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SPIN-ORBIT INTERACTION IN THE DEUTERON STRIPPING REACTION ON DEFORMED NUCLEI

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SPIN-ORBIT INTERACTION IN THE DEUTERON STRIPPING REACTION ON DEFORMED NUCLEI

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I. Introduction

By neglecting the spin-orbit interaction the deuteron stripping reaction on deformed nuclei is treated in ref./1/ and numerical results are given in ref./2/. However, from stripping reactions on spherical nuclei, it is well known that the agreement of the experimental data with the theoretical predictions could be improved taking into account the spin-orbit interaction/3/. A corresponding treatment for deformed nuclei is carried out in this paper. At present numerical results are not obtained.

2. The Differential Cross Section and the Polarization

In this section the expressions for the differential cross section and the polarization of the emergent protons including spin-orbit interaction in both the initial and final channels are obtained.

If a zero range force is taken for the neutron-proton interaction the amplitude of the $A(d,p)B$ reaction in terms of the DWBA is

$$T^{\Sigma_d' \Sigma_p'}_{M_A M_B} = \langle \Psi^{(-)}_{\vec{k}_p}(\vec{r}, \theta_1, s_p, \Sigma_p') \, \Phi^{J_B}_{M_B K_B}(\theta_1, \vec{r}_n) | \Phi^{J_A}_{M_A K_A}(\theta_1) \Psi^{(+)}_{\vec{k}_d}(\vec{r}, \theta_1, s_d, \Sigma_d') \rangle .$$

Here the wave functions in both initial and final channels are represented in the adiabatic approximation that means like a product of the wave functions of the interior and relative motions. The adiabatic approximation[4] can be used when the incident energy of the projectile is much higher than the excitation energies of the above rotational states and the excitation spectrum of the initial and final nuclei is closly that of an ideal rotational band. Therefore, we describe the initial and final nuclei as

$$\Phi^{J_A}_{M_A K_A}(\theta_i) = \mathfrak{D}^{J_A}_{M_A K_A}(\theta_i) \qquad (2)$$

and

$$\Phi^{J_B^*}_{M_B K_B}(\theta_i, r_n) = \mathfrak{D}^{J_B^*}_{M_B K_B}(\theta_i)\, \phi^*_\Omega(\vec{r}_n), \qquad (3)$$

respectively. Here is

$$\mathfrak{D}^{J}_{MK} = \sqrt{\frac{2J+1}{8\pi^2}}\, D^{J}_{MK} \qquad (4)$$

the wave function of the symmetric top and describes the rotational levels of the nucleus with the angular momentum J and its projections on the nuclear symmetric and the laboratory axis, with the quantum numbers K and M , respectively. θ_i stands for the Euler angles between the body-fixed and space-fixed coordinates and

$$\phi^*_\Omega(\vec{r}_n) = \sum_{n\,\nu_n\,\Sigma_n} C^{\Omega}_{n\nu_n} R_{n\nu_n}(r)(\ell_n s_n\, \Omega-\Sigma_n\, \Sigma_n | j_n \Omega)\, Y^*_{\ell_n\, \Omega-\Sigma_n}(\omega)\, \chi^*_{s_n \Sigma_n} \qquad (5)$$

is the wave function of the captures neutron in the deformed nucleus[5], at which the angles ω are referred to the nuclear symmetric axis $(\nu_i \equiv \ell_i j_i)$. The whole interaction to which an incident particle is subject is described by an optical model potential $V(r, \theta, \phi)$. This potential is usually complex and includes the spin-orbit interaction and, if the incident particle is charged, the Coulomb interaction too. As to the radial dependence of the potential we assume that of the Saxon-Woods form. If the nucleus is axially deformed we can expand the potential in powers of $\sum_\lambda \beta_\lambda Y_{\lambda 0}$ and obtain

$$V(r,\theta,\phi) = u_0(r) + \sum_\lambda u_\lambda(r)\, Y_{\lambda 0}(\theta). \tag{6}$$

