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EQUILIBRIUM DEFORMATIONS β_{20} AND β_{40} OF NUCLEI IN RARE EARTH AND TRANSURANIUM REGIONS AND β_{40}-DEPENDENCE OF SINGLE-PARTICLE CHARACTERISTICS

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EQUILIBRIUM DEFORMATIONS β_{20} AND β_{40} OF NUCLEI IN RARE EARTH AND TRANSURANIUM REGIONS AND β_{40}-DEPENDENCE OF SINGLE-PARTICLE CHARACTERISTICS

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At present it is firmly established that the nuclei of the rare-earth ($150 \leq A \leq 190$) and transuranium ($220 \leq A \leq 250$) regions possess a large static quadrupole deformation β_{20} . The rotational spectra and big electric quadrupole moments of these nuclei measured with a very high accuracy may be accounted for by assuming the existence of a large component $\beta_{20} Y_{20} (\theta)$ in the nucleus radius. The static deformation of the type $\beta_{2;\,2}$ for the nuclei in the middle of the rare-earth and transuranium regions appears to be 0 while for the nuclei at the edges of these regions as well as in the "new" deformation region (near Ba) $\beta_{2;\,2}$ may differ from zero /1,2/. The problem is whether there exist deformations of higher multipolarity. The hexadecapole deformation was introduced in ref. /3/ in the multipole expansion of the nuclear shape in order to explain the experimental intensities of alpha decay of even nuclei to the levels of the same rotational band of the daughter nucleus. The comparison of the results of theoretical calculations without the account of β_{40} with experiment leads to systematic disagreements: by a factor of several times for $0^+ \to 4^+$ transitions and by a factor of some dozens of times for $0^+ \to 6^+$ transitions. The β_{20} and β_{40} values were determined by equating the theoretical and experimental probabilities for $0^+ \to 2^+$ and $0^+ \to 4^+$ transitions. The theoretical analysis of alpha decay points to the necessity of introduction of β_{40} but unfortunately it does not give the β_{40} value with sufficient accuracy.

The study of the β_{40} [4-7] effect on the cross section for nucleon, deuteron and alpha particle scattering with excitation of 2^+, 4^+, 6^+ and sometimes 8^+ levels of rotational band of the ground state shows that the introduction of β_{40} changes very strongly the value and the shape of the differential cross sections $\sigma_{0^+ \to 4^+}$, $\sigma_{0^+ \to 4^+}$, ... in favour of the agreement between theoretical and experimental data. Apparently, the most convenient tool of determining the nuclear shape is alpha particles since they are strongly absorbed by the nucleus and interact with it mainly on the nuclear surface[6,7].

The calculations of the equilibrium β_{20} and β_{40} values were performed in refs.[8,9] by the Nilsson scheme, adding to the interaction hamiltonian a term proportional to $\epsilon_{40} Y_{40}(\theta)$.

The aim of the present paper is the calculation of the equilibrium β_{20} and β_{40} by the Woods-Saxon potential schemes[10,11] and the study of the hexadecapole β_{40} deformation effect on the properties of the single-particle states of deformed nuclei.

1. General Relations

We start from the assumption that the nuclear surface can be represented in the form

$$R(\theta) = R_0 [1 + \beta_0 + \beta_{20} Y_{20}(\theta) + \beta_{40} Y_{40}(\theta)] , \qquad (1)$$

where R_0 is the spherical nucleus radius, β_0 is a constant introduced for conservation of the nuclear volume with changing deformation /11/.

We determine the internal multipole moment of a rotational axially symmetric nucleus

$$Q_\lambda = 2z \sqrt{\frac{4\pi}{2\lambda+1}} \int_0^{\pi} \int_0^{2\pi} \int_0^{R(\theta)} r^\lambda Y_{\lambda 0}(\theta) \rho(\vec{r}) r^2 \, dr \, d\Omega . \qquad (2)$$

Let

$$\rho(\vec{r}) = \begin{cases} \rho_0 & r \le R(\theta) \\ 0 & r > R(\theta) . \end{cases} \qquad (3)$$

For the homogeneous constant distribution (3) the nuclear quadrupole moment is

$$Q_2 = \frac{3z R_0^2}{\sqrt{5\pi}} \beta_{20} (1 + 0{,}36 \beta_{20} + 0{,}97 \beta_{40} + 0{,}57 \frac{\beta_{40}^2}{\beta_{20}}) . \qquad (4)$$

The hexadecapole moment is

$$Q_4 = \frac{1}{\sqrt{\pi}} z R_0^4 \beta_{40} (1 + 1{,}32 \beta_{20} + 0{,}72 \frac{\beta_{20}^2}{\beta_{40}}) . \qquad (5)$$

In the case of the Saxon-Woods distribution

$$n(\vec{r}) = \frac{n_0}{1 + \exp\left(\frac{r - R(\theta)}{a}\right)} \, , \tag{6}$$

using the relation[11]

$$n(\vec{r}) = n(r) + \sum_{\lambda = 0} c_\lambda^0 \left(r, \sum_\nu \beta_{\nu 0}\right) Y_{\lambda 0}(\theta) \tag{7}$$

we obtain

$$Q_\lambda = 2z \sqrt{\frac{4\pi}{2\lambda + 1}} \int_0^\infty r^\lambda c_\lambda^0 \left(r, \sum_\nu \beta_{\nu 0}\right) r^2 \, dr \, . \tag{8}$$

It is seen from eqs. (4) and (5) that the determination of the β_{20} and β_{40} deformation parameters from the experimental multipole moments becomes noticeably complicated even for the distribution (3), while for (6) it needs cumbersome calculations.

2. Calculation of Equilibrium β_{20} and β_{40} Values

The equilibrium deformations β_{20} and β_{40} were calculated by the Strutinsky method[12]. The total energy of the ground state of an even-even nucleus is taken to be

$$\mathcal{E}_0(\beta_{20}, \beta_{40}) = \mathcal{E}_{drop}(\beta_{20}, \beta_{40}) + \Delta\mathcal{E}(\beta_{20}, \beta_{40}) \, , \tag{9}$$

where $\mathcal{E}_{drop}(\beta_{20}, \beta_{40})$ - liquid drop energy. In the expansion of $\mathcal{E}_{drop}$ into a series in powers of the deformation parameters β_{20} and β_{40} we take into account the following terms[13]

$$\mathcal{E}_{drop}(\beta_{20},\beta_{40}) \approx 0{,}4u\, \alpha_{20}^{2} + (-0{,}114285 + 0{,}076190u)\, \alpha_{20}^{3} +$$

$$+ (0{,}39183 - 0{,}416326u)\, \alpha_{20}^{4} + (-0{,}457144 + 0{,}342858u)\, \alpha_{20}^{2}\, \alpha_{40} + \qquad (10)$$

$$(0{,}629630 + 0{,}370370u)\, \alpha_{40}^{2} + (-0{,}090538 - 0{,}065306u)\, \alpha_{20}^{3}\, \alpha_{40} +$$

$$+ (-0{,}288600 + 0{,}23088u)\, \alpha_{20}\, \alpha_{40}^{2},$$

where $\alpha_{\lambda 0} = \left(\frac{2\lambda + 1}{4\pi}\right)^{1/2} \beta_{\lambda 0}$ and $u = 1 - \frac{z^2/A}{(z^2/A)_{cr.}}, (z^2/A)_{cr.} = 45.$

