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# THE SELF-CONSISTENT FIELD METHOD IN NUCLEAR THEORY

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At present it is difficult to imagine theoretical nuclear phyiscs without such a notion like the self-consistent field. Numerous experimental data point out that the nucleons in the nucleus behave in a certain approximation as independent particles moving in a common potential well. Owing to this fact it is reasonable to construct nuclear theory, at least theory of low-lying excited states of nuclei, basing on the concept of self-consistent field.

The self-consistent field method in the Bogolubov formulation /1/ is presented below . Equations describing the ground and low-lying states of nuclei are obtained on the basis of this method. We will show by various examples that these equations lead to the well-known results of the microscopic approach to the nuclear structure: secular equations for multipole and spin-multipole forces, equations for pairing vibration frequencies, equations of the finite Fermi system theory.

The total Hamiltonian of the system is taken in a rather general form:

$$H = \sum_{f,f'} T(f,f')\, a_f^+ a_{f'} - \frac{1}{4} \sum_{f_1,f_2,f_1',f_2'} G(f_1 f_2; f_2' f_1')\, a_{f_1}^+ a_{f_2}^+ a_{f_2'} a_{f_1'}$$

$$T(f,f') = I(f,f') - \lambda\, \delta_{ff'} , \qquad (1)$$

where $f$ is the set of quantum numbers describing a single particle state, $a_f^+$ , $a_f$ are the creation and annihilation fermion operators, $\lambda$ is the chemical potential, $I$ is the single particle Hamiltonian, $G$ is the interaction matrix.

From the anticommutation properties of the operators $a_f^+$, $a_f$ and the hermiticity of the Hamiltonian it follows that

$$I(f,f') = I^*(f',f)$$

$$G(f_1 f_2; f_2' f_1') = -G(f_2 f_1; f_2' f_1') = -G(f_1 f_2; f_1' f_2') = \qquad (2)$$

$$= G(f_2 f_1; f_1' f_2') = G^*(f_1' f_2'; f_2 f_1) .$$

Further we shall use also the representation $f = q, \sigma$ where $\sigma = \pm 1$ distinguish the states conjugated under time reversal

$$\hat{T}\, a_{q\sigma}^+\, \hat{T}^{-1} = S_\sigma\, a_{q-\sigma} . \qquad (3)$$

Here $\hat{T}$ is the time reversal operator and the coefficients $S_\sigma$ possess the following properties:

$$S_\sigma S_{-\sigma} = -1 , \quad S_\sigma^2 = 1 . \qquad (4)$$



Using eq. (3) it may be shown that from the invariance of the Hamiltonian under time reversal it follows

$$I(q\sigma, q'\sigma') = I^*(q-\sigma, q'-\sigma')\, S_\sigma S_{\sigma'} ,$$

$$G(q_1\sigma_1, q_2\sigma_2; q_2'\sigma_2', q_1'\sigma_1') = G^*(q_1-\sigma_1, q_2-\sigma_2; q_2'-\sigma_2', q_1'-\sigma_1') \times \qquad (5)$$

$$\times S_{\sigma_1} S_{\sigma_2} S_{\sigma_1'} S_{\sigma_2'} .$$

Let us introduce the functions

$$F(f_1 f_2) = \langle a_{f_1}^+ a_{f_2} \rangle , \quad \Phi(f_1 f_2) = \langle a_{f_1} a_{f_2} \rangle . \qquad (6)$$

The averaging is performed over the ground state of the system. We note that the equations of motion give the following exact relations for the functions $F$ , $\Phi$ :

$$i \frac{\partial}{\partial t} F(f_1 f_2) = \langle [ a_{f_1}^+ a_{f_2} , H ] \rangle \equiv \mathcal{B}(f_1 f_2) ,$$

$$\qquad (7)$$

$$i \frac{\partial}{\partial t} \Phi(f_1 f_2) = \langle [ a_{f_1} a_{f_2} , H ] \rangle \equiv \mathcal{U}(f_1 f_2) .$$

In the self-consistent field method $\mathcal{U}(f_1 f_2)$ and $\mathcal{B}(f_1 f_2)$ may be expressed in terms of the functions $F(f_1 f_2)$ , $\Phi(f_1 f_2)$ /1,2/.

$$\mathcal{M}(f_1 f_2) = \sum_f \{ \xi(f_1 f)\, \Phi(f f_2) + \xi(f_2 f)\, \Phi(f_1 f) \} -$$

$$- \frac{1}{2} \sum_{f f_1' f_2'} \Phi(f_2' f_1') \{ G(f_1 f; f_2' f_1')\, F(f f_2) +$$

$$+ G(f f_2; f_2' f_1')\, F(f f_1) \} + \frac{1}{2} \sum_{f_1' f_2'} \Phi(f_2' f_1')\, G(f_1 f_2; f_2' f_1'), \tag{8}$$

$$\mathcal{B}(f_1 f_2) = \sum_f \{ \xi(f_2 f)\, F(f_1 f) - \xi(f f_1)\, F(f f_2) \} +$$

$$+ \frac{1}{2} \sum_{f f_1' f_2'} \{ \Phi^*(f_1 f)\, G(f_2 f; f_2' f_1')\, \Phi(f_2' f_1') -$$

$$- \Phi(f_2 f)\, G(f_1 f; f_2' f_1')\, \Phi^*(f_2' f_1') \}, \tag{9}$$

where

$$\xi(f f') = T(f f') - \sum_{f_1 f_2} G(f f_1; f_2 f')\, F(f_1 f_2). \tag{10}$$

The functions $F$, $\Phi$ are not independent but connected with each other by the relations:

$$F(f_1 f_2) = \sum_f \{ F(f_1 f)\, F(f f_2) + \Phi^*(f f_1)\, \Phi(f f_2) \},$$

$$0 = \sum_f \{ F(f_1 f)\, \Phi(f f_2) + F(f_2 f)\, \Phi(f f_1) \}. \tag{11}$$



If we are interested in the time independent ground state we should solve instead of eqs. (3) the following equations

$$\mathcal{M}(f_1 f_2) = 0, \qquad \dot{\mathcal{B}}(f_1 f_2) = 0. \tag{12}$$

The solutions for eqs. (12) are then denoted by $F_0$ and $\Phi_0$. The same results may be obtained by assuming that the ground state of the system is the vacuum of quasiparticles connected with the usual Fermion operators by the Bogolubov general transformation

$$a_f = \sum_\nu \left( u(f\nu)\, \alpha_\nu + v(f\nu)\, \alpha_\nu^+ \right) \tag{13}$$

the coefficients of which obey the orthonormalization relations

$$\sum_\nu \left( u(f\nu)\, u^*(f'\nu) + v(f\nu)\, v(f'\nu) \right) = \delta_{ff'}$$

$$\sum_\nu \left( u(f\nu)\, v(f'\nu) + u(f'\nu)\, v(f\nu) \right) = 0. \tag{14}$$

If we are interested in the spectrum of elementary excitations due to small deviations from the ground state then we should consider small additions to the functions $F_0$ and $\Phi_0$:

$$F(ff') = F_0(ff') + \delta F(ff')$$

$$\Phi(ff') = \Phi_0(ff') + \delta \Phi(ff'), \ldots \tag{15}$$

The equations for $\delta F$ and $\delta\Phi$ can be derived from eqs. (7):

$$i\frac{\partial}{\partial t}\,\delta F(ff') = \delta\mathcal{B}(ff'),$$
$$i\frac{\partial}{\partial t}\,\delta\Phi(ff') = \delta\mathcal{M}(ff'). \tag{16}$$

In addition, $\delta F$ and $\delta\Phi$ are not independent, but they are related to each other by subsidiary relations following from eq. (11):

$$\delta\Big\{F(f_1f_2) - \sum_f F(f_1f)\,F(ff_2) - \sum_f \Phi^*(ff_1)\,\Phi(ff_2)\Big\} = 0$$

$$\delta\Big\{\sum_f F(f_1f)\,\Phi(ff_2) + \sum_f F(f_2f)\,\Phi(ff_1)\Big\} = 0. \tag{17}$$

Due to the connection between $\delta F$ and $\delta\Phi$ it is more convenient to express them via new independent unknown functions which satisfy automatically eqs. (17).

Employing the canonical transformation (13) we write $F$ and $\Phi$ in the form:

$$F(f_1f_2) = \langle a^+_{f_1} a_{f_2}\rangle = \sum_g v^*(f_1g)\,v(f_2g) +$$
$$+\sum_{g_1g_2}\Big\{u^*(f_1g_1)\,u(f_2g_2)\langle\alpha^+_{g_1}\alpha_{g_2}\rangle + v^*(f_1g_1)\,u(f_2g_2)\langle\alpha_{g_1}\alpha_{g_2}\rangle - \tag{18}$$
$$- v^*(f_1g_1)\,v(f_2g_2)\langle\alpha^+_{g_2}\alpha_{g_1}\rangle + u^*(f_1g_1)\,v(f_2g_2)\langle\alpha^+_{g_1}\alpha^+_{g_2}\rangle\Big\},$$



$$F_0(f_1f_2) = \sum_g v^*(f_1g)\,v(f_2g), \tag{19}$$

$$\Phi(f_1f_2) = \langle a_{f_1}a_{f_2}\rangle = \sum u(f_1g)\,v(f_2g) +$$
$$+\sum_{g_1g_2}\Big\{v(f_1g_1)\,u(f_2g_2)\langle\alpha^+_{g_1}\alpha_{g_2}\rangle - u(f_1g_1)\,v(f_2g_2)\langle\alpha^+_{g_2}\alpha_{g_1}\rangle + \tag{20}$$
$$+ u(f_1g_1)\,u(f_2g_2)\langle\alpha_{g_1}\alpha_{g_2}\rangle + v(f_1g_1)\,v(f_2g_2)\langle\alpha^+_{g_1}\alpha^+_{g_2}\rangle\Big\},$$

$$\Phi_0(f_1f_2) = \sum_g u(f_1g)\,v(f_2g). \tag{21}$$

In the method of time-independent self-consistent field the wave function of the ground state is not a quasi-particle vacuum. However, the average number of quasi-particles in the ground state is small, therefore it is assumed to be equal to zero:

$$\langle\alpha^+_g\alpha_{g'}\rangle = 0. \tag{22}$$

To characterize the deviation of the wave function from the quasi-particle vacuum we introduce the coefficients

$$\mu(gg') = \langle\alpha_g\alpha_{g'}\rangle, \tag{23}$$

satisfying the condition

$$\mu(gg') = -\mu(g'g). \tag{24}$$

We express $\delta F$ and $\delta \Phi$ through $\mu$ and $\mu^*$ :

$$\delta F(f_1 f_2) = \sum_{g_1 g_2} \left\{ v^*(f_1 g_1)\, u(f_2 g_2)\, \mu(g_1 g_2) + u^*(f_1 g_1)\, v(f_2 g_2)\, \mu^*(g_2 g_1) \right\} \tag{25}$$

$$\delta \Phi(f_1 f_2) = \sum_{g_1 g_2} \left\{ u(f_1 g_1)\, u(f_2 g_2)\, \mu(g_1 g_2) + v(f_1 g_1)\, v(f_2 g_2)\, \mu^*(g_2 g_1) \right\}. \tag{26}$$

