ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
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Дубна

E4 - 4901

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

I.Ž. Petkov

OPTICAL POTENTIAL FOR SCATTERING OF HIGH ENERGY PARTICLES

1970

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Submitted to "Теоретическая и математическая физика"

Introduction

The high-energy scattering of particles by nuclei gives as is known, the information about the nucleon-nucleon correlation function. An exact extraction of this information from experiment turns out to be extremely difficult since an infinite system of coupled equations should be solved (even if it is assumed that the nuclear states are known). Therefore it is very interesting to develop methods sufficiently well approximating the exact solution in which the scattering amplitude is more directly connected with the desired correlation function.

In ref.[1] it is shown that the amplitude of elastic scattering of particles by nuclei can be sought by solving a system of two equations which is equivalent to introducing in addition to elastic, only one inelastic channel taking into account effectively all inelastic channels. In this case the pairing correlation function is expressed through the coupling potentials in terms of which the effective optical potential is also defined. Earlier[2,3] a similar problem was formulated in order to clarify the role of virtual transitions in the nucleus in elastic electron scattering. However, in solving it the change in the average potential of interaction of a particle with the nucleus in the excited state and the excitation energy were neglected.

In the present paper a method is suggested of solving the system of two equations for high particle energies in the case of the central-symmetric coupling potential. An explicit expression for the optical potential is obtained in terms of the effective potentials of interaction of a particle with the nucleus, which depends on both the projectile energy and the effective nuclear excitation energy. The elastic scattering amplitude, which is also given, has a simple form identical with the Fourier-Bassel representation of the amplitude. The obtained results allow to treat easily the experimental data on elastic scattering of particles by nuclei at high energies and thus to obtain information on nucleon-nucleon correlation function.

2. Method of Solving

To the system of two equations under consideration

$$\begin{aligned} (\nabla^2 + p_0^2)\, \psi_0(\bar{r}) &= U_{00}(r)\, \psi_0(\bar{r}) + U_{01}(r)\, \psi_1(\bar{r}) \\ (\nabla^2 + p_1^2)\, \psi_1(\bar{r}) &= U_{10}(r)\, \psi_0(\bar{r}) + U_{11}(r)\, \psi_1(\bar{r}) \end{aligned} \qquad (1)$$

there corresponds the following optical potential operator

$$U_{opt} = U_{00} + U_{01} \frac{1}{p_1^2 + \nabla^2 - U_{11} + i\,\delta} U_{10} \,. \qquad (2)$$

Here $p_0 = \sqrt{2\mu E}$ - is the projectile momentum; $p_1 = \sqrt{2\mu(E-\epsilon)}$ is the momentum related to the effective excitation energy ϵ; $U_{nn} = 2\mu V_{nn}$ is the average value of the effective projectile-nucleon interaction[1] over the ground ($n = 0$) and excited ($n = 1$) nuclear states; $U_{nm} = 2\mu V_{nm}$ stands for the effective coupling potential. The correlation function is contained in the second terms of the right-hand side of eq. (2) and in the momentum representation is determined approximately by the formula $K(q, q') \approx U_{01}(q)\, U_{10}(q')$.

We find the optical potential by solving the system (1) (in the momentum representation) in the high-energy approximation $E \gg V$. To this end we write the elastic[x] scattering amplitude in the form

$$f_{00}(\theta) = -\frac{1}{4\pi}\int e^{i\bar{q}\bar{r}}\,[U_{00}(r)\,\phi_0(\bar{r}) + U_{01}(r)\phi_1(\bar{r})]\,d^3r \,, \quad (3)$$

where $\bar{q} = \bar{p}_0 - \bar{p}$, $\phi_n(r) = e^{-i\bar{p}_0\bar{r}}\psi_n(\bar{r})$ $(n = 0, 1)$, or introducing the appropriate Fourier representation (see also[4]):

$$f_{00}(\theta) = -\frac{1}{4\pi}\sum_n \int U_{0n}(|\bar{q} - \bar{\tau}|)\,\phi_n(\bar{\tau})\,d^3\tau \,, \quad (4)$$

where

$$\phi_n(\tau) = \delta_{n0}\delta(\bar{\tau}) - \frac{1}{|\bar{p}_0 - \bar{\tau}|^2 - p_n^2 - i\delta}\sum_m \int U_{nm}(|\bar{\tau} - \bar{\tau}'|)\,\phi_m(\bar{\tau}')\,\frac{d^3\tau'}{(2\pi)^3} \,. \quad (5)$$

Then we make use of the exact representation of the functions $U_{nm}(x)$ through the Bassel function

$$U_{nm}(x) = -4\pi\int_0^\infty J_0(\rho x)\,\chi_{nm}(\rho)\,\rho\,d\rho \,,$$
$$\chi_{nm}(\rho) = -\int_0^\infty U_{nm}\left(\sqrt{\rho^2 + t^2}\right) dt \,. \quad (6)$$

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[x] The scattering amplitude $f_{01}(\theta)$ defining the total inelastic cross section can be found by a similar method.