The shell correction to $\mathcal{E}_{drop}$ consists of neutron and proton terms

$$\Delta\mathcal{E} = \Delta\mathcal{E}(Z) + \Delta\mathcal{E}(N) \qquad (11)$$

and is determined by the one-particle energies as follows (e.g. for $\Delta\mathcal{E}(Z)$):

$$\Delta\mathcal{E}(Z) = \mathcal{E}(Z) - \bar{\epsilon}(Z), \qquad (12)$$

where

$$\mathcal{E}(Z) = \sum_{\mu} E(\mu)\, 2V_{\mu}^{2} - \frac{c_p^2}{G_z} \qquad (13)$$

$$\bar{\epsilon}(Z) = 2 \int_{-\infty}^{\lambda} E\, g(E)\, dE \qquad (14)$$

$$g(E) = \frac{1}{\gamma\sqrt{\pi}} \sum_{\nu} \exp\left[-\left(\frac{E - E_{\nu}}{\gamma}\right)^{2}\right] \qquad (15)$$

$$\gamma = 5 \div 10 \text{ MeV}.$$

In these expressions λ is the Fermi energy for "homogeneous" distribution of levels which is determined from

$$Z = 2 \int_{\infty}^{\lambda} g(E)\, dE \quad . \tag{16}$$

The pairing interaction constants G_z and G_N were chosen the same as in ref.[14]

$G_z = 33/A$ MeV, $G_N = 28/A$ MeV for nuclei of rare-earth and transuranium regions. In eqs. (13) - (15) the summation is performed over all the single-particle levels of the Saxon-Woods potential which were calculated by diagonalization of the energy matrix[10,11]. The introduction of the term $\beta_{40} Y_{40}$ in the expansion (1) does not change, in principle, the method developed earlier in ref.[10,11] for calculating the eigenenergies and eigenfunctions but essentially affects some of the one-particle characteristics of the nucleus (see § 3,4).

The numerical values of the average field parameters are the following[10]

Table 1

A	V_0	$\varkappa$	r_0	a	
181	43,4	0.37	1,26	0,63	neutrons
181	59,8	0.37	1,24	0,63	protons
237	46,4	0.38	1,26	0,63	neutrons
237	61	0.38	1,24	0,63	protons

In refs.[10,11] it is shown that the Saxon-Woods potential parameters are functional of the mass number A. Therefore in ref.[10] the one-particle spectrum for nuclei of the rare-earth region was calculated for three zones with A = 155, 165, 181. Such a division made it possible to improve the description of the experimental one-particle levels of odd nuclei. The one-particle states characterized by certain quantum numbers are A-dependent. This is essential for the specroscopy of deformed nuclei. However, it is known that the account of this dependence little affects the equilibrium deformation values[12,15]. In order to simplify calculations and economize computer time we have used the level scheme with A = 181 for nuclei in rare-earth and with A = 237 for those in transuranium regions. The calculations were performed for the rare-earth nuclei for the β_{40} values from - 0.10 to 0.10 and for the transuranium nuclei from 0 to 0.10 with $\Delta\beta_{40}$ = 0.02 (what, of course, restricts the accuracy of the obtained equilibrium values of β_{40}).

The result of calculations of the equilibrium deformation are presented in Tables 2 and 3. It is seen that the β_{40} values equal zero only for nuclei ^{170}Er, ^{172}Yb, $^{172,174}Hf$, but for the remaining ones $\beta_{40} \neq 0$. The equilibrium values of the hexadecapole deformation β_{40} lie in the limit 0.06 to - 0.06 in the transition from nuclei near Sm to those near Os and change from 0.085 to 0.025 in the transition from nuclei near Th to nuclei near Fm.

It is interesting to analyse with what certitude we may say that $\beta_{40} \neq 0$. Figs. 2, 3 give the curves of the total deformation energy $E(\beta_{20}=\beta_{20}^{eq})$ as a function of β_{40}. A clear and rather deep minimum of E is observed.

Thus our calculations show that the equilibrium values of β_{40} for the nuclei of the rare-earth and transuranium regions differ from zero. In ref.[8,9] the equilibrium ϵ_{20} and ϵ_{40} values were calculated by the Nilsson scheme and it was shown that $\epsilon_{40} \neq 0$. The question arises as to how to compare the equilibrium deformations of ϵ_{20} and ϵ_{40} in the Nilsson scheme and β_{20} and β_{40} in the Saxon-Woods potential scheme. It seems to us that in order to establish the connection between $\beta_{\lambda 0}$ and $\epsilon_{\lambda 0}$ the equality of the quadrupole and hexadecapole moments in different one-particle schemes should be required. This way would be most justified from the physical point of view since the multipole moments of nuclei can be extracted from the experimental data (for example, from the $E\lambda$ -transition probabilities). But here one encounters some calculation difficulties. To avoid them in estimating the connection between $\beta_{\lambda 0}$ and $\epsilon_{\lambda 0}$ one requires that the equipotential surfaces be the same in both models. Then we obtain

$$\beta_{20} \approx \left[\frac{2}{3} \epsilon_{20} + \frac{8}{7} \epsilon_{20} \left(\frac{\epsilon_{20}}{9} - \epsilon_{40} \right) \right] \sqrt{\frac{4\pi}{5}}$$

$$\beta_{40} \approx \left[- \epsilon_{40} + 8 \epsilon_{20} \left(\frac{\epsilon_{20}}{35} - \frac{10}{77} \epsilon_{40} \right) \right] \sqrt{\frac{4\pi}{9}} \qquad (17)$$

In Fig. 4 the continuous lines are the equilibrium β_{40} values calculated by the Nilsson scheme, with coupling between shell N and $N \pm 2$ [8b]. It should be noted that this coupling for $\beta_{40} \neq 0$ becomes even more important than for $\beta_{40} = 0$. So, for example, the neglect of the couplings between shells N and $N \pm 2$ leads to smaller values of β_{40} by a factor of two-three[8b]. In the same figures the dashed lines are the β_{40} values obtained in our calculations. We see that the results obtained in different approaches are very likely. This is not suprising since, as was noted earlier, the calculations of the equilibrium deformations are not very sensitive to small changes in the single-particle schemes. Analogous calculations of the equilibrium deformations β_{40} by the Nilsson scheme were carried out earlier in ref.[8a] without the account of pairing, Coulomb forces and couplings between shells N and $N \pm 2$. Since the couplings between shells N and $N\pm 2$, and Coulomb effects are balanced by the opposing effect of pairing, the results of ref.[8a] are very close to ours. Unfortunately, there is not a single physical phenomenon from which it would be possible to extract experimental values of β_{20} and β_{40}. However, sometimes it is possible to estimate the contribution of β_{20} and β_{40} taken separately, neglecting their interference term. For example, recently experiments on inelastic scattering of alpha particles on even-even nuclei of the rare-earth region have been performed[6,7]. A thorough analysis of the cross sections by the coupled channel method shows that the inclusion of β_{40} in eq.(1) strongly changes the form and the value of the scattering cross sections $\sigma_{0^+ \to 4^+}$ and $\sigma_{0^+ \to 6^+}$ improving the agreement between theory and experiment. The cross sections $\sigma_{0^+ \to 4^+}$ and $\sigma_{0^+ \to 6^+}$ are very sensitive to the magnitude and the sign of β_{40} therefore this method of determination of β_{40} may be considered as reliable. Fig. 4 gives the experimental β_{40}; our calculations well agree with these data.

