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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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THE ENERGY  
OF NONAXIAL DEFORMATION  
OF HEAVY NUCLEI

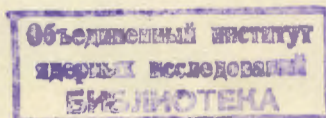
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Effects of the shell structure in the spectrum of single-particle excitations are important in the study of nuclear masses and nuclear fission. The deviation of the nuclear total energy from the values given by the liquid drop model, the so called shell correction, may be calculated by the Strutinsky method<sup>/1/</sup>. The shell correction to the liquid drop energy is associated with the difference between the level density of a single-particle spectrum and the density of the corresponding uniform distribution of levels.

The calculations of the shell correction performed by the Strutinsky method<sup>/2-5/</sup> show that in a number of nuclei of the transuranium region the total energy of nuclei as a function of the quadrupole deformation has two minima. The first one corresponds to the equilibrium nuclear deformation. The existence and the position of the second one was predicted in the Strutinsky's theory and recently received most immediate support from the sub-threshold fission experiments<sup>/6/</sup>.

The assumption of the existence of the second minimum in the deformation energy permits a likely explanation of the nature of spontaneously fissioning isomers discovered in the Laboratory of Nuclear Reactions of JINR (ref.<sup>/7/</sup>). Other experimental indications to the existence of the second minimum were discussed in the report by V.M.Strutinsky and S.Bjørnholm<sup>/8/</sup>.

Since spontaneously fissioning isomers are interpreted as nuclear states in the second well, it is essential to investigate the stability of a nucleus in the second well against a number of other deformations of the nuclear shape, in particular, against a nonaxial deformation. The present paper is devoted to the study of the dependence of the deformation energy of transuranium nuclei on nuclear shape, which is assumed to be approximated by a triaxial ellipsoid. The scheme of single-particle states used and the calculation method are described in the first and second sections. The third section presents the main results.

### 1. Single-Particle Levels

The levels of the single-particle spectrum in a nonaxial Newton potential<sup>/9/</sup> were used with the correction<sup>/10/</sup>

$$-D \langle \vec{l}^2 \rangle, \quad \langle \vec{l}^2 \rangle = N(N+3)/2$$

and parameters

$$\kappa = 0.0577, \quad \mu = 0.65$$

for protons and

$$\kappa = 0.0635, \quad \mu = 0.325$$

for neutrons.

All the levels with  $N \leq 8$  for protons and  $N \leq 9$  for neutrons were taken into account.

At large deformations ( $\beta > 0.5$ ) the inclusion of the shell  $N=9$  for protons and  $N=10$  for neutrons introduces some changes in the value of the deformation energy. However, as was shown by the tentative calculations with  $N=9$  for protons, this correction smoothly increasing with  $\beta$  does not exceed 1 MeV and is weakly  $\gamma$ -dependent. Consequently, it will not influence much the dependence of the deformation energy on  $\gamma$ .

### 2. Method of Calculating the Deformation Energy

The deformation energy equal to the sum of the liquid drop energy, shell correction and pairing energy was calculated by the Strutinsky method<sup>/11/</sup>. In fact, the routine for the computer made by Strutinsky was used. Only minor changes were introduced. Firstly, the dependence of the liquid drop energy on the nonaxial deformation was taken into account. Secondly, in the calculation of the pairing energy the equations for the gap and chemical potential were solved not in an approximate way, but exactly, which, however, only scarcely affects the results, when  $\beta > 0.1$ . The pairing interaction constants for protons and neutrons were calculated, following Strutinsky, starting from a gap in the corresponding uniform level distribution equal to 0.6 MeV. The calculation with a usual choice of constants<sup>/12/</sup>

$$G_p = 33/A, \quad G_n = 28/A$$

was made for  $^{288}\text{U}$ . The results are very much alike (compare the contour maps of the deformation energy for  $^{288}\text{U}$  in Figs. 1 and 4).

### 3. Discussion

Figs. 1-4 present the calculated deformation energy for various nuclei as topographic maps at  $0.0 \leq \beta \leq 0.9$  and  $0^\circ \leq \gamma \leq 60^\circ$ . The contours (solid lines) are drawn for the integral values of the deformation energy (in units of MeV) and sometimes to specify the relief for the half-integral values (dashed lines). The deformation energy at  $\beta = 0.0$  is taken to be equal to zero. The nuclei are indicated in figures.