The appropriate partial wave expansion for a particle moving in such a potential is

$$\Psi^{(+)}_{\vec{k}}(\vec{r},\theta_1,s,\Sigma') = \sum_{\nu\nu'\mu\mu'} R^{\mu}_{\nu\nu'}(r)\, Y^{*}_{\ell'\mu'-\Sigma'}(\hat{k})\,(\ell' s \mu'-\Sigma'\Sigma'|j'\mu')\, D^{j'}_{\mu'\mu}(\theta_1)\, \mathcal{Y}_{j\ell s\mu}(\omega) \tag{7}$$

with

$$\mathcal{Y}_{j\ell s\mu}(\omega) = \sum_{\Sigma} (\ell s \mu-\Sigma\Sigma|j\mu)\, Y_{\ell\mu-\Sigma}(\omega)\, \chi_{s\Sigma} \tag{8}$$

and

$$R^{\mu}_{\nu\nu'}(r) = 4\pi i^{\ell} e^{i\sigma_\ell}\, \frac{\phi^{\mu}_{\nu\nu'}(r)}{2kr} \tag{9}$$

and $\Psi^{(+)}_{\vec{k}}$ is essentially the same as used in ref./4/. The radial functions $\phi^{\mu}_{\nu\nu'}(r)$ satisfy the following coupled channel equations/4,6/ ($\hat{\ell} \equiv \sqrt{2\ell+1}$)

$$\left[\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - u_0(r) + k^2\right] \phi^{\mu}_{\nu\nu'}(r) = \sum_{\lambda\geq 2,\nu_1} u_\lambda(r) * \tag{10}$$

$$* \frac{\hat{\ell}_1 \hat{\ell} \hat{j}_1 \hat{j}}{2\lambda+1} (-1)^{s+\mu} (\ell\ell_1 00|\lambda 0)(j j_1 -\mu\mu|\lambda 0) *$$

$$* W(j_1 j \ell_1 \ell; \lambda s)\, \phi^{\mu}_{\nu_1\nu'}(r)$$

with the appropriate asymptotic

$$\phi^{\mu}_{\nu\nu'} \approx (F_\ell + i\, G_\ell)\,\delta_{\nu\nu'} + (F_\ell - i\, G_\ell)\, S^{\mu}_{\nu\nu'} , \qquad (11)$$

where $S^{\mu}_{\nu\nu'}$ are the elements of the scattering matrix. The transition amplitude can now be written as

$$T^{\Sigma'_d \Sigma'_p}_{M_A M_B} = \frac{\hat{J}_A \hat{J}_B}{\sqrt{4\pi}} \sum_J \frac{(-1)^{K_A - M_A}}{2J+1} (J_A J_{B'}, -M_A M_B | JM)(J_A J_{B'} - K_A K_B | J\Omega) *$$

$$* \sum_{n\nu_n\Sigma_n} C^{\Omega}_{n\nu_n} \sum_{\substack{\nu_d \nu'_d \mu_d \mu'_d \Sigma_d \\ \nu_p \nu'_p \mu_p \mu'_p \Sigma_p}} (-1)^{s_p - \mu'_p} \frac{\hat{\ell}_d \hat{s}_d}{\hat{\ell}_n \hat{s}_n} \hat{\ell}_p (\ell_d \ell_p 00 | \ell_n 0) *$$

$$* (\ell'_d s_d \mu'_d - \Sigma'_d \Sigma'_d | j'_d \mu'_d)(\ell'_p s_p \mu'_p - \Sigma'_p \Sigma'_p | j'_p \mu'_p) *$$

$$* (j'_d j'_p \mu'_d, -\mu'_p | JM)(j'_d j'_p \mu_d, -\mu_p | J\Omega) * \qquad (12)$$

$$* (\ell_d s_d \mu_d - \Sigma_d \Sigma_d | j_d \mu_d)(\ell_p s_p \mu_p - \Sigma_p \Sigma_p | j_p \mu_p) *$$

$$* (\ell_d \ell_p \mu_d - \Sigma_d, -\mu_p + \Sigma_p | \ell_n \Omega - \Sigma_n)(s_d s_p \Sigma_d - \Sigma_p | s_n \Sigma_n) *$$

$$* (\ell_n s_n \Omega - \Sigma_n \Sigma_n | j_n \Omega)\, I_R\, Y^{*}_{\ell'_d \mu'_d - \Sigma'_d}(\hat{k}_d)\, Y_{\ell'_p \mu'_p - \Sigma'_p}(\hat{k}_p)$$