In ref.[17] the values of β_{40} and of the deformation of higher multipolarities are supposed to be rather large for heavy nuclei. Our calculation are in agreement with these qualitative considerations. The β_{40} value for nuclei of the transuranium region is about 0.08. The experimental β_{60} are determined less accurately; their values are small (about - 0.02), therefore we have not considered the deformation β_{60} .

The equilibrium β_{20} values calculated by us relate to the form of the common average nucleus field (nucleus plus coulomb one) and are smaller than the experimental $\beta_{20}^{exp.}$ determined from the electric quadrupole moments due to Coulomb excitation. In obtaining $\beta_{20}^{exp.}$ from B(E2) the contribution of β_{40} and of the interference term $\beta_{20}\beta_{40}$ are neglected. This $\beta_{20}^{exp.}$ value is defined by the charge density distribution. On the other hand, our calculation results are in good agreement with quadrupole deformations extracted from the analysis of reactions (α, α') by the rare-earth nuclei /6/. This fact points out that the nucleon and charge distributions of the nucleus are not necessarily the same.

In the general case the quadrupole moments of Q_2 nuclei should be calculated by the microscopic model taking into account residual interactions. (Such calculations are performed in ref. /9/ by the Nilsson scheme. The theoretical Q_2 are found to be smaller than the experimental ones by 10-20 percent). However, in ref. 2 it is shown that the ratio of the quadrupole moment of an axially symmetric ellipsoid with proton distribution (3) to the microscopic quadrupole moment is constant and equals unity within the accuracy of 5%. Therefore we have used eq. (8) to estimate the multipole moments with the Saxon-Woods type proton distribution. The distribution parameters were taken the same as for the proton average field potential (see Table 1). Although the theoretical Q_2 well reproduce the dependence of the quadrupole moments on the mass number A , the absolute Q_2 are smaller than the experimental ones by 10-20% (see Tables 2,3).

The analysis of the calculated hexadecapole moments Q_4 (see Tables 2,3) shows that the experimental discovery of enhanced

The investigation of these changes is important, since most observable one-quasiparticle transitions are hindered with respect to asymptotic quantum numbers therefore the study of the probabilities of such transitions needs some improvement of the small wave function components.

Our calculations show that the energy spectrum is sensitive to the value of the hexadecapole deformation β_{40}. Fig.5 gives a part of the neutron spectrum for A = 181 at $\beta_{20} = 0$ for different β_{40}. It is seen that the splitting of the states for $\beta_{40} \neq 0$ is such that for $\beta_{40} > 0$ the lowest is the state with $\Omega_{max} = 13/2$, the next is the state with $\Omega_{min} = 1/2$, for $\beta_{40} < 0$ these states have the highest energy. When β_{40} changes in the interval $-0,10 \leq \beta_{40} \leq 0,10$ the eigenvalue for the states with $\Omega = 5/2$ is actually constant, while for the states with $\Omega = 1/2$, $\Omega = 13/2$ $\Delta E = 2$ MeV.

Fig.6 represents the neutron energy spectrum at $\beta_{20} = 0.214$ and Fig.7 the proton spectrum for $\beta_{20} = 0.28$, with A = 181 for different β_{40} in the interval $-0.10 \leq \beta_{40} \leq 0.10$. It is seen that with increasing $|\beta_{40}|$ the order of the levels as well as their local density change.

It is known that in the scheme of the Saxon-Woods potential, in the approximation of ref. /10/ there is a mixing of states with identical Ω and with the main quantum numbers N and $N \pm 2$. Such a mixing takes place for any values of β_{20} and is small what is proved by experiment (e.g. N -forbidden β-transitions). It is more important near the quasiintersection levels. Very strong mixing of one-particle states with identical Ω and quantum numbers N and $N \pm 2$ occurs in a narrow interval of β_{20} values ($\Delta\beta_{20} = 0.001 - 0.005$). In this case the state having before quasiintersection the quantum number N, will have after in the quantum number $N \pm 2$. In ref. /19/ one obtained the experimental data which testify the existence of this fact. It is clear that this effect can be observed only when the interval of β_{20}, where a strong mixing occurs, is not too small. The introduction of the hexadecapole deformation to the expansion (1) leads to the widening of this interval as compared to the calculations of ref. /10/. Fig.8 gives the structure of the wave function of the state $n(2d_{3/2})\,1/2^+$ for $A = 157$ depending on β_{20} for $\beta_{40} = 0$ (Fig. 8a) and $\beta_{40} = 0.1$ (Fig. 8b). Fig. 8c shows the energies of the quasiintersecting levels at $\beta_{40} = 0$ (dashed line) and $\beta_{40} = 0.1$ (continuous lines), and Fig. 8d the decoupling parameters respectively. One can see that the closest approach of the quasiintersecting levels at $\beta_{40} = 0$ is 0.04 MeV, at $\beta_{40} = 0.1$ it is 0.25 MeV and the corresponding $\Delta\beta_{20}$ intervals are 0.001 - 0.005 and 0.03 - 0.05. Summarizing we may say that the introduction of β_{40} leads to the increase of both the β_{20} interval where there occurs a strong mixing of two one-particle states and the energy interval of the closest approach of the quasiintersecting levels. The account of the quasiparticle-phonon interaction leads to the same effect /20/.

4. Decoupling Parameters "a"

The very important characteristic of the one-particle states with $\Omega = 1/2$ is the decoupling parameter "a" describing the connection between the rotational and internal motions of the nucleus in the first perturbation order. The parameter "a" enters the formula for the energy spectrum of the rotational band with

$$E_{rot.} = \frac{h^2}{2J}\left[I\,(I + 1) + a(-1)^{I+1/2}(I + 1/2) \right] \qquad (18)$$

and can be determined by means of (18) from the experimental data.

If the wave function of the one-particle state is determined in the ℓj representation then "a" has the following form

$$a = -\Sigma(-1)^{j+1/2}(j+1/2)\,|\,a^{\Omega=1/2}_{n\ell j}\,|^{2} \quad . \tag{19}$$

The comparison of the experimental "a" determined by means of eq. (18) and the theoretical ones calculated by eq. (19) is tentative. This is due to the fact that as is seen from (19), the decoupling parameter "a" is strongly affected by small changes in the structure of the wave function of the internal state. These changes are more essential for the coefficients $a^{\Omega=1/2}_{n\ell j}$ having large j. In addition it should be noted that the "a" for certain states (e.g. 1/2⁻[510] state) is very sensitive to small changes of the deformation parameter. The magnitude of "a" strongly depends on the average field parameters[15].

