectrum, by their nature are relevant to discrete spectrum and are considered as quasi-stationary states. The  $1^+$

states in the energy region in question give the main contribution to the sum rule. With further increase of the energy; the function  $\chi_n$  remains practically constant (see fig.1). Therefore, one can classify these states as belonging to the giant magnetic dipole resonance. The number of the states forming the resonance usually is not very large (3-4) and they are concentrated in the energy interval of the order of 1 MeV. The main maxima of the strength functions  $S_0$  and  $S_1$  as well as of the average reduced width  $k_{M1}$  are also to be found in the energy region under consideration. In fig. 2,3, their behaviour as a function of  $\kappa$  is shown. In fig.3 the density of  $1^+$  states as a function of the energy is also shown. One can see that the total density of these excitations remains almost constant in the energy region 5-13 MeV.

The calculated  $S_\mu$ ,  $\bar{k}_{M1}$  and  $\chi_n$  for the other rare-earth nuclei show the same features as those discussed for  $^{168}\text{Er}$ .

Recapitulating the results obtained it is possible to say that the spin-spin interaction in even-even nuclei considerably decreases the strength functions and average reduced width for M1 transitions for low-lying  $1^+$  states and concentrates the strength of M1 transition in the energy region of 10-13 MeV. Therefore, the possibility of observing a giant M1 resonance in the capture of slow neutrons<sup>[4]</sup> seems doubtful to us. It is possible to use either the processes of the scattering of fast neutrons or large-angle electron scattering for that purpose.

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#### References

1. Z. Bochnacki, S. Ogaza. *Nucl.Phys.*, 69, 186 (1965); 83, 619 (1966); A102, 529 (1967).  
A.B. Migdal. *Nucl.Phys.*, 75, 441 (1966).  
L.A. Rapoport, A.S. Chernyshev. *Yader.Fiz.*, 7, 309 (1968).



2. A.A. Kuliev, N.I. Pyatov. *Yader.Fiz.*, 9, 313, 955 (1969).  
A.A. Kuliev, N.I. Pyatov. *Phys.Lett.*, 28B, 443 (1969).
3. S.I. Gabrakov, A.A. Kuliev, N.I. Pyatov. *JINR Preprint E4-4774*,  
Dubna (1969).
4. C.S. Shapiro and G.T. Emery. *Phys.Rev.Lett.*, 23, 244 (1969).
5. I.M. Blatt and V.F. Weisskopf. *Theoretical Nuclear Physics*  
N.Y.L. (1952).
6. K.M. Zheleznova et al. *JINR Preprint D-2157*, Dubna (1965).
7. L.M. Bollinger. *Nuclear Structure: Dubna Symposium, 1968*, IAEA,  
Vienna, 1968, p.317.
8. L.V. Groshev et al. *Izv. Acad.Nauk USSR, ser.fys.*, 29, 772(1965).

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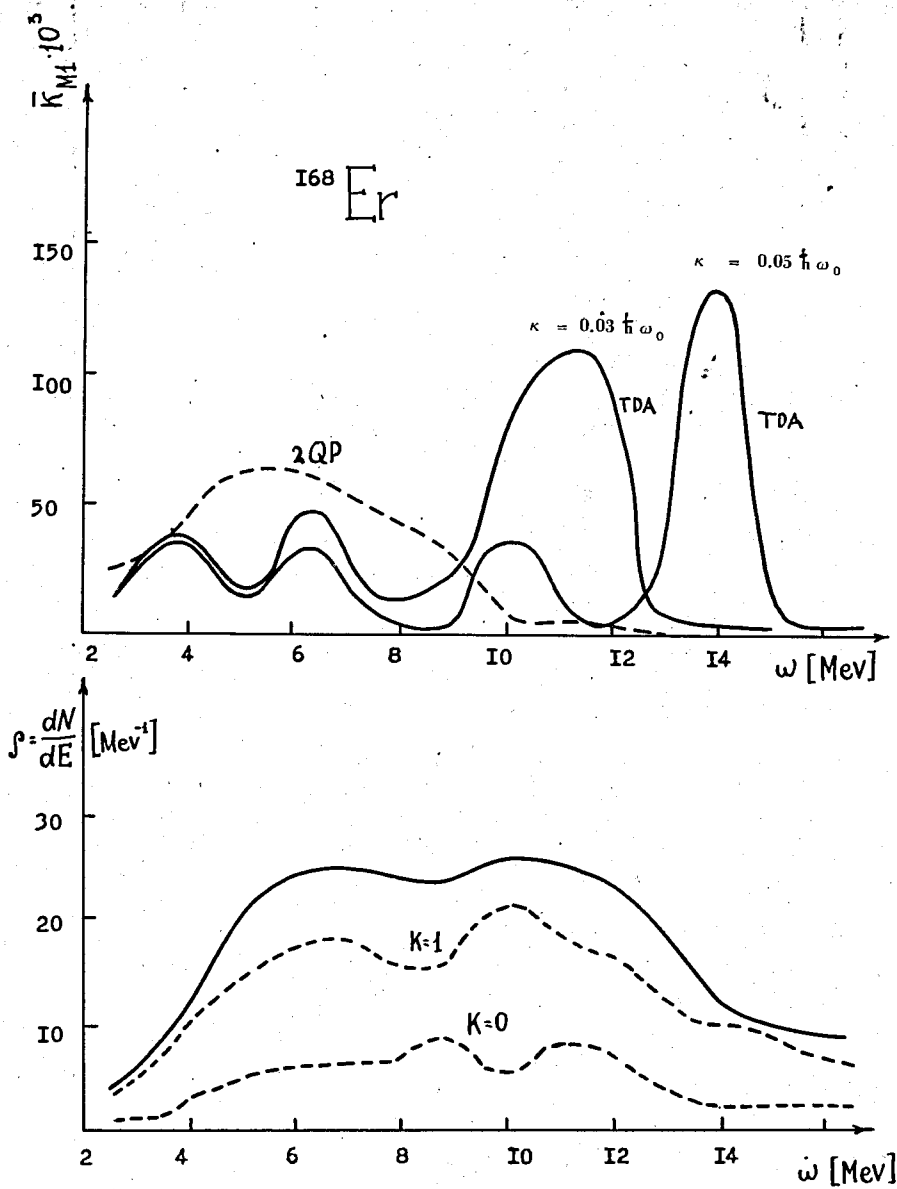


Fig.3. The  $\bar{K}_{M1}$  and the average  $1^+$  state density for  $^{168}\text{Er}$ . The significance of the labels 2QP and TDA is the same as in Fig.2. The solid curve corresponds to the total  $1^+$  state density ( $K=0$  and  $K=1$ ).