in which the subscripts p and d stand for proton and deuteron, respectively, and

$$I_R = \frac{(2\pi)^2}{k_p k_d}\, i^{\ell'_d - \ell'_p}\, e^{i(\sigma_{\ell'_d} + \sigma_{\ell'_p})} \int dr\, \phi^{\mu_p}_{\nu_p \nu'_p}(r)\, R_{n\nu_n}(r)\, \phi^{\mu_d}_{\nu_d \nu'_d}(r) \qquad (13)$$

is the overlap integral of the radial functions of the relative motion of the proton and deuteron with the bound state radial wave function. After some standard Racah algebra/7/ eq. (12) becomes

$$T^{\Sigma'_d \Sigma'_p}_{M_A M_B} = \frac{\hat{J}_A \hat{J}_B}{\sqrt{4\pi}} \sum_J \frac{(-1)^{K_A - M_A}}{2J+1} (J_A J_B, -M_A M_B | JM)(J_A J_B, -K_A K_B | J\Omega) *$$

$$* \sum_{n \nu_n \Sigma_n} C^{\Omega}_{n \nu_n} \sum_{\substack{\nu_d \nu'_d \mu_d \mu'_d \\ \nu_p \nu'_p \mu_p \mu'_p}} (-1)^{s_p - \mu'_p} (-1)^{\ell'_p + \ell_p + 2s_p - j'_p - j_p} \hat{\ell}_d \hat{j}_d \hat{\ell}_p \hat{j}_p \hat{s}_d *$$

$$* (\ell_d \ell_p\ 00 | \ell_n 0)(\ell'_d s_d \mu_d - \Sigma'_d \Sigma'_d | j'_d \mu'_d)(\ell'_p s_p, -\mu'_p + \Sigma'_p, -\Sigma'_p | j'_p, -\mu'_p) * \tag{14}$$

$$* (j'_d j'_p \mu'_d, -\mu'_p | JM)(j'_d j'_p \mu_d, -\mu_p | J\Omega) *$$

$$* (j_d j_p \mu_d - \mu_p | j_n \Omega) X(\ell_d s_d j_d; \ell_p s_p j_p; \ell_n s_n j_n) *$$

$$* I_R Y^*_{\ell'_d \mu'_d - \Sigma'_d}(\hat{k}_d) Y_{\ell'_p \mu'_p - \Sigma'_p}(\hat{k}_p).$$

The angular momentum transfer J is given by $\vec{J} = \vec{J}_A + \vec{J}_B$ and its projection Ω on the nuclear symmetric axis by $\Omega = K_A - K_B$. It should be noted that in general the angular momentum transfer J does not coincide with the nonconserved momentum j_n of the neutron captured on the Nilsson orbit $(\vec{j}_n = \vec{\ell}_n + \vec{s}_n)$.

For further investigations it is convenient to define the amplitude $\bar{T}^{\Sigma'_d \Sigma'_p}_J$ by means of

$$T^{\Sigma'_d \Sigma'_p}_{M_A M_B} = \sum_J (-1)^{K_A - K_A} (J_A J_B, -M_A M_B | JM)(J_A J_B, -K_A K_B | J\Omega) \bar{T}^{\Sigma'_d \Sigma'_p}_J \tag{15}$$

which does not depend on the projections M_A, M_B, K_A, K_B.

The resultant expression for the differential cross section and the polarization are related to the stripping amplitude by

$$\sigma(\theta) = c \sum_{M_A M_B \Sigma'_d \Sigma'_p} | T^{\Sigma'_d \Sigma'_p}_{M_A M_B} |^2 = c \sum_J (J_A J_B - K_A K_B | J\Omega)^2 \sum_{\Sigma'_d \Sigma'_p} | \bar{T}^{\Sigma'_d \Sigma'_p}_J |^2 \tag{16}$$

and

$$P = \frac{c\sum_{\bar{J}}(J_A J_B - K_A K_B | J\,\Omega)^2 \sum_{\Sigma'_d} [\,|\bar{T}_J^{-\Sigma'_d \frac{1}{2}}|^2 - |\bar{T}_J^{-\Sigma'_d - \frac{1}{2}}|^2\,]}{\sigma(\theta)}, \qquad (17)$$

where

$$C = [(2J_A+1)(2s_d+1)]^{-1} \frac{\bar{\mu}_d \bar{\mu}_p}{(2\pi h^2)^2} \frac{k_p}{k_d} A^2 \qquad (18)$$

with the reduced masses $\bar{\mu}_d$ and $\bar{\mu}_p$ and the strength A of the zero range force[9].