It is also important to know how definitely we may interpret an experimentally observed state as the single-particle one. Most states for which the parameter "a" is experimentally determined are highly excited states and therefore may contain considerable admixture from the interaction of quasiparticles with quadrupole and octupole vibrations of the core. For such states of importance is the account of the interaction of quasiparticles with phonons which decreases the absolute value of the decoupling parameter but conserves its sign[21]. Its influence on "a" for the ground and the lowest excited states is insignificant. However, as is shown in refs.[22,23] it is necessary to take into consideration the spin polarization of the core due to an odd particle. This effect especially strongly influences the states with the asymptotic quantum number $\Lambda = 0$ [22]. Even the sign of the decoupling parameter can alter in this case, besides, eq. (18) for "a" is obtained in the first perturbation order and, as is shown in ref./24/, it is not always valid. Therefore we will make the qualitative comparison of the theoretical and experimental "a".

In the all known one-particle schemes the decoupling parameter for the $1/2^-[510]$ state is negative in the rare-earth region of nuclei. The available experimental data show that "a" for this state is here positive except ^{161}Gd /19/. Faessler and Sheline have obtained a right value of "a" for ^{183}W /16/. It should be however noted that this result is perhaps a consequence of the neglect of the interaction between shells and of the influence of the quasidiscrete spectrum on the weakly bound states.

Table 4 gives the calculated "a" for the $1/2^-[510]$ state with $A = 181$ and $A = 157$ for different β_{20} and β_{40}. It is seen that "a" changes sharply with changing β_{20} and β_{40} and for the equilibrium β_{20} and β_{40} the parameter "a" is positive. It turns out that the parameter "a" for certain states (for example, $p\,1/2^+[411]$, $p\,1/2^-[541]$, $n\,1/2^-[521]$ /25/) little changes depending on β_{20} and β_{40}. Since the effect of the spin polarization of the core due to an odd particle on "a" is small for states with asymptotic quantum numbers $\Lambda = 1$ and the interaction of quasiparticles with phonons decrease the absolute value of the decoupling parameter then the correlation between the one-particle estimates of "a" for $p\,1/2^+[411]$, $n\,1/2^-[521]$ and $p\,1/2^-[541]$ states and the experimental data is expected to be large.

As is known there is a quasiintersection of the $1/2^+[400]$ and $1/2^+[660]$ states at the beginning of the rare-earth region for neutrons and at the end for protons. In Fig. 8d the calculated "a" for the $n\,1/2^+[400]$ and $n\,1/2^+[660]$ are shown by the dashed line at $\beta_{40} = 0$ and by the continuous line at $\beta_{40} = 0.10$. It is seen that at $\beta_{40}= 0.10$ the quasiintersection interval $\Delta\beta_{20}$ essentially increases. The same is for the proton p $1/2^{+}[400]$ and p $1/2^{+}[660]$ states.

Thus , the introduction of the hexadecapole deformation β_{40} to the expansion (1) noticeably changes certain one-particle characteristics of deformed nuclei (density of one-particle levels near the Fermi surface, decoupling parameters, deformation interval $\Delta\beta_{20}$ of strong mixing of quasiintersecting levels).

The calculations of the equilibrium deformations β_{20} and β_{40} by the Strutinsky method for nuclei in the rare-earth and transuranium regions show that $\beta_{20}\neq 0$ and $\beta_{40}\neq 0$ and the equilibrium values of β_{20} and β_{40} obtained by us are in good agreement with the experimental data[6] and with the results of similar calculations with the Nilsson scheme /8,9/.

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Table 2

Equilibrium Values β_{20} , β_{40} , quadrupole and hexadecapole moments respectively

Nuclei	β_{20}	β_{40}	Q_2^{theor} $10^{-24}\,sm^2$	Q_2^{exp} $10^{-24}\,sm^2$	Q_4^{theor} $10^{-48}\,sm^4$
Sm^{152}	0.I7	0.045	4.I	5.96±0.II*	0.7
Sm^{154}	0.I9	0.05	4.7	6.73±0.09*	0.9
Sm^{156}	0.20	0.055	5.0		I.0
Gd^{152}	0.I5	0.04	3.7	3.27±0.28*	0.6
Gd^{154}	0.I8	0.04	4.5	6.I8±0.II*	0.7
Gd^{156}	0.20	0.045	5.I	6.74±0.I3*	0.9
Gd^{158}	0.2I	0.045	5.4	7.24±0.I3*	0.9
Gd^{160}	0.23	0.045	6.0	7.54±0.I3*	I.0
Gd^{162}	0.25	0.045	6.7		I.I
Dy^{156}	0.I8	0.03	4.7	6.I8±0.25*	0.6
Dy^{158}	0.20	0.035	5.3	6.85±0.30*	0.8
Dy^{160}	0.2I	0.03	5.6	6.88±0.I7*	0.8
Dy^{162}	0.23	0.035	6.2	7.I2±0.09*	0.9
Dy^{164}	0.25	0.04	6.8	7.50±0.I6*	I.I
Er^{160}	0.I9	0.03	5.2		0.7
Er^{162}	0.22	0.02	6.0	7.02±0.I8*	0.7
Er^{164}	0.23	0.02	6.3	7.05±0.20*	0.7
W^{182}	0.2I	-0.04	6.3	6.46±0.II*	-0.2
W^{184}	0.2I	-0.05	6.3	6.23±0.II*	-0.4
W^{186}	0.2I	-0.06	6.2	5.95±0.I2*	-0.6
Os^{182}	0.I8	-0.03	5.6		-0.2
Os^{184}	0.I8	-0.04	5.6	5.56±0.I[illegible]**	-0.4
Os^{186}	0.I8	-0.04	5.6	5.54±0.I5**	-0.4
Os^{188}	0.I7	-0.05	5.3	4.96±0.08**	-0.6
Os^{190}	0.I7	-0.05	5.3	5.I5±0.I6**	-0.6
Pt^{194}	0.I4	-0.04	4.5		-0.5
Pt^{196}	0.I3	-0.04	4.2		-0.5

Nuclei	β_{20}	β_{40}	Q_2^{theor} $10^{-24}\,sm^2$	Q_2^{exp} $10^{-24}\,sm^2$	Q_4^{theor} $10^{-48}\,sm^4$
Er^{166}	0.25	0.02	7.0	7.58±0.I4*	0.8
Er^{168}	0.27	0.0I	7.5	7.6I±0.II*	0.8
Er^{170}	0.28	0	7.8	7.4I±0.I0*	0.7
Yb^{166}	0.22	0.02	6.3		0.7
Yb^{168}	0.24	0.02	6.9	7.43±0.I8*	0.8
Yb^{170}	0.26	0.0I	7.5	7.48±0.I5*	0.8
Yb^{172}	0.27	0	7.8	7.73±0.I3*	0.7
Yb^{174}	0.27	-0.0I	7.8	7.6I±0.II*	0.5
Yb^{176}	0.27	-0.02	7.7	7.45±0.II*	0.4
Hf^{170}	0.22	0.0I	6.5		0.6
Hf^{172}	0.23	0	6.8		0.5
Hf^{174}	0.24	0	7.I	7.28±0.24*	0.5
Hf^{176}	0.25	-0.0I	7.5	7.33±0.I5*	0.6
Hf^{178}	0.25	-0.02	7.4	6.80±0.II*	0.3
Hf^{180}	0.25	-0.04	7.3	6.74±0.I2*	-0.I
W^{178}	0.2I	-0.02	6.3		0.I
W^{180}	0.2I	-0.02	6.4	6.39±0.24*	-0.I

* P.H.Stelson, L.Grodzins, Nucl.Data 1(1965)21.