To obtain equations for $\mu$ we write $\mu$ in terms of $\delta F$ and $\delta \Phi$. To this end we multiply eq. (25) by $v(f_1 g)$ and eq. (26) by $u^*(f_1 g)$ add to each other and take the sum over $f_1$. Using the orthonormalization conditions (14) we get

$$\begin{aligned} &\sum_{f_1} \left\{ v(f_1 g)\, \delta F(f_1 f_2) + u^*(f_1 g)\, \delta \Phi(f_1 f_2) \right\} = \\ &= \sum_{f_1 g_1 g_2} \Big\{ \left[ v(f_1 g)\, v^*(f_1 g_1) + u^*(f_1 g)\, u(f_1 g_1) \right] u(f_2 g_2)\, \mu(g_1 g_2) + \\ &\quad + \left[ v(f_1 g)\, u^*(f_1 g_1) + u^*(f_1 g)\, v(f_1 g_1) \right] v(f_2 g_2)\, \mu^*(g_2 g_1) \Big\} = \\ &= \sum_{g_2} u(f_2 g_2)\, \mu(g g_2). \end{aligned} \tag{27}$$

In a similar way we multiply eq. (25) by $u(f_1 g)$ and eq. (26) by $v^*(f_1 g)$, add to each other, take the sum over $f_1$ and using eq. (14) we get

$$\sum_{f_1} \left\{ u^*(f_1 g)\, \delta F^*(f_1 f_2) + v(f_1 g)\, \delta \Phi^*(f_1 f_2) \right\} = -\sum_{g_2} v^*(f_2 g_2)\, \mu(g g_2). \tag{28}$$



Let us use this procedure once more: multiply eq. (27) by $u^*(f_2 g')$ and eq. (28) by $v(f_2 g')$, subtract eq. (28) from eq. (27) and take the sum over $f_2$. Then

$$\begin{aligned} \mu(g g') = \sum_{f_1 f_2} \Big\{ & v(f_1 g)\, u^*(f_2 g')\, \delta F(f_1 f_2) + u^*(f_1 g)\, u^*(f_2 g')\, \delta \Phi(f_1 f_2) - \\ & - u^*(f_1 g)\, v(f_2 g')\, \delta F^*(f_1 f_2) - v(f_1 g)\, v(f_2 g')\, \delta \Phi^*(f_1 f_2) \Big\}. \end{aligned}$$

Differentiating this expression with respect to $t$ and taking into account eq. (16) we obtain the equation for $\mu$ :

$$\begin{aligned} i \frac{\partial}{\partial t} \mu(g_1 g_2) = \sum_{f_1 f_2} \Big\{ & u^*(f_1 g_1)\, u^*(f_2 g_2)\, \delta \mathcal{M}(f_1 f_2) + \\ & + v(f_1 g_1)\, v(f_2 g_2)\, \delta \mathcal{M}^*(f_1 f_2) + v(f_1 g_1)\, u^*(f_2 g_2)\, \delta \mathcal{B}(f_1 f_2) + \\ & + u^*(f_1 g_1)\, v(f_2 g_2)\, \delta \mathcal{B}(f_1 f_2) \Big\}. \end{aligned} \tag{29}$$

We get the equations for $\mu$ in an explicit form. To this end we express $\delta \mathcal{M}$ and $\delta \mathcal{B}$ in terms of $\mu$. From eqs.(8) and (9) we find

$$\begin{aligned} \delta \mathcal{M}(f_1 f_2) = \sum_{f} \Big\{ & \delta \xi(f_1 f)\, \Phi_0(f f_2) + \xi_0(f_1 f)\, \delta \Phi(f f_2) + \\ & + \delta \Phi(f_1 f)\, \xi_0(f_2 f) + \Phi_0(f_1 f)\, \delta \xi(f_2 f) \Big\} - \frac{1}{2} \sum_{f f_1' f_2'} \delta \Phi(f_2' f_1') \times \\ \times \Big\{ & G(f_1 f; f_2' f_1')\, F_0(f f_2) + G(f f_2; f_2' f_1')\, F_0(f f_1) \Big\} - \\ - \frac{1}{2} & \sum_{f f_1' f_2'} \Phi_0(f_2' f_1') \Big\{ G(f_1 f; f_2' f_1')\, \delta F(f f_2) + G(f f_2; f_2' f_1')\, \delta F(f f_1) \Big\} + \\ + \frac{1}{2} & \sum_{f_1' f_2'} \delta \Phi(f_2' f_1')\, G(f_1 f_2; f_2' f_1') \,, \end{aligned} \tag{30}$$

$$\delta\mathcal{B}(f_1 f_2)=\sum_{f}\Big\{\delta\xi(f_2 f)\,F_0(f_1 f)+\xi_0(f_2 f)\,\delta F(f_1 f)-$$
$$-\delta\xi(f f_1)\,F_0(f f_2)-\xi_0(f f_1)\,\delta F(f f_2)\Big\}+\frac{1}{2}\sum_{f f_1' f_2'}G(f_2 f;f_2' f_1')\times$$
$$\times\Big\{\delta\Phi^*(f_1 f)\Phi_0(f_2' f_1')+\Phi_0^*(f_1 f)\,\delta\Phi(f_2' f_1')\Big\}-\frac{1}{2}\sum_{f f_1' f_2'}G(f_1 f;f_2' f_1')\times \tag{31}$$
$$\times\Big\{\delta\Phi(f_2 f)\,\Phi_0^*(f_2' f_1')+\Phi_0(f_2 f)\,\delta\Phi^*(f_2' f_1')\Big\};$$

where
$$\delta\xi(f f')=-\sum_{f_1' f_2'}G(f f_1';f_2' f')\,\delta F(f_1' f_2'),$$
$$\xi_0(f f')=T(f f')-\sum_{f_1' f_2'}G(f f_1';f_2' f')\,F_0(f_1' f_2').$$

We insert eqs. (30) and (31) in eq. (29) and utilize eqs. (25) and (26). Grouping the terms of similar structure and using the orthonormalization relations, after long calculations we get the following equations:

$$i\frac{\partial}{\partial t}M(g_1 g_2)=\sum_{g'}\Big(\Omega(g_2 g')M(g_1 g')-\Omega(g_1 g')M(g_2 g')\Big)+$$
$$+\sum_{g_1' g_2'}\Big\{X(g_1 g_2;g_1' g_2')M(g_1' g_2')+Y(g_1 g_2;g_1' g_2')M^*(g_2' g_1')\Big\}, \tag{32}$$

$$-i\frac{\partial}{\partial t}M^*(g_1 g_2)=\sum_{g'}\Big\{\Omega^*(g_2 g')M^*(g_1 g')-\Omega^*(g_1 g')M^*(g_2 g')\Big\}$$
$$+\sum_{g_1' g_2'}\Big\{X^*(g_1 g_2;g_1' g_2')M^*(g_1' g_2')+Y^*(g_1 g_2;g_1' g_2')M(g_2' g_1')\Big\}, \tag{33}$$



$$\Omega(g g')=\sum_{f f'}\xi_0(f f')\Big\{\mathcal{U}^*(f g)\,\mathcal{U}(f' g')-\mathcal{V}^*(f g)\,\mathcal{V}(f' g')\Big\}- \tag{34}$$
$$-\sum_{f_1 f_2}\Big\{C^{\circ}_{f_1 f_2}\,\mathcal{U}^*(f_1 g)\,\mathcal{V}^*(f_2 g')+C^{\circ *}_{f_1 f_2}\,\mathcal{V}(f_2 g)\,\mathcal{U}(f_1 g')\Big\},$$

where
$$C^{\circ}_{f_1 f_2}=\frac{1}{2}\sum_{f f'}G(f_1 f_2;f' f)\,\Phi_0(f' f),$$

$$X(g_1 g_2;g_1' g_2')=-\frac{1}{2}\sum_{f_1 f_2 f_1' f_2'}G(f_1 f_2;f_2' f_1')\Big\{\mathcal{U}(f_1 g_2)\mathcal{U}(f_2 g_1)\mathcal{U}(f_1' g_2')\mathcal{U}(f_2' g_1')+$$
$$+\mathcal{V}(f_1' g_1)\mathcal{V}(f_2' g_2)\mathcal{V}(f_1 g_1')\mathcal{V}(f_2 g_2')+\Big(\mathcal{V}(f_1' g_1)\,\mathcal{U}(f_1 g_2)-\mathcal{U}(f_1 g_1)\,\mathcal{V}(f_1' g_2)\Big)\times \tag{35}$$
$$\times\Big(\mathcal{V}(f_2 g_1')\,\mathcal{U}(f_2' g_2')-\mathcal{V}(f_2 g_2')\,\mathcal{U}(f_2' g_1')\Big)\Big\},$$

$$Y(g_1 g_2;g_1' g_2')=-\frac{1}{2}\sum_{f_1 f_2 f_1' f_2'}G(f_1 f_2;f_2' f_1')\Big\{\mathcal{U}(f_2 g_1)\mathcal{U}(f_1 g_2)\mathcal{V}(f_2' g_1')\mathcal{V}(f_1' g_2')+$$
$$+\mathcal{V}(f_1' g_1)\mathcal{V}(f_2' g_2)\mathcal{U}(f_2 g_2')\mathcal{U}(f_1 g_1')+\Big(\mathcal{V}(f_1' g_1)\,\mathcal{U}(f_1 g_2)- \tag{36}$$
$$-\mathcal{V}(g_2 f_1')\,\mathcal{U}(f_1 g_1)\Big)\Big(\mathcal{U}(f_2 g_1')\,\mathcal{V}(f_2' g_2')-\mathcal{U}(f_2 g_2')\,\mathcal{V}(f_1' g_1')\Big)\Big\}.$$

The solutions for the homogeneous equations (32) and (33) are sought in the form:

$$\mu(g_1 g_2) = \sum_{\omega} e^{-i\omega t} \psi_\omega(g_1 g_2),$$
$$\mu^*(g_1 g_2) = \sum_{\omega} e^{-i\omega t} \varphi_\omega(g_1 g_2), \tag{37}$$

where $\varphi_\omega = \psi^*_{-\omega}$.