Stability to  $\gamma$ -deformation is apparent for both the first and second minima, when the latter is present. Stiffness of  $\gamma$ -vibrations in the first minimum, which is small in the Th isotopes, increases with atomic number. The energy increment  $\Delta E$  for  $\Delta\gamma = 10^\circ$  grows with  $A$  from  $\approx 0.1$  MeV to  $\approx 0.7$  MeV in the nuclei inves-

tigated. Stiffness of oscillation in the second minimum appears to be larger as compared with the stiffness in the first one. The energy increment  $\Delta E$  for  $\Delta\gamma = 5^\circ$  in the Th and Pu isotopes amounts to about 1-2 MeV.

It is interesting to note that the saddle point between the first and second minima, which for the first four nuclei occurs at the axial-symmetric deformation, shifts more and more into the region of the nonaxial deformation ( $\gamma = 5^\circ$ ). The energy of the saddle is lower than the maximum energy between the two minima at  $\gamma = 0^\circ$  by 0.4, 0.4, 1.2, 1.7, 2.0 and 2.1 MeV for  $^{240}\text{Pu}$ ,  $^{242}\text{Pu}$ ,  $^{244}\text{Cm}$ ,  $^{250}\text{Cf}$ ,  $^{254}\text{Fm}$  and  $^{256}\text{Fm}$ , respectively.

The calculations performed suggest that it is important to take nonaxial deformation into account, when treating nuclear fission. On the other hand, a conclusion may be drawn that only the region of small deformations ( $\gamma \approx 5-10^\circ$ ) is important and, consequently, it is not improbable that the required calculations of single-particle levels in a more realistic potential may be performed in the framework of the perturbation theory.

The author is grateful to Professor V.M.Strutinsky for numerous illuminating discussions and furnishing the computer program for the calculation of the shell correction and also to Dr. Yu.A. Muzychka for useful discussion and to Professor V.G.Soloviev for the continuous interest in this work.

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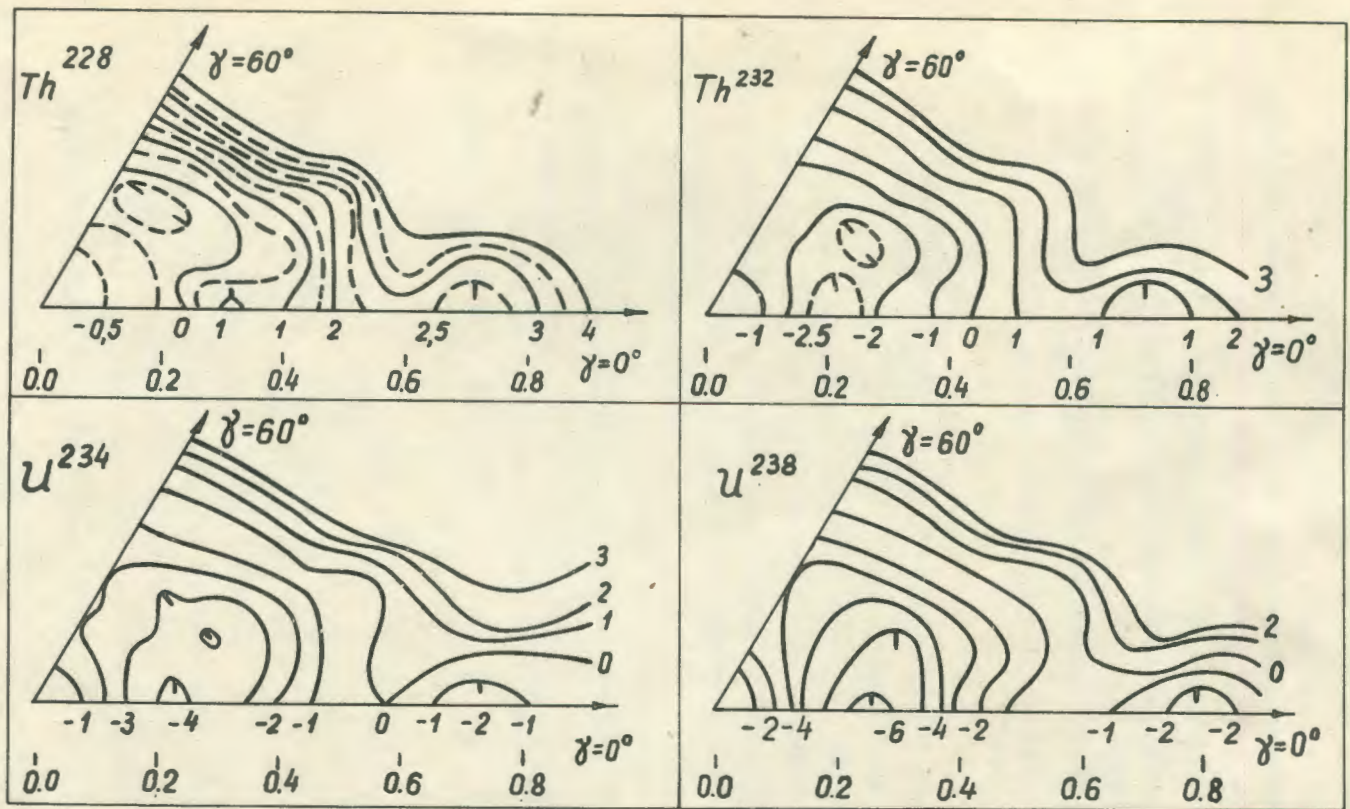