Note that nowhere we use a particularly chosen corrdinate system contrary to the procedure in quasiclassical methods. Here ρ coincides formally with the impact parameter.

Inserting eq. (5) into eq. (4) and taking into account eq.(6) we get

$$f_{00}(\theta) = \sum_n \int_0^\infty M_{0n}(q,\rho)\, \chi_{0n}(\rho)\, \rho \, d\rho \,, \qquad (7)$$

where the "local" amplitudes $M_{0n} \equiv \int J_0(\rho\,|\bar{q}-\bar{r}|)\,\phi_n(\bar{r})\, d^3r$ satisfy the following integral eqiations

$$M_{0n}(\rho q) = J_0(q\rho)\,\delta_{0n} + \frac{1}{2\pi^2}\sum_m \int_0^\infty \rho' d\rho' \chi_{nm}(\rho') \int \frac{J_0(\rho\,|\bar{q}-\bar{r}|)\, M_{0m}(\rho' r)}{|\bar{p}_0-\bar{r}|^2 - p_n^2 - i\delta}\, d^3r \,.$$

As before/4/ we solve eq. (8) at high energies and small scattering angles. Then

$$M_{0n}(q\rho) = J_0(\rho q)\, \Gamma_n(\rho)$$

$$f_{00}(\theta) = \sum_n \int_0^\infty J_0(q\rho)\, \chi_{0n}(\rho)\, \Gamma_n(\rho)\, \rho\, d\rho \qquad (9)$$

and the functions Γ_n should be found from the system of equations

$$\Gamma_n(\rho) = \delta_{n0} + \frac{1}{2\pi^2}\sum_m \int_0^\infty G_n(\rho,\rho')\chi_{nm}(\rho')\, \Gamma_m(\rho')\, \rho'\, d\rho' \,. \qquad (10)$$

The kernel of the integral equation (10)

$$G_n(\rho,\rho') = \int \frac{J_0(\rho\,|\bar{p}_0-\bar{r}|)\, J_0(\rho'|\bar{p}_0-\bar{r}|)}{r^2 - p_n^2 - i\delta}\, d^3r \qquad (10')$$

was investigated[4] for $n=0$:

$$G_0(\rho,\rho') = i\frac{\pi^2}{p_0}\left[\frac{1}{\rho}\delta(\rho-\rho') + 0\left(\frac{1}{p}\right)\right]. \qquad (11)$$

For $n \neq 0$

$$G_n(\rho,\rho') = i\frac{\pi^2}{p_0}\frac{1}{\rho}\delta(\rho-\rho') - i\frac{\pi^2}{p_0}\int_0^{p_0-p_n} J_0(\rho w)\, J_0(\rho' w)\, w\, dw$$

or within the accuracy up to the terms $(\Delta p)^2 = (p_0 - p_n)^2$

$$G_n(\rho,\rho') = i\frac{\pi^2}{p_0}\left[\frac{1}{\rho}\delta(\rho-\rho') - \frac{1}{2}(\Delta p)^2\right]. \qquad (12)$$

With the account of eqs. (12) and (11) the final form of the system (10) is

$$\begin{aligned} \Gamma_0 &= 1 + a\,\Gamma_0 + b\,\Gamma_1 \\ \Gamma_1 &= c\,.\,\Gamma_0 + d\,.\,\Gamma_1 - \gamma\,, \end{aligned} \qquad (13)$$

where the following notation is introduced

$$a = \frac{i}{2p_0}\chi_{00}\,,\quad b = \frac{i}{2p_0}\chi_{01}\,,\quad c = \frac{i}{2p_0}\chi_{10}\,,\quad d = \frac{i}{2p_0}\chi_{11}$$

$$\gamma = i\frac{(\Delta p)^2}{4p_0}\left[\int_0^\infty \Gamma_1\,\chi_{11}\rho\, d\rho + \int_0^\infty \Gamma_0\,\chi_{10}\rho\, d\rho\right]. \qquad (14)$$

The exact solution of eq. (13) reads

$$\begin{aligned} \Gamma_0 &= \Delta^{-1}(1 - d - \gamma\,.\,b) \\ \Gamma_1 &= \Delta^{-1}(c + \gamma\,.\,a - \gamma)\,. \end{aligned} \qquad (15)$$

Here $\Delta = (1-a)(1-d) - b.c$ and the constant γ is defined by

$$\gamma = \frac{a_0}{1 + a_1 + a_2}, \tag{16}$$

where

$$a_0 = \frac{(\Delta p)^2}{2} \int_0^\infty \Delta^{-1} c.\rho \; d\rho , \quad a_1 = \frac{(\Delta p)^2}{2} \int_0^\infty \Delta^{-1} (1-a).d.\rho. \; d\rho$$

$$a_2 = \frac{(\Delta p)^2}{2} \int_0^\infty \Delta^{-1} . b. c. \rho. \; d\rho . \tag{17}$$

For simplicity we have neglected here the contribution of the principal value of the integral (10) which was considered approximately in the former paper of the author /4/. An exact account of it as well as the account of higher orders of Δp require obviously the numerical solution of the system (10). Since, however, the integration out the energy shell changes also the scattering amplitude at small angles which is mainly valid in this paper, we should bear in mind the possibility of introducing a correction in the elastic channel where it is more essential (see ref. /4/).