The functions $\psi_\omega$ and $\varphi_\omega$ have the properties:

$$\psi_\omega(g_1 g_2) = -\psi_\omega(g_2 g_1), \quad \varphi_\omega(g_1 g_2) = -\varphi_\omega(g_2 g_1). \tag{38}$$

We substitute eq. (37) in eqs. (32) and (33) and obtain the equations for the determination of the elementary excitation spectrum

$$\omega \psi_\omega(g_1 g_2) = \sum_{g'} \left\{ \Omega(g_2 g') \psi_\omega(g_1 g') - \Omega(g_1 g') \psi_\omega(g_2 g') \right\} + \tag{39}$$
$$+ \sum_{g_1' g_2'} \left\{ X(g_1 g_2; g_1' g_2') \psi_\omega(g_1' g_2') - Y(g_1 g_2; g_1' g_2') \varphi_\omega(g_1' g_2') \right\},$$

$$-\omega \varphi_\omega(g_1 g_2) = \sum_{g'} \left\{ \Omega^*(g_2 g') \varphi_\omega(g_1 g') - \Omega^*(g_1 g') \varphi_\omega(g_2 g') \right\} + \tag{40}$$
$$+ \sum_{g_1' g_2'} \left\{ X^*(g_1 g_2; g_1' g_2') \varphi_\omega(g_1' g_2') - Y(g_1 g_2; g_1' g_2') \psi_\omega(g_1' g_2') \right\}.$$

These equations were first derived in ref.[1]. Note that if $\psi_\omega$, $\varphi_\omega$ and $\omega$ are the solutions for eqs. (39) and (40) then the transformation



$$\omega \to -\omega, \quad \psi_\omega \to \varphi^*_\omega, \quad \varphi_\omega \to \psi^*_\omega$$

leads again to the solution of the same system.

Eqs. (39) and (40) are derived without any assumptions on the character of the particle interaction and on the structure of the ground state. In what follows we shall assume that the functions $I(ff')$, $G(ff'; g'g)$ are real and the functions $\mathcal{U}(fg)$, $\mathcal{V}(fg)$, $\xi_0(ff')$ and $C^0_{ff'}$ are real and diagonal:

$$\mathcal{U}(fg) = \mathcal{U}_f \delta_{fg}, \quad \mathcal{U}_f \equiv \mathcal{U}_{q\sigma} = \mathcal{U}_q, \quad \mathcal{U}^*_q = \mathcal{U}_q,$$
$$\mathcal{V}(fg) = \mathcal{V}_f \delta_{-fg}, \quad \mathcal{V}_f \equiv \mathcal{V}_{q\sigma} = S_\sigma \mathcal{V}_q, \quad \mathcal{V}^*_q = \mathcal{V}_q, \tag{41}$$

$$\xi_0(ff') = \xi(f) \delta_{ff'}, \quad \xi(f) = \xi^*(f), \tag{42}$$

$$C^0_{ff'} = C^{0*}_{ff'} = C_f \delta_{-ff'},$$
$$C_f = \frac{1}{2} \sum_{f'} G(f, -f; -f', f') \mathcal{U}_{f'} \mathcal{V}_{f'}. \tag{43}$$

Then

$$\Omega(gg') = \delta_{gg'} \left\{ \xi(g)(\mathcal{U}_g^2 - \mathcal{V}_g^2) + 2 C_g \mathcal{U}_g \mathcal{V}_g \right\}.$$

Using the results of the superfluid nuclear model we get

$$\Omega(gg') = \delta_{gg'}\,\mathcal{E}(g) \quad , \quad \mathcal{E}(g) = \sqrt{C_g^2 + \xi^2(g)} \; . \tag{44}$$

In the same approximation

$$\begin{aligned} X(g_1g_2;g_1'g_2') = &-\tfrac{1}{2}G(g_1g_2;g_2'g_1')\,u_{g_1}u_{g_2}u_{g_2'}u_{g_1'} - \\ &-\tfrac{1}{2}G(-g_1-g_2;-g_2'-g_1')\,v_{g_1}v_{g_2}v_{g_1'}v_{g_2'} - \tfrac{1}{2}G(g_1-g_2';g_1'-g_2)\,u_{g_1}v_{g_2}u_{g_1'}v_{g_2'} - \\ &-\tfrac{1}{2}G(-g_1g_2';-g_1'g_2)\,v_{g_1}u_{g_2}v_{g_1'}u_{g_2'} + \tfrac{1}{2}G(g_1-g_1';g_2'-g_2)\,u_{g_1}v_{g_2}u_{g_2'}v_{g_1'} + \\ &+\tfrac{1}{2}G(-g_1g_1';-g_2'g_2)\,v_{g_1}u_{g_2}v_{g_2'}u_{g_1'} \; , \end{aligned} \tag{45}$$

$$\begin{aligned} Y(g_1g_2;-g_1'-g_2') = &-\tfrac{1}{2}G(g_1g_2;g_2'g_1')\,u_{g_1}u_{g_2}v_{g_1'}v_{g_2'} - \\ &-\tfrac{1}{2}G(-g_1-g_2;-g_2'-g_1')\,v_{g_1}v_{g_2}u_{g_1'}u_{g_2'} - \tfrac{1}{2}G(g_1-g_2';g_1'-g_2)\,u_{g_1}v_{g_2}u_{g_2'}v_{g_1'} + \\ &+\tfrac{1}{2}G(-g_1g_2';-g_1'g_2)\,v_{g_1}u_{g_2}u_{g_1'}v_{g_2'} - \tfrac{1}{2}G(g_1-g_1';g_2'-g_2)\,u_{g_1}v_{g_2}u_{g_1'}v_{g_2'} - \\ &-\tfrac{1}{2}G(-g_1g_1';-g_2'g_2)\,v_{g_1}u_{g_2}v_{g_1'}u_{g_2'} \; , \end{aligned} \tag{46}$$

$$X^* = X \; , \quad Y^* = Y \; .$$



After inserting eq. (44), eqs. (39) and (40) take on the form

$$\begin{aligned} \omega\,\psi_\omega(g_1g_2) = &\left(\mathcal{E}(g_1) + \mathcal{E}(g_2)\right)\psi_\omega(+g_1,+g_2) + \\ &+\sum_{g_1'g_2'}\left\{X(g_1g_2;g_1'g_2')\,\psi_\omega(g_1'g_2') - Y(g_1g_2;-g_1'-g_2')\,\varphi_\omega(-g_1'-g_2')\right\} , \end{aligned} \tag{47}$$

$$\begin{aligned} -\omega\,\varphi_\omega(-g_1-g_2) = &\left(\mathcal{E}(g_1) + \mathcal{E}(g_2)\right)\varphi_\omega(-g_1-g_2) + \\ &+\sum_{g_1'g_2'}\left\{X(-g_1-g_2;-g_1'-g_2')\,\varphi_\omega(-g_1'-g_2') - Y(-g_1-g_2;g_1'g_2')\,\psi_\omega(g_1'g_2')\right\} . \end{aligned} \tag{48}$$

Let us now introduce new functions

$$Z^{(\pm)}(g_1g_2) = \tfrac{1}{2}\left\{\psi_\omega(g_1g_2) \pm \varphi_\omega(-g_1-g_2)\right\} . \tag{49}$$

the equations for which are of the form

$$\begin{aligned} \omega\,Z^{(\mp)}(g_1g_2) = &\left(\mathcal{E}(g_1) + \mathcal{E}(g_2)\right)Z^{(\pm)}(g_1g_2) + \\ &+\tfrac{1}{2}\sum_{g_1'g_2'}\left\{\left[X(g_1g_2;g_1'g_2') + X(-g_1-g_2;-g_1'-g_2')\right] \mp \right. \\ &\left. \mp\left[Y(g_1g_2;-g_1'-g_2') + Y(-g_1-g_2;g_1'g_2')\right]\right\}Z^{(\pm)}(g_1'g_2') + \\ &+\tfrac{1}{2}\sum_{g_1'g_2'}\left\{\left[X(g_1g_2;g_1'g_2') - X(-g_1-g_2;-g_1'-g_2')\right] \pm \right. \\ &\left. \pm\left[Y(g_1g_2;-g_1'-g_2') - Y(-g_1-g_2;g_1'g_2')\right]\right\}Z^{(\mp)}(g_1'g_2') \; . \end{aligned} \tag{50}$$

where

$$\left[X(g_1g_2;g_1'g_2')+X(-g_1-g_2;-g_1'-g_2')\right]\mp\left[Y(g_1g_2;-g_1'-g_2')+Y(-g_1-g_2;g_1'g_2')\right]=$$

$$=-\frac{1}{2}\left[G(g_1g_2;g_2'g_1')+G(-g_1-g_2;-g_2'-g_1')\right]v^{(\pm)}_{g_1g_2}\,v^{(\pm)}_{g_1'g_2'}\;- \tag{51'}$$