** Т.Бэдике и др. Препринт ОИЯИ 6-4350, Дубна, 1969 .

Table 3.

Equilibrium values β_{20}, β_{40}, quadrupole and hexadecapole moments respectively

Nuclei	β_{20}	β_{40}	Q_2^{theor}, 10^{-24} см^2	Q_2^{exp}, 10^{-24} см^2	Q_4^{theor}, 10^{-48} см^4
Th^{224}	0.13	0.07	6.0		2.3
Th^{226}	0.14	0.07	6.5	8.25±0.46*	2.4
Th^{228}	0.15	0.07	7.0	8.46±0.45*	2.5
Th^{230}	0.16	0.07	7.6	8.91±0.45*	2.6
Th^{232}	0.17	0.08	8.3	9.88±0.25*	2.7
Th^{234}	0.18	0.07	8.2	8.97±0.56*	2.5
Th^{236}	0.18	0.06	8.9		2.3
U^{228}	0.15	0.08	7.4		2.6
U^{230}	0.16	0.08	7.9	9.46±0.69*	2.7
U^{232}	0.17	0.08	8.5	9.48±0.60*	2.8
U^{234}	0.18	0.08	9.1	9.77±0.38*	2.9
U^{236}	0.19	0.08	9.8	10.36±0.44*	3.0
U^{238}	0.20	0.075	10.4	10.52±0.48*	3.2
U^{240}	0.21	0.07	10.9		3.3
U^{242}	0.22	0.06	10.9		2.7
Pu^{234}	0.19	0.085	9.6		3.0
Pu^{236}	0.20	0.085	10.2		3.2

Nuclei	β_{20}	β_{40}	Q_2^{theor}, 10^{-24} см^2	Q_2^{exp}, 10^{-24} см^2	Q_4^{theor}, 10^{-48} см^4
Pu^{238}	0.21	0.08	10.7	10.94+0.69*	3.3
Pu^{240}	0.22	0.08	11.3	11.30±0.18*	3.4
Pu^{242}	0.23	0.075	12.1	11.82±0.51*	3.6
Pu^{244}	0.24	0.07	12.7		3.8
Pu^{246}	0.25	0.06	12.8		3.2
Cm^{238}	0.21	0.08	10.9		3.4
Cm^{240}	0.21	0.075	11.5		3.5
Cm^{242}	0.23	0.075	12.4		3.7
Cm^{244}	0.24	0.07	13.0	13.49±0.74*	3.8
Cm^{246}	0.24	0.065	12.9		3.2
Cm^{248}	0.25	0.05	13.6		3.3
Cm^{250}	0.24	0.04	12.7		2.5
Cf^{244}	0.24	0.07	12.7		3.1
Cf^{246}	0.25	0.065	13.4		3.3
Cf^{248}	0.25	0.055	13.9		3.4
Cf^{250}	0.25	0.045	13.8		2.7
Cf^{254}	0.25	0.03	13.7		2.7
Fm^{246}	0.25	0.06	14.2		3.4
Fm^{248}	0.26	0.055	14.6		3.6
Fm^{250}	0.26	0.05	14.3		2.8
Fm^{252}	0.26	0.04	14.6		2.9
Fm^{254}	0.26	0.035	14.6		2.9
Fm^{256}	0.26	0.025	14.3		2.1

*P.H.Stelson, L.Grodzins, Nucl. Data 1(1965)21.

T a b l e 4

Decoupling parameter a for $1/2^-$ 510 as a function β_{20} and β_{40}

State	β_{20} / β_{40}	0,I35	0,2I4	0,28	0,345	0,4I
				A = 181		
$I/2^-$ 5I0	0,I	0,45	0,08	-0,II	-0,29	-0,47
	0,08	0,40	0,05	-0,I2	-0,28	-0,42
	0,06	0,37	0,04	-0,I3	-0,26	-0,38
	0,04	0,36	0,03	-0,I2	-0,24	-0,34
	0,02	0,36	0,03	-0,I2	-0,22	-0,3I
	0	0,38	0,04	-0,I0	-0,2I	-0,29
	-0,02	0,42	0,06	-0,09	-0,I9	-0,27
	-0,04	0,49	0,09	-0,07	-0,I7	-0,25
	-0,06	0,58	0,I2	-0,04	-0,I5	-0,23
	-0,08	0,72	0,I6	-0,02	-0,I3	-0,20
	-0,I	0,93	0,2I	0,02	-0,I0	-0,I7
				A = I57		
	0,I	0,9I	0,44	0,I7	0,03	-0,28
	0,08	0,88	0,42	0,I9	0,07	0,02
	0,06	0,86	0,4I	0,2I	0,II	0,06
	0,04		0,43	0,24	0,I5	0,I0
	0		0,48	0,32	0,23	0,I7

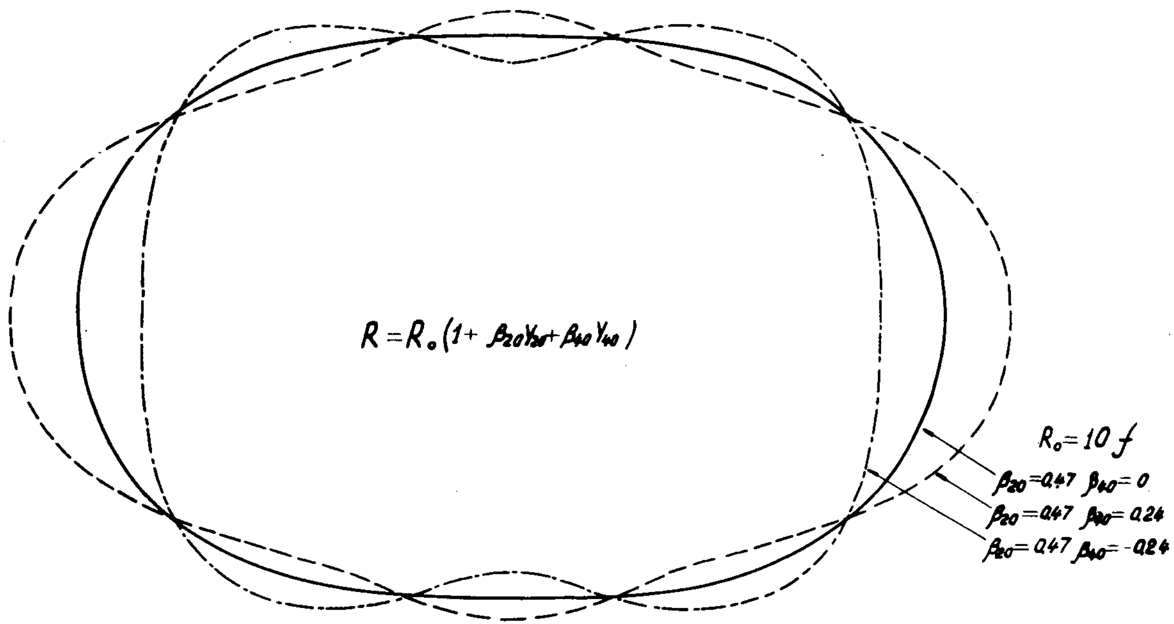

Fig.1. Nuclear surface as a function β_{20} and β_{40}.

