ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E4 - 4743

J.Czerwonko

ON THE STATIC PROPERTIES OF THE FERROMAGNETIC FERMI LIQUID WITH THE CONTRIBUTION OF THE ELECTRON-PHONON INTERACTION

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

1969

E4 - 4743

J.Czerwonko*

ON THE STATIC PROPERTIES OF THE FERROMAGNETIC FERMI LIQUID WITH THE CONTRIBUTION OF THE ELECTRON-PHONON INTERACTION

Submitted to "Физика твердого тела".

* Institute of Mathematics and Theoretical Physics, Technical University of Wroclaw, Poland.

Червонко Е. E4-4743

О статических свойствах ферромагнитной ферми-жидкости с учётом вклада от электрон-фононного взаимодействия

Дается формула для сжимаемости и продольной восприимчивости ферромагнитной ферми-жидкости со сферическими поверхностями Ферми для каждого спина с учётом вклада от электрон-фононного взаимодействия.

Препринт Объединенного института ядерных исследований.
Дубна, 1969

Czerwonko J. E4-4743

On the Static Properties of the Ferromagnetic Fermi Liquid with the Contribution of the Electron-Phonon Interaction

The formulae for the compressibility and the longitudinal magnetic susceptibility of the ferromagnetic Fermi liquid with spherical Fermi surface for both spins are given. The contribution of the electron-phonon interaction to the above quantities is taken into account.

Preprint. Joint Institute for Nuclear Research.
Dubna, 1969

The static properties of the simple model of the ferromagnetic Fermi liquid with the spherical Fermi surface for both spins were investigated in the paper of Kondratenko[1] (cf. with the paper of the author[2]). In these papers the electron-phonon interaction (EPI) was neglected. It could be considered as unimportant since for normal spin-compensated systems EPI does not change the static properties. This fact was proved firstly by Migdal[3] for the system in which only EPI is of importance. Subsequently this proof was given also for more complicated systems by Prange, Kadanoff and Sachs,[4,5] (cf. with the recent Leggett's paper[6]). It was proved recently by us that the above situation does not appear for the ferromagnetic Fermi liquid without additional restriction for the effective masses or Landau parameters[7].

Following the paper[6] we shall express all quantities by Legendre amplitudes of the spin matrices $\tilde{\tilde{\Gamma}}_{\omega}(00)$ and $\tilde{\tilde{\Gamma}}_{c\omega}$. Here $\tilde{\tilde{\Gamma}}_{c\omega}$ denotes the effective interaction of screened quasiparticles when EPI disappears, whereas in $\tilde{\tilde{\Gamma}}_{\omega}(00)$ appears the contribution of the EPI. It will be convenient to apply sometimes the scattering amplitude of screened quasiparticles on the Fermi surface in the absence of EPI, $\tilde{\tilde{\Gamma}}_{Ck}$. This amplitude is simply related to $\tilde{\tilde{\Gamma}}_{C\omega}$ /1,2/. The dimensionless amplitudes, $\tilde{\tilde{f}}_{\omega}$, $\tilde{\tilde{f}}_{C\omega}$ and $\tilde{\tilde{f}}_{Ck}$ will be applied sometimes. They will be defined as follows

$$\tilde{\tilde{f}}_{\omega} = \sqrt{N^*}\, z\, \tilde{\tilde{\Gamma}}_{\omega}(00)\, z\, \sqrt{N^*} \qquad (1)$$

$$\tilde{\tilde{f}}_{C\omega} = \sqrt{N_C}\, z_C\, \tilde{\tilde{\Gamma}}_{C\omega}\, z_C \sqrt{N_C}$$

$$\tilde{\tilde{f}}_{Ck} = \sqrt{N_C}\, z_C\, \tilde{\tilde{\Gamma}}_{Ck}\, z_C \sqrt{N_C},$$

where N^* and N_C denote spin-diagonal matrices giving the level density for electrons with a given spin on the Fermi surface respectively without and with EPI. In these formulae z and z_C denote diagonal spin-matrices defining the discontinuity of the density of electrons with a given spin on the Fermi surface with and without EPI. Let us write the formulae

$$\frac{N^*}{N_C} = \frac{z_C}{z} = \frac{m^*}{m_C} \qquad (2)$$

which were proved in /4/ and /5/ for the normal nonferromagnetic liquid. These formulae are valid also for the ferromagnetic liquid /7/. Here m^*/m_C denotes the spin-diagonal matrix giving the ratio of effective masses of electrons after and before switching on EPI. The amplitudes $\tilde{\tilde{\Gamma}}_{\omega}(00)$ and $\tilde{\tilde{\Gamma}}_{Ck}$ are not completely independent.