$$-\frac{1}{2}\Big\{\left[G(g_1-g_2';g_1'-g_2)\mp G(g_1-g_1';g_2'-g_2)\right]+\left[G(-g_1g_2';-g_1'g_2)\mp\right.$$

$$\left.\mp G(-g_1g_1';-g_2'g_2)\right]\Big\}\,u^{(\pm)}_{g_1g_2}\,u^{(\pm)}_{g_1'g_2'}\;,$$

$$\left[X(g_1g_2;g_1'g_2')-X(-g_1-g_2;-g_1'-g_2')\right]\pm\left[Y(g_1g_2;-g_1'-g_2')-Y(-g_1-g_2;g_1'g_2')\right]=$$

$$=-\frac{1}{2}\left[G(g_1g_2;g_2'g_1')-G(-g_1-g_2;-g_2'-g_1')\right]v^{(\pm)}_{g_1g_2}\,v^{(\mp)}_{g_1'g_2'}\;- \tag{51''}$$

$$-\frac{1}{2}\Big\{\left[G(g_1-g_2';g_1'-g_2)\pm G(g_1-g_1';g_2'-g_2)\right]-$$

$$-\left[G(-g_1g_2';-g_1'g_2)\pm G(-g_1g_1';-g_2'g_2)\right]\Big\}\,u^{(\pm)}_{g_1g_2}\,u^{(\mp)}_{g_1'g_2'}\;,$$

$$u^{(\pm)}_{gg'}=u_g v_{g'}\pm u_{g'}v_g\;,\quad v^{(\pm)}_{gg'}=u_g u_{g'}\mp v_g v_{g'}. \tag{52}$$



Thus the basic equation way be written in the form:

$$\omega\, Z^{(\mp)}(g_1g_2)=\left(\varepsilon(g_1)+\varepsilon(g_2)\right)Z^{(\pm)}(g_1g_2)-\frac{1}{4}\sum_{g_1'g_2'}\left[G(g_1g_2;g_2'g_1')+\right.$$
$$\left.+G(-g_1-g_2;-g_2'-g_1')\right]v^{(\pm)}_{g_1g_2}\,v^{(\pm)}_{g_1'g_2'}\,Z^{(\pm)}(g_1'g_2')-\frac{1}{4}\sum_{g_1'g_2'}\Big\{\left[G(g_1-g_2';g_1'-g_2)\mp\right.$$
$$\left.\mp G(g_1-g_1';g_2'-g_2)\right]+\left[G(-g_1g_2';-g_1'g_2)\mp G(-g_1g_1';-g_2'g_2)\right]\Big\}\,u^{(\pm)}_{g_1g_2}\times$$
$$\times u^{(\pm)}_{g_1'g_2'}\,Z^{(\pm)}(g_1'g_2')-\frac{1}{4}\sum_{g_1'g_2'}\left[G(g_1g_2;g_2'g_1')-G(-g_1-g_2;-g_2'-g_1')\right]\times \tag{53}$$
$$\times v^{(\pm)}_{g_1g_2}\,v^{(\pm)}_{g_1'g_2'}\,Z^{(\mp)}(g_1'g_2')-\frac{1}{4}\sum_{g_1'g_2'}\Big\{\left[G(g_1-g_2';g_1'-g_2)\pm\right.$$
$$\left.\pm G(g_1-g_1';g_2'-g_2)\right]-\left[G(-g_1g_2';-g_1'g_2)\pm G(-g_1g_1';-g_2'g_2')\right]u^{(\pm)}_{g_1g_2}\times$$
$$\times u^{(\mp)}_{g_1'g_2'}\,Z^{(\mp)}(g_1'g_2')\,.$$

We rewrite eq. (53) in the $q,\sigma$ representation using eqs. (5)

$$\omega\, Z^{(\mp)}(q_1\sigma_1,q_2\sigma_2)=\left(\varepsilon(q_1)+\varepsilon(q_2)\right)Z^{(\pm)}(q_1\sigma_1,q_2\sigma_2)\;-$$
$$-\frac{1}{4}\sum_{q_1'\sigma_1'q_2'\sigma_2'}G(q_1\sigma_1,q_2\sigma_2;q_2'\sigma_2',q_1'\sigma_1')\left[1+S_{\sigma_1}S_{\sigma_2}S_{\sigma_1'}S_{\sigma_2'}\right]v^{(\pm)}_{q_1\sigma_1,q_2\sigma_2}\times$$
$$\times v^{(\pm)}_{q_1'\sigma_1',q_2'\sigma_2'}\,Z^{(\pm)}(q_1'\sigma_1',q_2'\sigma_2')-\frac{1}{4}\sum_{q_1'\sigma_1'q_2'\sigma_2'}G(q_1\sigma_1,q_2\sigma_2;q_2'\sigma_2',q_1'\sigma_1')\times$$
$$\times\left[1-S_{\sigma_1}S_{\sigma_2}S_{\sigma_1'}S_{\sigma_2'}\right]v^{(\pm)}_{q_1\sigma_1,q_2\sigma_2}\,v^{(\mp)}_{q_1'\sigma_1',q_2'\sigma_2'}\,Z^{(\mp)}(q_1'\sigma_1',q_2'\sigma_2')\;- \tag{54}$$
$$-\frac{1}{4}\sum_{q_1'\sigma_1',q_2'\sigma_2'}\left[G(q_1\sigma_1,q_2'-\sigma_2';q_1'\sigma_1',q_2'-\sigma_2)\mp G(q_1\sigma_1,q_1'-\sigma_1';q_2'\sigma_2',q_2-\sigma_2)\right]\times$$
$$\times\left[1+S_{\sigma_1}S_{\sigma_2}S_{\sigma_2'}S_{\sigma_1'}\right]u^{(\pm)}_{q_1\sigma_1,q_2\sigma_2}\,u^{(\pm)}_{q_1'\sigma_1',q_2'\sigma_2'}\,Z^{(\pm)}(q_1'\sigma_1',q_2'\sigma_2')\;-$$

$$-\frac{1}{4}\sum_{q_1'\sigma_1',\, q_2'\sigma_2'} \left[G(q_1\sigma_1, q_2'-\sigma_2'; q_1'\sigma_1', q_2-\sigma_2) \pm G(q_1\sigma_1, q_1'-\sigma_1'; q_2'\sigma_2', q_2-\sigma_2)\right] \times$$

$$\times\, U^{(\pm)}_{q_1\sigma_1, q_2\sigma_2}\, U^{(\mp)}_{q_1'\sigma_1', q_2'\sigma_2'}\, Z^{(\mp)}(q_1'\sigma_1', q_2'\sigma_2').$$

Let us write eq. (54) for the cases $\sigma_1=\sigma_2$ and $\sigma_1=-\sigma_2$ using eqs.(41) and the properties of the coefficients $S_\sigma$. From eq. (41) it follows that

$$U^{(\pm)}_{q\sigma, q'\sigma} = S_\sigma (U_q V_{q'} \pm U_{q'} V_q) = S_\sigma U^{(\pm)}_{qq'},$$

$$U^{(\pm)}_{q\sigma, q'-\sigma} = S_{-\sigma} (U_q V_{q'} \mp U_{q'} V_q) = S_{-\sigma} U^{(\mp)}_{qq'},$$

$$V^{(\pm)}_{q\sigma, q'\sigma} = U_q U_{q'} \mp V_q V_{q'} = V^{(\pm)}_{qq'},$$

$$V^{(\pm)}_{q\sigma, q'-\sigma} = U_q U_{q'} \pm V_q V_{q'} = V^{(\mp)}_{qq'}.$$

Employing these equalities we get:

$$\omega Z^{(\mp)}(q_1\sigma, q_2\sigma) = (\varepsilon(q_1)+\varepsilon(q_2)) Z^{(\pm)}(q_1\sigma, q_2\sigma) - \frac{1}{2}\sum_{q_1'q_2'\sigma'} G(q_1\sigma, q_2\sigma; q_2'\sigma', q_1'\sigma_1') \times$$

$$\times\, V^{(\pm)}_{q_1q_2} V^{(\pm)}_{q_1'q_2'} Z^{(\pm)}(q_1'\sigma', q_2'\sigma') - \frac{1}{2}\sum_{q_1'q_2'\sigma'} G(q_1\sigma, q_2\sigma; q_2'-\sigma', q_1'\sigma') \times$$



$$\begin{aligned}
&\times\, V^{(\pm)}_{q_1q_2} V^{(\pm)}_{q_1'q_2'} Z^{(\mp)}(q_1'\sigma', q_2'-\sigma') - \frac{1}{2}\sum_{q_1'q_2'\sigma'}\left[G(q_1\sigma, q_2'-\sigma'; q_1'\sigma', q_2-\sigma) \mp\right.\\
&\left.\mp\, G(q_1\sigma, q_1'-\sigma'; q_2'\sigma', q_2-\sigma)\right] S_\sigma S_{\sigma'} U^{(\pm)}_{q_1q_2} U^{(\pm)}_{q_1'q_2'} Z^{(\pm)}(q_1'\sigma', q_2'\sigma') - \\
&- \frac{1}{2}\sum_{q_1'q_2'\sigma'}\left[G(q_1\sigma, q_2'\sigma'; q_1'\sigma', q_2-\sigma) \pm G(q_1\sigma, q_1'-\sigma'; q_2'-\sigma', q_2-\sigma)\right] \times \\
&\times\, S_\sigma S_{-\sigma'} U^{(\pm)}_{q_1q_2} U^{(\pm)}_{q_1'q_2'} Z^{(\pm)}(q_1'\sigma', q_2'\sigma'),
\end{aligned} \tag{55}$$