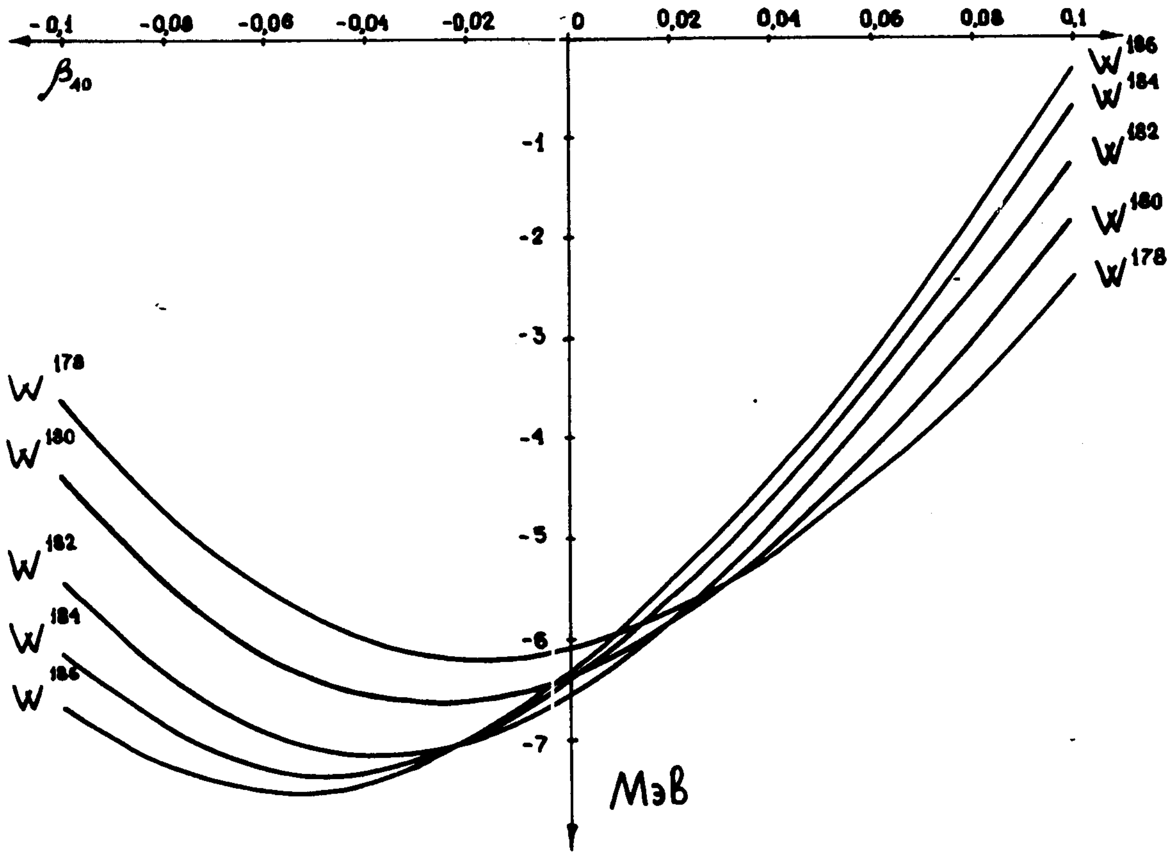

Fig.2. Curves of the total deformation energy as a function of β_{40} for W .

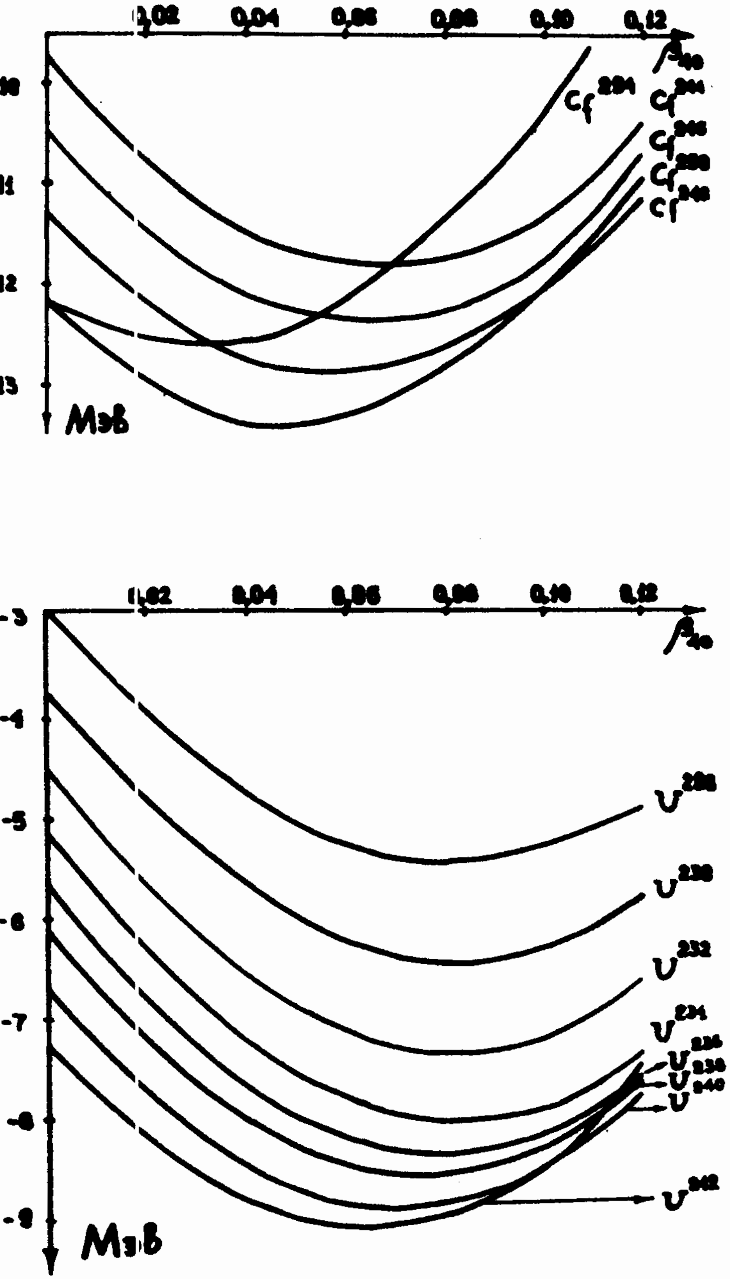

Fig.3. Curves of the total deformation energy as a function of β_{40} for Cf and U .