Analogously, as for the paramagnetic liquid in [6], we have proved that the dependence can be expressed in the form (cf. with [7])

$$\left(\frac{m_{Ca}}{m^*_a}\right)\left(1+\tilde{\tilde{f}}{}^{aa}_{\omega 0}\right)\left(1-\tilde{\tilde{f}}{}^{aa}_{Ck,0}\right)-\tilde{\tilde{f}}{}^{1\bar{1}}_{\omega 0}\tilde{\tilde{f}}{}^{1\bar{1}}_{Ck,0}\left(\frac{m_{C1}m_{C\bar{1}}}{m^*_1 m^*_{\bar{1}}}\right)^{1/2}=1$$

$$\tilde{\tilde{f}}{}^{1\bar{1}}_{\omega 0}=\left(\frac{m^*_a m_{C\bar{a}}}{m_{Ca}\, m^*_{\bar{a}}}\right)^{1/2}\left(1+\tilde{\tilde{f}}{}^{aa}_{\omega 0}\right)+\left(\frac{m^*_a m_{C\bar{a}}}{m_{Ca}\, m^*_{\bar{a}}}\right)\tilde{\tilde{f}}{}^{1\bar{1}}_{\omega 0}\tilde{\tilde{f}}{}^{a\bar{a}}_{Ck,0}\,, \tag{3}$$

where the upper indices are the spin ones, $a=\pm 1$ and $\bar{a}\equiv -a$. On the other hand, the bottom index zero denotes that we take zeroth Legendre amplitudes of the functions $\tilde{\tilde{f}}_{\omega}$ and $\tilde{\tilde{f}}_{Ck}$. It is worth to mention that the formulae (3) are equivalent to the formulae for dimension amplitudes $\tilde{\tilde{\Gamma}}_{\omega 0}(00)$ and $\tilde{\tilde{\Gamma}}_{Ck,0}$ [7]. From the first formula (3) it can be proved

$$\frac{m^*_a}{m_{Ca}}=\frac{1}{R_{\bar{a}}}\left[R_1 R_{\bar{1}}-C'\left(R_1 R_{\bar{1}}\right)^{1/2}\right]. \tag{4}$$

where

$$R_a=\left(1+\tilde{\tilde{f}}{}^{aa}_{\omega 0}\right)\left(1-\tilde{\tilde{f}}{}^{aa}_{Ck,0}\right),\quad C'=\tilde{\tilde{f}}{}^{1\bar{1}}_{\omega 0}\tilde{\tilde{f}}{}^{1\bar{1}}_{Ck,0} \tag{5}$$

Substituting (4) into the second formula (3) we obtain the relation between the amplitudes $\tilde{\tilde{f}}_{\omega 0}$ and $\tilde{\tilde{f}}_{Ck,0}$ where the ratios of effective masses disappear.

Taking into account the results of our paper[7] and applying the transformation of the autocorrelation functions given in[6] we can express all basic thermodynamic formulae in the following form

$$u = \tilde{\tilde{\mathrm{Tr}}}\, \{ \frac{A}{z_C} [1 - N_C z_C^2 \; \tilde{\tilde{\Gamma}}_{Ck,0}][1 + N^* z^2 \; \tilde{\tilde{\Gamma}}_{\omega 0}(00)] \times$$

$$\times N^* z^2 [1 - \tilde{\tilde{\Gamma}}_{Ck,0} N_C z_C^2] \frac{B}{z_C} \} ; \qquad (6)$$