Now we are going over to the special case when the spin-orbit interaction in the optical potential for the deuteron and proton is omitted[1,2]. For this reason we write once more the wave function (7) for the initial channel:

$$\Psi_{\vec{k}}^{(+)} = \sum_{\nu\nu'\mu\mu'\Sigma} R^{\mu}_{\ell\ell'} Y^{*}_{\ell'\mu'-\Sigma'}(\hat{k}) D^{j'}_{\mu'\mu}(\theta)(\ell' s \mu'-\Sigma'\Sigma'|j'\mu')(\ell s \mu-\Sigma\Sigma|j\mu) Y_{\ell\mu-\Sigma}(\omega)\chi_{s\Sigma} \qquad (19)$$

and by means of

$$D^{j'}_{\mu'\mu}(\theta_1) = \sum_{\Sigma_1\Sigma_2} (\ell' s \mu'-\Sigma_1\Sigma_1|j'\mu')(\ell' s \mu-\Sigma_2\Sigma_2|j'\mu) D^{\ell'}_{\mu'-\Sigma_1\,\mu-\Sigma_2}(\theta_1) D^{s}_{\Sigma_1\Sigma_2}(\theta_1)$$

may be derived

$$\Psi_{\vec{k}}^{(+)} = \sum_{\substack{\ell\ell' jj'\mu\mu' \\ \Sigma\Sigma_1\Sigma_2}} R^{\mu}_{\ell\ell'} Y^{*}_{\ell'\mu'-\Sigma'}(\hat{k}) D^{\ell'}_{\mu'-\Sigma_1\mu-\Sigma_2}(\theta_1)(\ell' s\mu'-\Sigma'\Sigma'|j'\mu')(\ell' s\mu'-\Sigma_1\Sigma_1|j'\mu')\,* \qquad (21)$$

$$*\,(\ell' s\mu-\Sigma_2\Sigma_2|j'\mu)(\ell s\mu-\Sigma\Sigma|j\mu) Y_{\ell\mu-\Sigma}(\omega) D_{\Sigma_1\Sigma_2}\chi_{s\Sigma}\,. \qquad (22)$$

We note that this wave function does not agree with that used in[1], because we have proceeded from the wave function (7) represented by a superposition of different spin states. With regard to this we must set the primes in the last Clebsch-Gordon coefficient. Performing the sums over j and j' we obtain the resultant expression for the wave function ($m \equiv \mu - \Sigma$)

$$\Psi_{\vec{k}}^{(+)} = \sum_{\ell\ell' mm'} R^{m}_{\ell\ell'} Y^{*}_{\ell' m'}(\hat{k}) D^{\ell'}_{m'm}(\theta_1) Y_{\ell m}(\omega) \sum_{\Sigma} D^{s}_{\Sigma'\Sigma}\chi_{s\Sigma}\,, \qquad (23)$$

which is now in agreement with that used in ref.[1] and with the common practice. With respect to that which was mentioned above and using

$$(\ell_1 s_1 m_1 \Sigma_1 | j_1 \mu_1)(\ell_2 s_2 m_2 \Sigma_2 | j_2 \mu_2)(j_1 j_2 \mu_1 \mu_2 | JM) = \sum_{LS} \hat{L}\hat{S}\hat{j}_1\hat{j}_2 \; *$$

$$* (\ell_1 \ell_2 m_1 m_2 | Lm)(s_1 s_2 \Sigma_1 \Sigma_2 | S\Sigma)(L S m \Sigma | JM) X(\ell_1 s_1 j_1; \ell_2 s_2 j_2; LSJ) \qquad (24)$$

and the orthogonality relation for the X -coefficients we obtain from (12) the same expression for the cross section