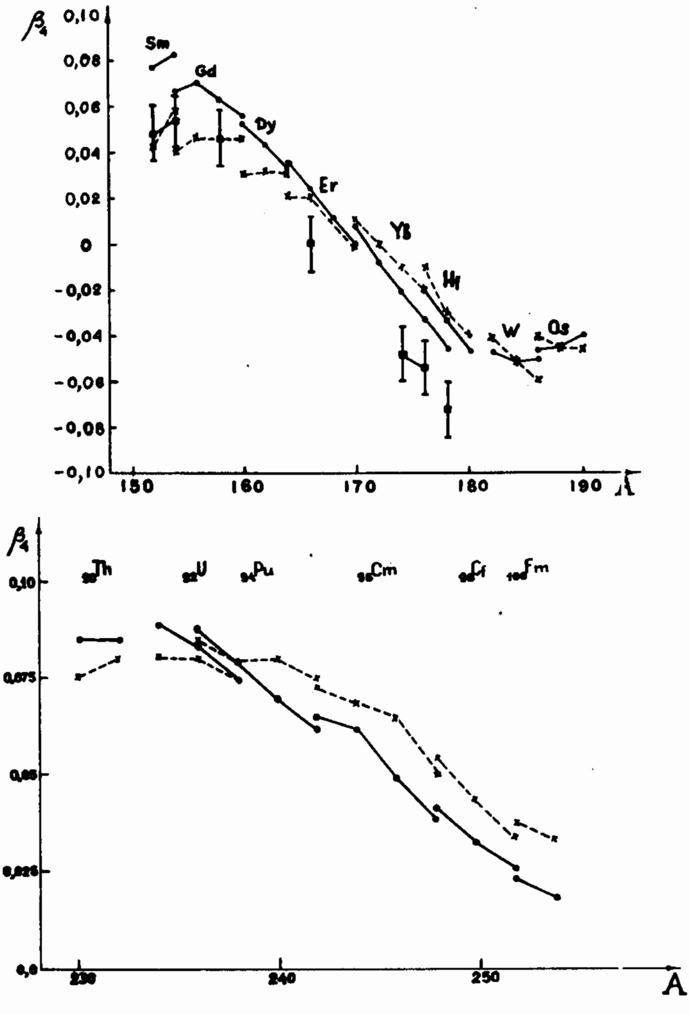

Fig.4. Equilibrium β_{40} value, calculated by the Nilsson scheme (continuous lines) and equilibrium β_{40} value calculated by Woods-Saxon potential schemes (Dashed lines).

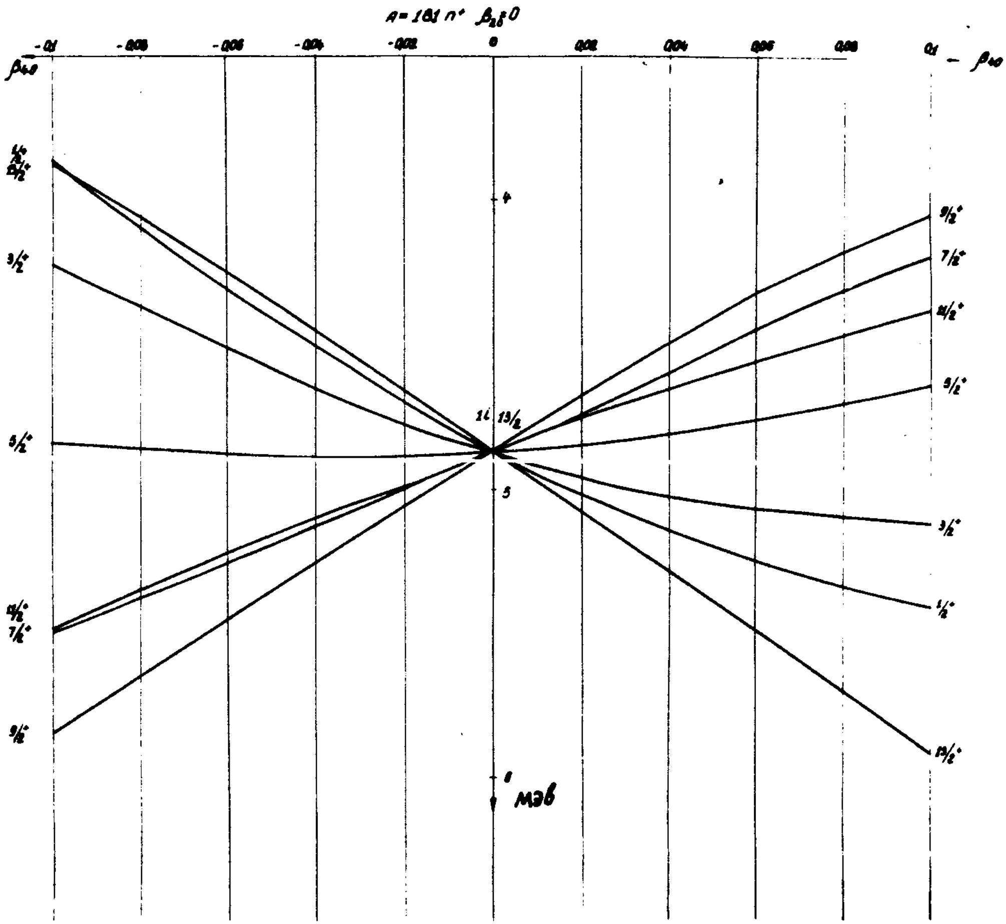

Fig.5. Part of the neutron spectrum for $A = 181$ at $\beta_{20} = 0$ for different β_{40}.