where all quantities appearing under the symbol Tr are spin-matrices. Their multiplication has the matrix character, i.e. with the summation over the intermediate spin. The symbol Tr denotes the sum of all elements of a given matrix. This symbol should not be treated as the trace operation. Let us mention that the matrices N^*, N_C, z, z_C, A, B are diagonal ones. The unity denotes the unit matrix. The meaning of the formula (6) is given by the following prescriptions

i) if $A = B = 1$ then $u = (\partial N / \partial \mu)_H$,

ii) if $A = B = \sigma^z$ (σ^z - the Pauli matrix in the usual form) then $u = (\partial M / \partial H)_\mu / \mu_B^2$,

iii) if $A = 1$, $B = \sigma^z$ or $A = \sigma^z$ $B = 1$ then

$$u = -(\partial N / \partial H)_\mu / \mu_B = -(\partial M / \partial \mu)_H / \mu_B ,$$

where we have used the following notation: N -denotes the density of particles, M -magnetization per unit volume, μ -the chemical potential, H -the external magnetic field and μ_B -Bohr's magneton. Let us note that in proof of the prescriptions i) - iii) the main role is played by the identities for correlation functions being the consequences of the Ward identities (cf. with [2]).

Using (1) and (2) we can express (6) with the help of the dimensionless amplitudes. We have

$$u = \tilde{\tilde{\mathrm{T}}}\mathrm{r}\,\{ A\sqrt{N_C}\,[1-\tilde{\tilde{f}}_{Ck,0}]\left(\frac{m_C}{m^*}\right)^{1/2}[1+\tilde{\tilde{f}}_{\omega 0}]\left(\frac{m_C}{m^*}\right)^{1/2}[1-\tilde{\tilde{f}}_{Ck,0}]\sqrt{N_C}B\} \quad (7)$$

and our prescriptions i) - iii) remain valid. Let us note that $(1-\tilde{\tilde{f}}_{Ck,0}) = (1+\tilde{\tilde{f}}_{C\omega,0})^{-1}$ and the equality has the matrix character. If we want to apply the correct level density, N^* then we must use the formula $(N_C)^{1/2} = (N^*)^{1/2}(m_C/m^*)^{1/2}$. Among our formulae disappear the formula for the specific heat. It is clear that the term linear with respect to the temperature in the low-temperature expansion of the specific heat is proportional to $(m^*_{\uparrow} p_{\uparrow} + m^*_{\downarrow} p_{\downarrow})$ where p_a denotes the Fermi momentum for electrons with the spin a. It is not difficult to see that for nonferromagnetic systems (7) tends to well-known results given by Landau[8]. The reason of such an identification consists in the fact that then $(m_C/m^*)^{1/2}$ and N_C are proportional to the unit matrix and can be put on an arbitrary place. On the other hand, for the nonferromagnetic system we have $(1+\tilde{\tilde{f}}_{\omega 0})(1-\tilde{\tilde{f}}_{Ck,0}) = (m^*/m_C)$ (cf. with[6]). The above properties preserve the static properties of nonferromagnetic normal system with EPI compared to the system without EPI.

This paper was performed during the stay of the author in the Laboratory of Theoretical Physics of the Joint Institute of Nuclear Research, Dubna, USSR. The author is greatly indebted to Professors V.G.Soloviev and D.N.Zubarev for their kind hospitality.

References

1. P.S.Kondratenko. Zhur. Exper.Theor.Fiz., 46, 1438 (1964).
2. J.Czerwonko. Acta Phys.Polon., on press (1969).
3. A.B.Migdal. Zhur. Eksperim. Teor. F uz., 34, 1438 (1958).
4. R.E.Prange, L.P.Kadanoff. Phys.Rev., 134, A566 (1964).
5. R.E.Prange., A.Sachs. Phys.Rev., 158, 672 (1967).
6. A.J.Leggett. Ann.Phys., (N.Y.), 46, 76 (1968).
7. J.Czerwonko. Physica, in press (1970).
8. L.D.Landau. Zhur. Eksper. Teor. Fiz., 30, 1058 (1956).

Received by Publishing Department
on October 10, 1969.