Inserting eq. (15) to eq. (9) and taking into account eq.(14) we get the final form of the elastic scattering amplitude

$$f_{00}(\theta) = \int_0^\infty J_0(\rho q) \frac{\chi_{00}(1-\frac{i}{2p_0}\chi_{11}) + \frac{i}{2p_0}\chi_{01}\chi_{10} - \gamma \chi_{01}}{(1-\frac{i}{2p_0}\chi_{00})(1-\frac{i}{2p_0}\chi_{11}) + \frac{1}{4p_0^2}\chi_{01}\chi_{10}} \rho . d\rho . \tag{18}$$

From this equation for a purely potential scattering follows the known result /5/

$$f(\theta) = \int_0^\infty J_0(q\rho) \frac{\chi(\rho)}{1 - \frac{i}{2p}\chi(\rho)} \rho . d\rho .$$

3. Optical Potential

We obtain the optical potential comparing eq. (18) with the formula

$$f_{00}(\theta) = \int_0^\infty J_0(q\rho) \frac{X_{opt}(\rho)}{1 - \frac{i}{2p} X_{opt}(\rho)} \rho \, d\rho . \qquad (19)$$

The amplitude (19) corresponds to the solution of the Schroedinger equation with the potential

$$U_{opt}(r) = \frac{1}{\pi r} \frac{d}{dr} \int_r^\infty \frac{X_{opt}(\rho)}{\sqrt{\rho^2 - r^2}} \rho \, d\rho \qquad (20)$$

in the high-energy approximation[4,5]. As a result we obtain

$$X_{opt} = \frac{\chi_{00}\left(1 - \frac{i}{2p_0}\chi_{11}\right) + \frac{i}{2p_0}\chi_{01}\chi_{10} - \gamma\,\chi_{01}}{1 - \frac{i}{2p_0}\chi_{11} - \frac{i}{2p_0}\gamma\,\chi_{01}} . \qquad (21)$$

This expression determining the optical potential (see eq. (20)) is the main result of this paper. To explain the structure of the optical potential we write it for a simple case $\Delta p = 0$, $\mathrm{Im}\,\chi_{nm} = 0$. (we recall that in the general case the functions χ_{nm} are complex[1]). Then $\gamma = 0$ and after simple transformations we get

$$R_e X_{opt} = \chi_{00} - \frac{1}{4p_0^2}\chi_{11} \frac{\chi_{01}\chi_{10}}{1 + \frac{1}{4p_0^2}\chi_{11}^2} \qquad (22)$$

$$\mathrm{Im}\,X_{opt} = \frac{1}{2p_0} \frac{\chi_{01}\chi_{10}}{1 + \frac{1}{4p_0^2}\chi_{11}^2} .$$

4. Discussion

The channel coupling potential χ_{01} enters in (22) quadratically, since only two-particle correlations in the nucleus are taken into account /1/. It is seen that the optical potential (21) is local in the approximation used, is complex and depends on both the projectile energy and the effective excitation energy. The imaginary part of the potential (22) is completely due to the introduction of the inelastic channel and is larger in magnitude than the second term of the real part.

Note that the sign of the imaginary part of (22) corresponds to the absorption since $\mathrm{Im}\ X_{opt} = -\int_0^\infty \mathrm{Im}\ U_{opt}(\rho, \iota)\, d\iota$ and, consequently $\mathrm{Im}\ U_{opt} < 0$.

Now we clarify qualitatively the role of the effective excitation energy Δp in elastic scattering of electrons by nuclei. For a rough estimation we write the amplitude (18) as

$$f_{00}(\theta) \approx \int_0^\infty J_0(q\rho)\, \chi_{00}(\rho)\, \rho\, d\rho - \gamma \int_0^\infty J_0(q\rho)\, \chi_{01}(\rho)\, \rho\, d\rho \, .$$

In the case of uniform electric charge distribution the first term is proportional to the form factor $F_{00}(qR) = \frac{3}{(qR)^2}\left(\frac{\sin qR}{qR} - \cos qR\right)$. If we assume /3/ that the transition density is $\rho_{01}(r,R) \approx \frac{d}{dR}\rho_{00}(r,R)$, where ρ_{00} is the charge density in the ground state, then the second term is proportional to the form factor $F_{01}(qR) \approx \frac{d}{dR}F_{00}(qR)$.

Now it is easy to show that in this case the curves of the squares of the form factors will be in the "counter-phase". This means that the factor γ can lead to the diffractional cross section minima being filled partially. This important fact will be further investigated in more detail by numerical calculation.

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Received by Publishing Department
on January, 22, 1970.