$$\begin{aligned}
&\omega Z^{(\pm)}(q_1\sigma, q_2-\sigma) = (\varepsilon(q_1)+\varepsilon(q_2)) Z^{(\mp)}(q_1\sigma, q_2-\sigma) - \\
&- \frac{1}{2}\sum_{q_1'q_2'\sigma'} G(q_1\sigma, q_2-\sigma; q_2'-\sigma', q_1'\sigma_1') V^{(\pm)}_{q_1q_2} V^{(\pm)}_{q_1'q_2'} Z^{(\mp)}(q_1'\sigma', q_2'-\sigma') - \\
&- \frac{1}{2}\sum_{q_1'q_2'\sigma'} G(q_1\sigma, q_2-\sigma; q_2'\sigma', q_1'\sigma') V^{(\pm)}_{q_1q_2} V^{(\pm)}_{q_1'q_2'} Z^{(\pm)}(q_1'\sigma', q_2'\sigma') - \\
&- \frac{1}{2}\sum_{q_1'q_2'\sigma'}\left[G(q_1\sigma, q_2'\sigma'; q_1'\sigma', q_2\sigma) \pm G(q_1\sigma, q_1'-\sigma'; q_2'-\sigma', q_2\sigma)\right] \times \\
&\times\, S_\sigma S_{\sigma'} U^{(\pm)}_{q_1q_2} U^{(\pm)}_{q_1'q_2'} Z^{(\mp)}(q_1'\sigma', q_2'-\sigma') - \\
&- \frac{1}{2}\sum_{q_1'q_2'\sigma'}\left[G(q_1\sigma, q_2'-\sigma'; q_1'\sigma', q_2\sigma) \pm G(q_1\sigma, q_1'\sigma'; q_2'\sigma', q_2\sigma)\right] \times \\
&\times\, S_\sigma S_{-\sigma'} U^{(\pm)}_{q_1q_2} U^{(\pm)}_{q_1'q_2'} Z^{(\pm)}(q_1'\sigma', q_2'\sigma').
\end{aligned} \tag{56}$$

In deriving eqs. (55) and (56) we have made use of the fact that in eq. (54) in summing over $\sigma_1'$, $\sigma_2'$ half of the terms vanishes owing to the multipliers $(1 \pm S_{\sigma_1} S_{\sigma_2} S_{\sigma_1'} S_{\sigma_2'})$. With the aid of eqs. (4) and (5) it may be shown that the coefficients

$$\sum_\sigma G(q_1\sigma, q_2\sigma; q_2'\sigma') \quad , \quad \sum_\sigma G(q_1\sigma, q_2\sigma; q_2'-\sigma', q_1'\sigma') S_{\sigma'},$$

$$\sum_\sigma G(q_1\sigma, q_2-\sigma; q_2'\sigma', q_1'\sigma') \quad , \quad \sum_\sigma S_\sigma G(q_1\sigma, q_2-\sigma; q_2'-\sigma', q_1'\sigma') S_{\sigma'}, \tag{57}$$

$$\sum_\sigma G(q_1\sigma, q_2'\sigma'; q_1'\sigma', q_2\sigma) \quad , \quad \sum_\sigma G(q_1\sigma, q_2'-\sigma'; q_1'\sigma', q_2\sigma) S_{-\sigma'},$$

$$\sum_\sigma G(q_1\sigma, q_2'\sigma'; q_1'\sigma', q_2-\sigma) S_{-\sigma} \quad , \quad \sum_\sigma S_{-\sigma} G(q_1\sigma, q_2'-\sigma'; q_1'\sigma', q_2-\sigma) S_{-\sigma'}.$$

are independent of $\sigma'$ . For example,

$$\sum_\sigma S_{-\sigma} G(q_1\sigma, q_2'\sigma'; q_1'\sigma', q_2-\sigma) = \sum_\sigma S_{-\sigma} S_\sigma S_{\sigma'} S_{\sigma'} S_{-\sigma} G(q_1-\sigma, q_2'-\sigma', q_1'-\sigma', q_2\sigma) =$$

$$= \sum_\sigma S_\sigma G(q_1-\sigma, q_2'-\sigma'; q_1'-\sigma', q_2\sigma) = \sum_\sigma S_{-\sigma} G(q_1\sigma, q_2'-\sigma'; q_1'\sigma', q_2-\sigma) .$$

Therefore for the coefficients (57) we introduce new notations reflecting this property:

$$\frac{1}{2}\sum_\sigma G(q_1\sigma, q_2\sigma; q_2'\sigma', q_1'\sigma') = G^{\xi}(q_1+, q_2+; q_2'+, q_1'+),$$

$$\frac{1}{2}\sum_\sigma G(q_1\sigma, q_2\sigma; q_2'-\sigma', q_1'\sigma') S_{\sigma'} = G^{\xi}(q_1+, q_2+; q_2'-, q_1'+),$$

$$\frac{1}{2}\sum_\sigma S_\sigma G(q_1\sigma, q_2-\sigma; q_2'\sigma', q_1'\sigma') = G^{\xi}(q_1+, q_2-; q_2'+, q_1'+), \tag{58}$$

$$\frac{1}{2}\sum_\sigma S_\sigma G(q_1\sigma, q_2-\sigma; q_2'-\sigma', q_1'\sigma') S_{\sigma'} = G^{\xi}(q_1+, q_2-; q_2'-, q_1'+),$$



$$\frac{1}{2}\sum_\sigma G(q_1\sigma, q_2'\sigma'; q_1'\sigma', q_2\sigma) = G^{\omega}(q_1+, q_2+; q_1'+, q_2'+),$$

$$\frac{1}{2}\sum_\sigma G(q_1\sigma, q_2'-\sigma'; q_1'\sigma', q_2\sigma) S_{-\sigma'} = G^{\omega}(q_1+, q_2+; q_2'-, q_1'+),$$

$$\frac{1}{2}\sum_\sigma G(q_1\sigma, q_2'\sigma'; q_1'\sigma', q_2-\sigma) S_{-\sigma} = G^{\omega}(q_1+, q_2-; q_2'+, q_1'+), \tag{58}$$

$$\frac{1}{2}\sum_\sigma S_{-\sigma} G(q_1\sigma, q_2'-\sigma'; q_1'\sigma', q_2-\sigma) S_{-\sigma'} = G^{\omega}(q_1+, q_2-; q_2'-, q_1'+) .$$

We sum up eq. (55) over $\sigma$ and multiply eq. (56) by $S_\sigma$ and also sum it over $\sigma$ . As a result we get:

$$\omega \sum_\sigma \overset{(\mp)}{Z}(q_1\sigma, q_2\sigma) = (\varepsilon(q_1) + \varepsilon(q_2)) \sum_\sigma \overset{(\pm)}{Z}(q_1\sigma, q_2\sigma) -$$

$$- \sum_{q_1'q_2'} G^{\xi}(q_1+ q_2+; q_2'+, q_1'+) \overset{(\pm)}{v}_{q_1q_2} \overset{(\pm)}{v}_{q_1'q_2'} \sum_\sigma \overset{(\pm)}{Z}(q_1'\sigma, q_2'\sigma) -$$

$$- \sum_{q_1'q_2'} G^{\xi}(q_1+ q_2+; q_2'-, q_1'+) \overset{(\pm)}{v}_{q_1q_2} \overset{(\pm)}{v}_{q_1'q_2'} \sum_\sigma S_\sigma \overset{(\mp)}{Z}(q_1'\sigma, q_2'-\sigma) - \tag{59}$$

$$- \sum_{q_1'q_2'} [G^{\omega}(q_1+ q_2-; q_2'-, q_1'+) \mp G^{\omega}(q_1+ q_2-; q_1'-, q_2'+)] \overset{(\pm)}{u}_{q_1q_2} \overset{(\pm)}{u}_{q_1'q_2'} \sum_\sigma \overset{(\pm)}{Z}(q_1'\sigma, q_2'\sigma) -$$

$$-\sum_{q_1' q_2'} \left[G^{\omega}(q_1^+ q_2^-; q_2'^+ q_1'^+) \pm G^{\omega}(q_1^+ q_2^-; q_1'^+ q_2'^+)\right] u^{(\pm)}_{q_1 q_2} u^{(\pm)}_{q_1' q_2'} \sum_\sigma S_\sigma \overset{(\mp)}{Z}(q_1'\sigma, q_2'-\sigma),$$

$$\omega \sum_\sigma S_\sigma \overset{(\pm)}{Z}(q_1\sigma, q_2-\sigma) = \left(\varepsilon(q_1)+\varepsilon(q_2)\right) \sum_\sigma S_\sigma \overset{(\mp)}{Z}(q_1\sigma, q_2-\sigma) -$$

$$-\sum_{q_1' q_2'} G^{\xi}(q_1^+ q_2^-; q_2'^+ q_1'^+)\, v^{(\pm)}_{q_1 q_2} v^{(\pm)}_{q_1' q_2'} \sum_\sigma \overset{(\pm)}{Z}(q_1'\sigma, q_2'\sigma) - \qquad (60)$$

$$-\sum_{q_1' q_2'} G^{\xi}(q_1^+ q_2^-; q_2'^+ q_1'^+)\, v^{(\pm)}_{q_1 q_2} v^{(\pm)}_{q_1' q_2'} \sum_\sigma S_\sigma \overset{(\mp)}{Z}(q_1'\sigma, q_2'-\sigma') -$$

$$-\sum_{q_1' q_2'} \left[G^{\omega}(q_1^+ q_2^+; q_2'^+ q_1'^+) \pm G^{\omega}(q_1^+ q_2^+; q_1'^+ q_2'^+)\right] u^{(\pm)}_{q_1 q_2} u^{(\pm)}_{q_1' q_2'} \sum_\sigma S_\sigma \overset{(\mp)}{Z}(q_1'\sigma, q_2'-\sigma') -$$

$$-\sum_{q_1' q_2'} \left[G^{\omega}(q_1^+ q_2^+; q_2'^- q_1'^+) \mp G^{\omega}(q_1^+ q_2^+; q_1'^- q_2'^+)\right] u^{(\pm)}_{q_1 q_2} u^{(\pm)}_{q_1' q_2'} \sum_\sigma \overset{(\pm)}{Z}(q_1'\sigma, q_2'\sigma).$$

Since the coefficients $\sum_\sigma \overset{(\pm)}{Z}(q\sigma, q'\sigma)$ are antisymmetrical and the coefficients $\sum_\sigma S_\sigma \overset{(\pm)}{Z}(q\sigma, q'-\sigma)$ symmetrical with respect to permutation of the indices $q, q'$ and for each excitation mode only one of the coefficients $\sum_\sigma \overset{(\pm)}{Z}(q\sigma, q'\sigma)$ and $\sum_\sigma S_\sigma \overset{(\pm)}{Z}(q\sigma, q'-\sigma)$ differs from zero for fixed $q\, q'$ then eqs. (59) and (60) can be essentially simplified. Firstly, the terms containing $\pm G^{\omega}$ may be excluded by using the symmetry properties of the coefficients. Secondly, the unknown $\sum_\sigma \overset{(\pm)}{Z}(q\sigma, q'\sigma)$ and $\sum_\sigma S_\sigma \overset{(\pm)}{Z}(q\sigma, q'-\sigma)$ under the sign of summation over $q, q'$ can be replaced by their sum since for each set of $q, q'$ only one of the terms will differ from zero.