$$\sigma(\theta) = [(2J_A+1)(2s_d+1)]^{-1} \sum_{M_A M_B \Sigma'_d \Sigma'_p} |T^{\Sigma'_d \Sigma'_p}_{M_A M_B}|^2$$

$$T^{\Sigma'_d \Sigma'_p}_{M_A M_B} = \frac{\hat{J}_A \hat{J}_B}{\sqrt{4\pi}} \sum_{LJ} \frac{(-1)^{K_A - M_A}}{2J+1} (J_A J_B, -K_A K_B | J\Omega)(J_A J_B, -M_A M_B | JM) \, *$$

$$* \sum_{n \nu_n \Sigma_n} c^{\Omega}_{n \nu_n} \sum_{\substack{\ell_d \ell'_d m_d m'_d \\ \ell_p \ell'_p m_p m'_p}} (-1)^{s_p - m'_p + \Sigma'_p} \frac{\hat{\ell}_d \hat{s}_d}{\hat{\ell}_n \hat{s}_n} \hat{\ell}_p (\ell_d \ell_p 00 | \ell_n 0)(\ell_d \ell_p m_d, -m_p | \ell_n \Omega - \Sigma_n) \, *$$

$$* (s_d s_p \Sigma'_d - \Sigma'_p | s_n \Sigma_n)(\ell_n s_n \Omega - \Sigma_n \Sigma_n | j_n \Omega)(\ell'_d \ell'_p m'_d - m'_p | LM - \Sigma_n)(\ell'_d \ell'_p m_d, -m_p | L\Omega - \Sigma_n) \, *$$

$$* (L s_n M - \Sigma_n \Sigma_n | J M)(L s \; \Omega - \Sigma_n \Sigma_n | J\Omega) Y^*_{\ell'_d m'_d}(\hat{k}_d) Y_{\ell'_p m'_p}(\hat{k}_p) I_R \begin{pmatrix} \ell_d \ell'_d m_d \\ \ell_p \ell'_p m_p \end{pmatrix} \qquad (25)$$

given in ref.[1].

The well-known differential cross section derived by Satchler[8] follows from eq. (25) if central field distorted waves are used ($\ell_p = \ell'_p$, $\ell_d = \ell'_d$, $R^{m_p}_{\ell_p \ell'_p} = R_{\ell_p}$, $R^{m_d}_{\ell_d \ell'_d} = R_{\ell_d}$). In this case the cross section can be written using the equations

$$\sum_{m_p m_d} (\ell'_d \ell'_p m_d, -m_p | \ell_n \Omega - \Sigma_n)(\ell'_d \ell'_p m_d, -m_p | L \Omega - \Sigma_n) = \delta_{\ell_n L}$$

$$\sum_{\Sigma_n} (\ell_n s_n \Omega - \Sigma_n \Sigma_n | j_n \Omega)(\ell_n s_n \Omega - \Sigma_n \Sigma_n | J\Omega) = \delta_{j_n J} \qquad (25)$$

as

$$\sigma(\theta) = [(2J_A+1)(2s_d+1)]^{-1} \sum_{n\, j_n \ell_n \Sigma_n} (c^{\Omega}_{n\nu_n})^2 (J_A J_B - K_A K_B | j_n \Omega)^2 *$$

$$* \left| \frac{\hat{J}_A \hat{J}_B}{\sqrt{4\pi}} \sum_{\ell_d' m_d' \ell_p' m_p'} (-1)^{m_p'} \frac{\hat{\ell}_d' \hat{s}_d}{\hat{\ell}_n \hat{s}_n} \frac{\hat{\ell}_p'}{\hat{j}_n^2} (\ell_d' \ell_p' 00 | \ell_n 0) * \right.$$

$$* (\ell_d' \ell_p' m_d', -m_p' | \ell_n m_d' - m_p')(\ell_n s_n m_d' - m_p' \Sigma_n | j_n m_d' - m_p' + \Sigma_n) * \tag{26}$$

$$\left. * Y^*_{\ell_d' m_d'}(\hat{k}_d) Y_{\ell_p' m_p'}(\hat{k}_p) I_R(\ell_p' \ell_d') \right|^2$$

In this expression for the cross section the angular momentum transfer J coincides with that of the Nilsson orbit j_n .

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