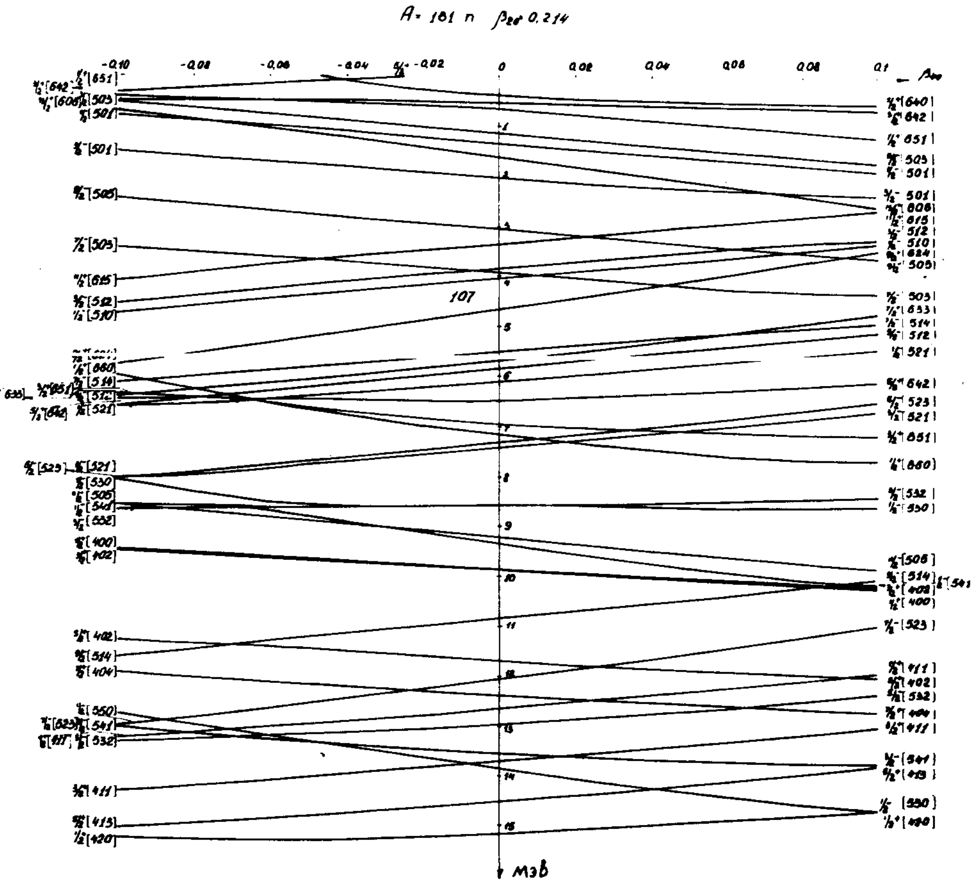

Fig.6. Neutron energy spectrum at β_{20} = 0.214 for A = 181.

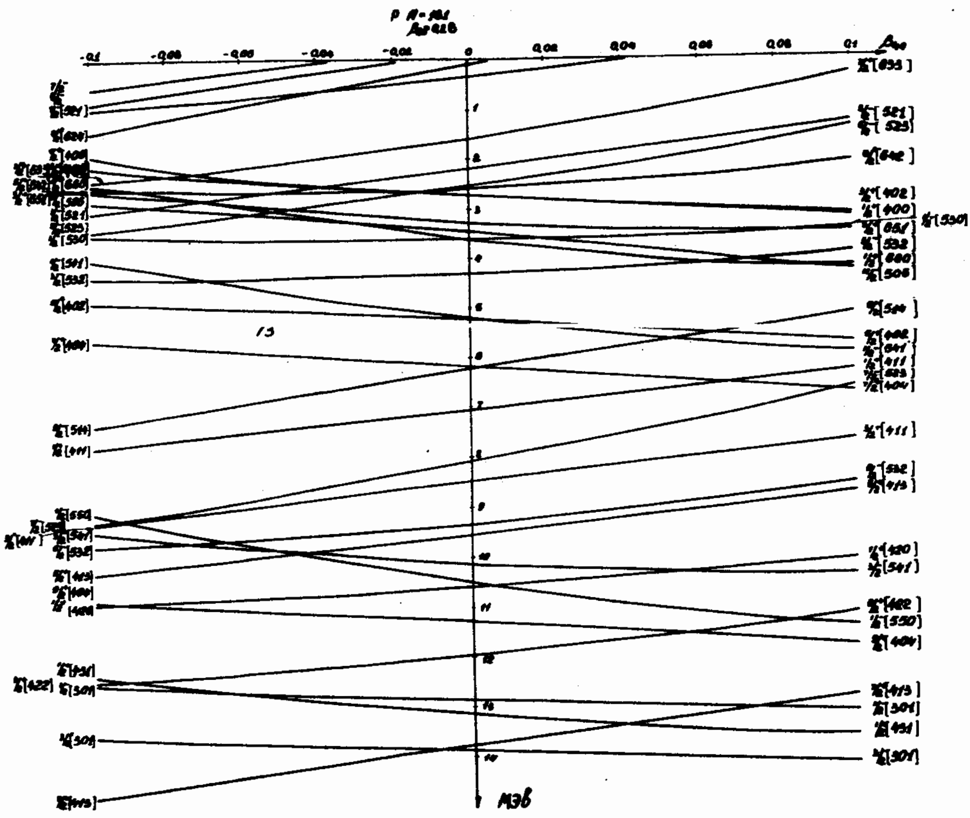

Fig.7. Proton energy spectrum at $\beta_{20} = 0.28$ for A =181.

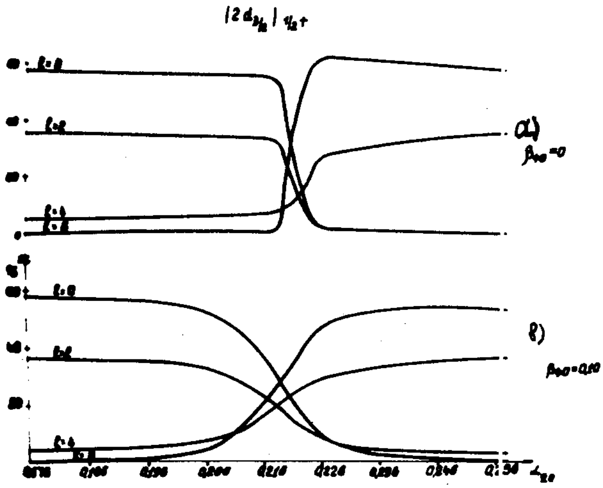

Fig.8. Structure of the wave function of the state $n(2d_{3/2})_{1/2}+$ for A = 157. depending on $\alpha_{20} = \sqrt{\frac{5}{4\pi}}\,\beta_{20}$ for $\beta_{40} = 0$ (8a) and $\beta_{40} = 0.1$ (8b).

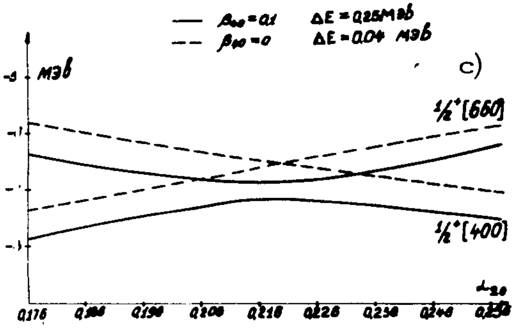

Fig.8c. Energy of the quasiintersecting levels at $\beta_{40}=0$ (dashed lines) and $\beta_{40}=0.1$ (continuous lines).

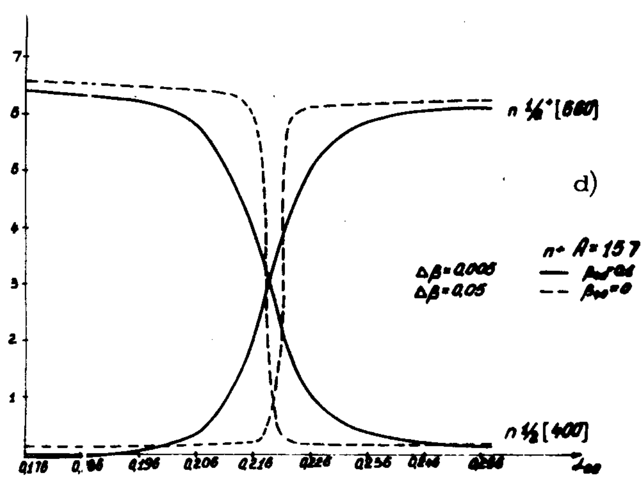

Fig.8d. "a"-decoupling parameters.