In this case, instead of eqs. (59), (60), we have

$$\omega \sum_\sigma \overset{(\mp)}{Z}(q_1\sigma, q_2\sigma) = \left(\varepsilon(q_1)+\varepsilon(q_2)\right) \sum_\sigma \overset{(\pm)}{Z}(q_1\sigma, q_2\sigma) -$$

$$-\sum_{q_1' q_2'} \left[G^{\xi}(q_1^+, q_2^+; q_2'^+ q_1'^+) + G^{\xi}(q_1^+ q_2^+; q_2'^- q_1'^+)\right] v_{q_1 q_2} v_{q_1' q_2'} \times$$

$$\times\left(\sum_\sigma \overset{(\pm)}{Z}(q_1'\sigma, q_2'\sigma) + \sum_\sigma S_\sigma \overset{(\mp)}{Z}(q_1'\sigma, q_2'-\sigma)\right) - \qquad (61)$$

$$-2\sum_{q_1' q_2'} \left[G^{\omega}(q_1^+ q_2^-; q_2'^- q_1'^+) + G^{\omega}(q_1^+ q_2^-; q_2'^+ q_1'^+)\right] u^{(\pm)}_{q_1 q_2} u^{(\pm)}_{q_1' q_2'} \times$$

$$\times\left(\sum_\sigma \overset{(\pm)}{Z}(q_1'\sigma, q_2'\sigma) + \sum_\sigma S_\sigma \overset{(\mp)}{Z}(q_1'\sigma, q_2'-\sigma)\right),$$

$$\omega \sum_\sigma \overset{(\pm)}{Z}(q_1\sigma, q_2-\sigma) S_\sigma = \left(\varepsilon(q_1)+\varepsilon(q_2)\right) \sum_\sigma S_\sigma \overset{(\mp)}{Z}(q_1\sigma, q_2-\sigma) -$$

$$-\sum_{q_1' q_2'} \left[G^{\xi}(q_1^+ q_2^-; q_2'^- q_1'^+) + G^{\xi}(q_1^+ q_2^-; q_2'^+ q_1'^+)\right] v^{(\pm)}_{q_1 q_2} v^{(\pm)}_{q_1' q_2'} \times$$

$$\times\left(\sum_\sigma \overset{(\pm)}{Z}(q_1'\sigma, q_2'\sigma) + \sum_\sigma S_\sigma \overset{(\mp)}{Z}(q_1'\sigma, q_2'-\sigma)\right) - \qquad (62)$$

$$-2\sum_{q_1' q_2'} \left[G^{\omega}(q_1^+ q_2^+; q_2'^+ q_1'^+) + G^{\omega}(q_1^+ q_2^+; q_2'^- q_1'^+)\right] u^{(\pm)}_{q_1 q_2} u^{(\pm)}_{q_1' q_2'} \times$$

$$\times\left(\sum_\sigma \overset{(\pm)}{Z}(q_1'\sigma, q_2'\sigma) + \sum_\sigma S_\sigma \overset{(\mp)}{Z}(q_1'\sigma, q_2'-\sigma)\right).$$

Adding eqs. (61) and (62), introducing new variables

$$\overset{(\pm)}{R}(q q') = \sum_{\sigma} \overset{(\pm)}{Z}(q\sigma, q'\sigma) + \sum_{\sigma} S_{\sigma} \overset{(\mp)}{Z}(q\sigma, q'-\sigma),$$

and notations

$$G^{\xi}(q_1 q_2; q_2' q_1') \equiv G^{\xi}(q_1+q_2+; q_2'+q_1'+) + G^{\xi}(q_1+q_2+; q_2'-q_1'+) +$$
$$+ G^{\xi}(q_1+q_2-; q_2'+q_1'+) + G^{\xi}(q_1+q_2-; q_2'-q_1'+),$$

$$G^{\omega}(q_1 q_2; q_2' q_1') \equiv G^{\omega}(q_1+q_2-; q_2'-, q_1'+) + G^{\omega}(q_1+q_2-; q_2'+q_1'+) +$$
$$+ G^{\omega}(q_1+q_2+; q_2'-q_1'+) + G^{\omega}(q_1+q_2+; q_2'+q_1'+).$$

we obtain equations for new variables

$$\omega \overset{(\mp)}{R}(q_1 q_2) = (\varepsilon(q_1)+\varepsilon(q_2)) \overset{(\pm)}{R}(q_1 q_2) -$$
$$- \sum_{q_1' q_2'} G^{\xi}(q_1 q_2; q_2' q_1') \overset{(\pm)}{v}_{q_1 q_2} \overset{(\pm)}{v}_{q_1' q_2'} \overset{(\pm)}{R}(q_1' q_2') - \qquad (63)$$
$$- 2\sum_{q_1' q_2'} G^{\omega}(q_1 q_2; q_2' q_1') \overset{(\pm)}{u}_{q_1 q_2} \overset{(\pm)}{u}_{q_1' q_2'} \overset{(\pm)}{R}(q_1' q_2').$$

The interaction in the particle-particle channel affects the properties of the collective states through the terms of eq. (63) proportional to $\overset{(\pm)}{v}_{q q'}$. The contribution of the interaction in the particle-hole channel is contained in the terms proportional to $\overset{(\pm)}{u}_{q q'}$.



In studying the properties of the low-lying states of nuclei we should bear in mind that the interaction $G(q_1 q_2; q_2' q_1')$ is used for different moment of colliding particles. Some collective effects associated with quadrupole, octupole and others correlations in the particle-hole channel are defined by the interaction with small momentum transfer (we imply here $G^{\omega}(q_1 q_2; q_2' q_1')$ ). The other effects associated with the superconducting pairing correlations are defined by the interaction with the zero total momentum of colliding particles (we imply here $G^{\xi}(q_1 q_2; q_2' q_1')$ ). These both interactions should be considered as independent.

In deriving eqs. (63) the particle interaction was taken in a general form. However, it is known that the appearance of vibrational states in nulcei is mainly due to the interaction in the particle-hole channel which gives the coherent contribution. Therefore as an example we consider the case when the effect of the interaction in particle-particle channel on the vibrational state properties may be neglected. The interaction in the particle-hole channel is taken as a sum of the multipole and spin-multipole interactions

$$G^{\omega}(q_1 q_2; q_2' q_1') = \varkappa_f f(q_1 q_2) f(q_1' q_2') + \varkappa_t t(q_1 q_2) t(q_1' q_2'), \qquad (64)$$

where $f, t$ are the single-particle matrix elements of the operators of the multipole and spin-multipole momenta respectively.

In this case eqs. (63), taking into account the symmetry properties of the coefficients $\overset{(\pm)}{R}(q_1 q_2)$ take the form

$$\omega R^{(-)}(q_1q_2) = (\varepsilon(q_1)+\varepsilon(q_2))\, R^{(+)}(q_1q_2) -$$
$$-2\varkappa_f\, f(q_1q_2)\, u^{(+)}_{q_1q_2} \sum_{q_1'q_2'} f(q_1'q_2')\, u^{(+)}_{q_1'q_2'}\, R^{(+)}(q_1'q_2'), \tag{65}$$

$$\omega R^{(+)}(q_1q_2) = (\varepsilon(q_1)+\varepsilon(q_2))\, R^{(-)}(q_1q_2) -$$
$$-2\varkappa_t\, t(q_1q_2)\, u^{(-)}_{q_1q_2} \sum_{q_1'q_2'} t(q_1'q_2')\, u^{(-)}_{q_1'q_2'}\, R^{(-)}(q_1'q_2'). \tag{66}$$

Eqs. (65) and (66) do not include the terms containing

$$\sum_{q_1'q_2'} t(q_1'q_2')\, u^{(+)}_{q_1'q_2'}\, R^{(+)}(q_1'q_2'), \quad \sum_{q_1'q_2'} f(q_1'q_2')\, u^{(-)}_{q_1'q_2'}\, R^{(-)}(q_1'q_2').$$

since these sums vanish due to the symmetry properties of the coefficients $f, t, R^{(\pm)}, u^{(\pm)}$ with respect to permutation of indices $q_1'$, $q_2'$. The coefficients $R^{(\pm)}(q_1'q_2')$ are antisymmetrical with respect to permutation of indices if for the mode of excitation considered to given $q_1'$, $q_2'$ there correspond identical $\sigma_1'$ and $\sigma_2'$. The coefficients $R^{(\pm)}(q_1'q_2')$ are symmetrical with respect to permutation of the indices in the opposite case. These symmetry properties are the same those for the coefficients $f(q_1'q_2')$ and opposite to those for the coefficients $t(q_1'q_2')$.

We transform eqs. (65) and (66). To this end we introduce the following notations



$$V^{(\pm)} = 2\begin{Bmatrix}\varkappa_f\\ \varkappa_t\end{Bmatrix} \sum_{qq'} \begin{Bmatrix} f(qq')\\ t(qq')\end{Bmatrix} u^{(\pm)}_{qq'}\, R^{(\pm)}(qq'), \tag{67}$$

and rewrite eqs. (65) and (66) in the form

$$(\varepsilon(q)+\varepsilon(q'))\, R^{(\pm)}(qq') - \omega R^{(\mp)}(qq') = \begin{pmatrix} f(qq')\\ t(qq')\end{pmatrix} u^{(\pm)}_{qq'}\, V^{(\pm)}. \tag{68}$$

Hence, we find

$$R^{(\pm)}(qq') = \frac{\begin{pmatrix} f(qq')\\ t(qq')\end{pmatrix} u^{(\pm)}_{qq'}(\varepsilon(q)+\varepsilon(q'))\, V^{(\pm)} + \begin{pmatrix} t(qq')\\ f(qq')\end{pmatrix} u^{(\mp)}_{qq'}\, \omega\, V^{(\mp)}}{(\varepsilon(q)+\varepsilon(q'))^2 - \omega^2}. \tag{69}$$

Inserting eq. (69) in eq. (67) and putting that the determinant of the system of linear equations derived is zero we get a secular equation for determining the frequencies of collective vibrations:

$$\left(1 - 2\varkappa_f \sum_{qq'} \frac{f^2(qq')\, u^{(+)2}_{qq'}\,(\varepsilon(q)+\varepsilon(q'))}{(\varepsilon(q)+\varepsilon(q))^2 - \omega^2}\right) \times$$

$$\times\left(1-2\varkappa_t\sum_{qq'}\frac{t^2(qq')\,u_{qq'}^{(-)2}\,(\varepsilon(q)+\varepsilon(q'))}{(\varepsilon(q)+\varepsilon(q'))^2-\omega^2}\right)=$$

$$=4\varkappa_f\varkappa_t\left(\sum_{qq'}\frac{f(qq')\,t(qq')\,u_{qq'}^{(+)}\,u_{qq'}^{(-)}\,\omega}{(\varepsilon(q)+\varepsilon(q'))^2-\omega^2}\right)^2 .$$

This equation is studied in ref./3/ when investigating the quadrupole states in deformed nuclei.

Assuming $\varkappa_t=0$ we get the well-known secular equation for the case of the multipole-multipole interaction:

$$1=2\varkappa_f\sum_{qq'}\frac{f^2(qq')\,u_{qq'}^{(+)2}\,(\varepsilon(q)+\varepsilon(q'))}{(\varepsilon(q)+\varepsilon(q'))^2-\omega^2} .$$

The roots of this equation are the energies of the vibrational states. The solutions for the equations of this type are found in studying the vibrational states in spherical/4/ and deformed/5/ nuclei.

In addition to the vibrational levels which are mainly due to the interaction in particle-hole channel, in nuclei there exist collective states the properties of which are mainly due to the interaction in particle-particle channel. An example of such states is pairing vibrations.

To consider the properties of pairing vibrations we put $G^{\omega}(q_1q_2;q_2'q_1')=0$ and

$$G^{\xi}(q_1q_2;q_2'q_1')=G\,\delta_{q_1q_2}\,\delta_{q_1'q_2'} \tag{70}$$



In this case eqs. (63) take on the form

$$\omega R_{(qq)}^{(\mp)}=2\varepsilon(q)R_{(qq)}^{(\pm)}-G\,v_{(qq)}^{(\pm)}\sum_{q'}v_{q'q'}^{(\pm)}R_{(q'q')}^{(\pm)} . \tag{71}$$

We introduce the notations

$$d^{(\pm)}=G\sum_{q}v_{qq}^{(\pm)}R_{(qq)}^{(\pm)} . \tag{72}$$

Then eqs. (71) read

$$2\varepsilon(q)R_{(qq)}^{(\pm)}-\omega R_{(qq)}^{(\mp)}=v_{qq}^{(\pm)}d^{(\pm)} . \tag{73}$$

It follows from (73)

$$R_{(qq)}^{(\pm)}=\frac{2\varepsilon(q)v_{qq}^{(\pm)}d^{(\pm)}+\omega v_{qq}^{(\mp)}d^{(\mp)}}{4\varepsilon^2(q)-\omega^2} . \tag{74}$$

We substitute (74) in (72), put that the determinant of the system of linear equations derived is zero and obtain an equation for determining the frequency of pairing vibrations:

$$\left(\frac{1}{G}-\sum_{q}\frac{2\varepsilon(q)\,v_{qq}^{(+)2}}{4\varepsilon^2(q)-\omega^2}\right)\left(\frac{1}{G}-\sum_{q}\frac{2\varepsilon(q)}{4\varepsilon^2(q)-\omega^2}\right)=$$
$$=\omega^2\left(\sum_{q}\frac{v_{qq}^{(+)}}{4\varepsilon^2(q)-\omega^2}\right)^2 . \tag{75}$$

Such type equations were derived in ref.[6] for spherical nuclei and in ref.[7] for deformed ones. These equations underlie the theory of pairing vibrations[8,9].

Now we consider the more general case and put

$$G^{\omega}(q_1 q_2; q_2' q_1') = \varkappa f(q_1 q_2) f(q_1' q_2')$$

$$G^{\xi}(q_1 q_2; q_2' q_1') = G \delta_{q_1 q_2} \delta_{q_1' q_2'} .$$

Then eqs. ( 63) take on the form:

$$\omega R^{(-)}(q_1 q_2) = (\varepsilon(q_1) + \varepsilon(q_2)) R^{(+)}(q_1 q_2) -$$
$$- \varkappa f(q_1 q_2) u^{(+)}_{q_1 q_2} \sum_{q_1' q_2'} f(q_1' q_2') u^{(+)}_{q_1' q_2'} R^{(+)}(q_1' q_2') -$$
$$- G v^{(+)}_{q_1 q_2} \delta_{q_1 q_2} \sum_{q'} v^{(+)}_{q'q'} R^{(+)}(q'q') , \qquad (76)$$
$$\omega R^{(+)}(q_1 q_2) = (\varepsilon(q_1) + \varepsilon(q_2)) R^{(-)}(q_1 q_2) - G \delta_{q_1 q_2} \sum_{q'} R^{(-)}(q'q') v^{(-)}_{q'q'} .$$

We introduce the notations

$$V^{(+)} = \varkappa \sum_{qq'} f(qq') u^{(+)}_{qq'} R^{(+)}(qq') ,$$
$$d^{(\pm)} = G \sum_{q} v^{(\pm)}_{qq} R^{(\pm)}(qq) . \qquad (77)$$

Then eqs. (76) read



$$(\varepsilon(q) + \varepsilon(q')) R^{(+)}(qq') - \omega R^{(-)}(qq') = f(qq') u^{(+)}_{qq'} V^{(+)} + \delta_{qq'} v^{(+)}_{qq} d^{(+)} ,$$
$$- \omega R^{(+)}(qq') + (\varepsilon(q) + \varepsilon(q')) R^{(-)}(qq') = \delta_{qq'} d^{(-)} . \qquad (78)$$

It follows from (78)

$$R^{(+)}(qq') = \frac{(\varepsilon(q)+\varepsilon(q')) f(qq') u^{(+)}_{qq'} V^{(+)} + 2\varepsilon(q) v^{(+)}_{qq} d^{(+)} \delta_{qq'} + \omega d^{(-)} \delta_{qq'}}{(\varepsilon(q)+\varepsilon(q'))^2 - \omega^2} ,$$
$$(79)$$
$$R^{(-)}(qq') = \frac{\omega f(qq') u^{(+)}_{qq'} V^{(+)} + \omega \delta_{qq'} v^{(+)}_{qq} d^{(+)} + 2\varepsilon(q) \delta_{qq'} d^{(-)}}{(\varepsilon(q)+\varepsilon(q'))^2 - \omega^2} .$$

We substitute (79) in (77), put that the determinant of the system of linear equations derived is zero and obtain an equation for determining the energy of the collective excitation:

$$
\det\begin{vmatrix}
\left(\varkappa\sum\limits_{qq'}\frac{\varepsilon(qq')f^2(qq')\,u_{qq'}^{(+)2}}{\varepsilon^2(qq')-\omega^2}-1\right) & \frac{G}{2}\sum\limits_{q}\frac{f(qq)u_{qq}^{(+)}\omega}{4\varepsilon^2(q)-\omega^2} & \frac{G}{2}\sum\limits_{q}\frac{2\varepsilon(q)u_{qq}^{(+)}v_{qq}^{(+)}f(qq)}{4\varepsilon^2(q)-\omega^2} \\
\varkappa\sum\limits_{q}\frac{\omega u_{qq}^{(+)}f(qq)}{4\varepsilon^2(q)-\omega^2} & \left(\frac{G}{2}\sum\limits_{q}\frac{2\varepsilon(q)}{4\varepsilon^2(q)-\omega^2}-1\right) & \frac{G}{2}\sum\limits_{q}\frac{\omega v_{qq}^{(+)}}{4\varepsilon^2(q)-\omega^2} \\
\varkappa\sum\limits_{q}\frac{2\varepsilon(q)u_{qq}^{(+)}v_{qq}^{(+)}f(qq)}{4\varepsilon^2(q)-\omega^2} & \frac{G}{2}\sum\limits_{q}\frac{\omega v_{qq}^{(+)}}{4\varepsilon^2(q)-\omega^2} & \left(\frac{G}{2}\sum\limits_{q}\frac{2\varepsilon(q)v_{qq}^{(+)2}}{4\varepsilon^2(q)-\omega^2}-1\right)
\end{vmatrix}=0
$$

$$\varepsilon(qq')\equiv\varepsilon(q)+\varepsilon(q').$$

Such type equations were derived in ref./8/ for deformed nuclei.

We have deduced the equations for the proper vibrations of the system. Now we consider vibrations induced by a weak external fied. To this end we add to the Hamiltonian (1) the term

$$\sum_{ff'}\delta I(ff')\,a_f^+a_{f'}, \qquad (80)$$

where the function $\delta I(ff')=\delta I^*(f'f)$ describes the external field. The expressions $\delta\mathcal{U}(f_1f_2)$ and $\delta\mathcal{B}(f_1f_2)$ presented by eqs. (30) and (31) should be supplemented with the terms

$$\delta\mathcal{U}_{ex}(f_1f_2)=\sum_f\left\{\delta I(f_1f)\,\Phi(ff_2)+\delta I(f_2f)\,\Phi(f_1f)\right\}, \qquad (81')$$



$$\delta\mathcal{B}_{ex}(f_1f_2)=\sum_f\left\{\delta I(f_2f)F(f_1f)-\delta I(ff_1)\,F(ff_2)\right\}. \qquad (81'')$$

Using

$$\delta I(ff')=\sum_\omega e^{-i\omega t}\,\delta I_\omega(ff'),$$

$$\delta I^*(ff')=\sum_\omega e^{-i\omega t}\,\delta I^*_{-\omega}(ff'). \qquad (82)$$

in the presence of the external field eqs. (39) and (40) are written in the form

$$
\begin{aligned}
\omega\Psi_\omega(g_1g_2)&=\sum_{g'}\left\{\Omega(g_2g')\Psi_\omega(g_1g')-\Omega(g_1g')\Psi_\omega(g_2g')\right\}+\\
&+\sum_{g_1'g_2'}\left\{X(g_1g_2;g_1'g_2')\Psi_\omega(g_1'g_2')-Y(g_1g_2;g_1'g_2')\varphi_\omega(g_1'g_2')\right\}+\\
&+\sum_{ff'}\left\{v(f'g_1)u^*(fg_2)-u^*(fg_1)v(f'g_2)\right\}\delta I_\omega(ff'),
\end{aligned}
\qquad (83)
$$

$$
\begin{aligned}
-\omega\varphi_\omega(g_1g_2)&=\sum_{g'}\left\{\Omega^*(g_2g')\varphi_\omega(g_1g')-\Omega^*(g_1g')\varphi_\omega(g_2g')\right\}+\\
&+\sum_{g_1'g_2'}\left\{X^*(g_1g_2;g_1'g_2')\varphi_\omega(g_1'g_2')-Y^*(g_1g_2;g_1'g_2')\Psi_\omega(g_1'g_2')\right\}+\\
&+\sum_{ff'}\left\{v^*(f'g_1)u(fg_2)-u(fg_1)v^*(f'g_2)\right\}\delta I^*_{-\omega}(ff').
\end{aligned}
\qquad (84)
$$

We rewrite eqs. (83) and (84) in the approximation of eqs.(41),(42) and (43). We introduce the functions $R^{(\pm)}(qq')$, perform the same calculations as in deriving eqs. (63) and get:

$$\omega R^{(\mp)}(q_1q_2) = (\varepsilon(q_1)+\varepsilon(q_2)) R^{(\pm)}(q_1q_2) - \\ - \sum_{q_1'q_2'} G^{\xi}(q_1q_2;q_2'q_1')\, v^{(\pm)}_{q_1q_2} v^{(\pm)}_{q_1'q_2'} R^{(\pm)}(q_1'q_2') - \\ - 2\sum_{q_1'q_2'} G^{\omega}(q_1q_2;q_2'q_1')\, u^{(\pm)}_{q_1q_2} u^{(\pm)}_{q_1'q_2'} R^{(\pm)}(q_1'q_2') - \\ - \frac{1}{2} u^{(\pm)}_{q_1q_2} [\delta I_\omega(q_1q_2) \pm \delta I^{*}_{-\omega}(q_1q_2)] , \qquad (85)$$

where

$$\delta I_\omega(qq') = \sum_{\sigma} \left( \delta I_\omega(q\sigma, q'\sigma) - S_\sigma\, \delta I_\omega(q\sigma, q'-\sigma) \right),$$

$$\delta I^{*}_{-\omega}(qq') = \sum_{\sigma} \left( \delta I^{*}_{-\omega}(q\sigma, q'\sigma) - S_\sigma\, \delta I^{*}_{-\omega}(q\sigma, q'-\sigma) \right).$$

We write eqs. (85) in a form close to that which is given in the finite Fermi system theory/10/. To this end we introduce the functions

$$d^{(\pm)}(q_1q_2) = \sum_{q_1'q_2'} G^{\xi}(q_1q_2;q_2'q_1')\, v^{(\pm)}_{q_1'q_2'} R^{(\pm)}(q_1'q_2') , \qquad (86)$$

$$V^{(\pm)}(q_1q_2) = 2\sum_{q_1'q_2'} G^{\omega}(q_1q_2;q_2'q_1')\, u^{(\pm)}_{q_1'q_2'} R^{(\pm)}(q_1'q_2') + V_0^{(\pm)}(q_1q_2) \qquad (87)$$



$$V_0^{(\pm)}(q_1q_2) = \frac{1}{2}\left( \delta I_\omega(q_1q_2) \pm \delta I^{*}_{-\omega}(q_1q_2) \right). \qquad (88)$$

Then

$$(\varepsilon(q)+\varepsilon(q'))R^{(\pm)}(qq') - \omega R^{(\mp)}(qq') = v^{(\pm)}_{qq'} d^{(\pm)}(qq') + \\ + u^{(\pm)}_{qq'} V^{(\pm)}(qq'). \qquad (89)$$

Hence, it follows

$$R^{(\pm)}(qq') = \frac{1}{(\varepsilon(q)+\varepsilon(q'))^2-\omega^2} \Big\{ (\varepsilon(q)+\varepsilon(q'))[u^{(\pm)}_{qq'} V^{(\pm)}(qq') + v^{(\pm)}_{qq'} d^{(\pm)}(qq')] + \qquad (90) \\ + \omega [u^{(\mp)}_{qq'} V^{(\mp)}(qq') + v^{(\mp)}_{qq'} d^{(\mp)}(qq')] \Big\}.$$

Inserting eq. (90) to (86) and (87) we obtain

$$V^{(\pm)}(q_1q_2) = V_0^{(\pm)}(q_1q_2) + 2\sum_{q_1'q_2'} G^{\omega}(q_1q_2;q_2'q_1') \frac{u^{(\pm)}_{q_1'q_2'}}{(\varepsilon(q_1')+\varepsilon(q_2'))^2-\omega^2} \times \\ \times \Big\{ (\varepsilon(q_1')+\varepsilon(q_2'))[u^{(\pm)}_{q_1'q_2'} V^{(\pm)}(q_1'q_2') + v^{(\pm)}_{q_1'q_2'} d^{(\pm)}(q_1'q_2')] + \qquad (91) \\ + \omega [u^{(\mp)}_{q_1'q_2'} V^{(\mp)}(q_1'q_2') + v^{(\mp)}_{q_1'q_2'} d^{(\mp)}(q_1'q_2')] \Big\}.$$

$$
\overset{(\pm)}{d}(\varrho_1\varrho_2) = \sum_{\varrho_1'\varrho_2'} G^{\xi}(\varrho_1\varrho_2;\varrho_2'\varrho_1') \frac{\overset{(\pm)}{v}_{\varrho_1'\varrho_2'}}{(\varepsilon(\varrho_1')+\varepsilon(\varrho_2'))^2-\omega^2} \times
$$

$$
\times \Big\{ (\varepsilon(\varrho_1')+\varepsilon(\varrho_2'))\Big[\overset{(\pm)}{u}_{\varrho_1'\varrho_2'} \overset{(\pm)}{V}(\varrho_1'\varrho_2') + \overset{(\pm)}{v}_{\varrho_1'\varrho_2'} \overset{(\pm)}{d}(\varrho_1'\varrho_2')\Big] + \tag{92}
$$

$$
+\omega\Big[\overset{(\mp)}{u}_{\varrho_1'\varrho_2'} \overset{(\mp)}{V}(\varrho_1'\varrho_2') + \overset{(\mp)}{v}_{\varrho_1'\varrho_2'} \overset{(\mp)}{d}(\varrho_1'\varrho_2')\Big]\Big\}.
$$

We have deduced the equations for four unknown functions $V^{(\pm)}$, $d^{(\pm)}$. However only two functions from them are independent, as it follows from eqs. (63). By this reason it is more convenient to use eqs. (63) instead of eqs. (91) and (92).

Thus we have derived from the equations of the self-consistent field method the equations of the finite Fermi system theory which were usually obtained by the Green function technique. Yet, the equations of the self-consistent field method have been written for the distribution functions $\langle \overset{+}{\Psi}(t\tau_1)\Psi(t\tau_2)\rangle$ and $\langle \Psi(t\tau_1)\Psi(t\tau_2)\rangle$. Nevertheless the result obtained should not be considered as unexpected. It follows from the general theorem on the variation of the mean value of a dynamic variable [11]:

$$
\delta\langle A(t)\rangle = \langle A(t)\rangle_{H+\delta H} - \langle A(t)\rangle_H =
$$

$$
= 2\pi\left\{ e^{-iEt}\langle\langle A,B\rangle\rangle_E\,\delta\xi + e^{iEt}\langle\langle A,B\rangle\rangle^*_{-E}\,\delta\xi^*\right\},
$$

where $A(t)$ is a certain dynamic quantity in the Heisenberg representation;

$B$ is a time independent operator;

$\delta\xi$ is an infinitesimal $c$-number;

$\langle\langle A,B\rangle\rangle_E$ is the Green function in the $E$-representation [11].

This theorem relates the variations of the distribution functions to the appropriate Green functions. Using this theorem, introducing in the Hamiltonian weak external fileds and varying with respect to a small parameter we can always obtain from the equations of the distribution functions the equations for the Green functions.

We have shown that among the mathematical methods of the microscopic nuclear theory the self-consistent field method is the most general one. Under certain assumptions, one may derive from its basic equations both the equations for effective fields of the finite Fermi system theory and the secular equations of the model with pairing and multipole-multipole forces. However, the self-consistent field method in itself is not free of restrictions. The fact that we use simple rules of splitting of the average values of the products of four Fermi operators and assume the matrix elements $\langle \overset{+}{\alpha}_g \alpha_{g'}\rangle$ to be equal to zero means that the nonlinear effects are not taken into account in this approach too. For this reason the self-consistent field method turns out to be actually equivalent to the quasi-boson approximation. Besides, it is more convenient from the practical point of view to use the wave functions of the nucleus as in the approximate second quantization method rather than the average values of the operators as in the self-consistent field method.

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