Fig.1. Contour maps of the deformation energy for  $^{228}\text{Th}$ ,  $^{232}\text{Th}$ ,  $^{234}\text{U}$ ,  $^{238}\text{U}$  in the range  $0^\circ \leq \gamma \leq 60^\circ$ ,  $0.0 \leq \beta \leq 0.9$ . The values of the constant along the contours deformation energy are given in units of MeV. The nuclei are indicated in the figure.

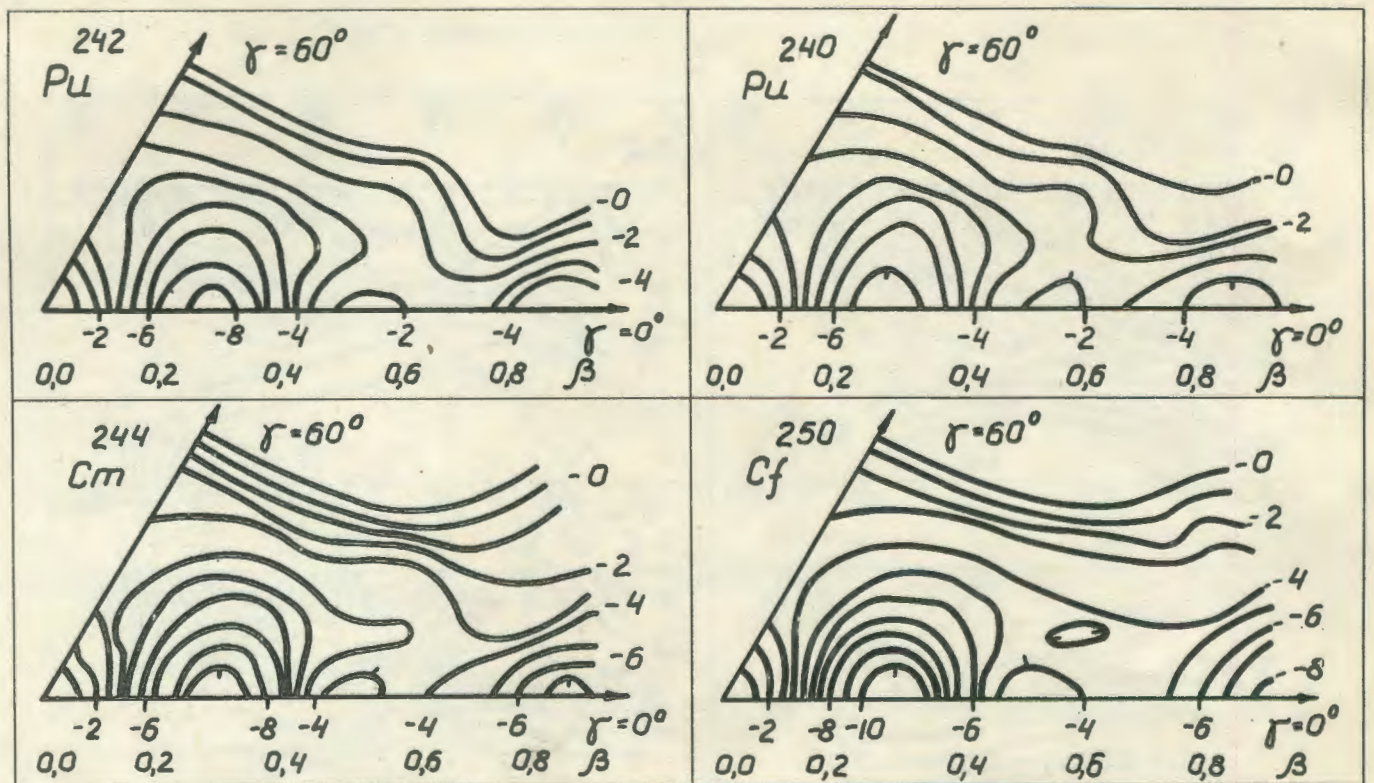


Fig.2. Same as fig.1 for  $^{242}\text{Pu}$ ,  $^{240}\text{Pu}$ ,  $^{244}\text{Cm}$ ,  $^{250}\text{Cf}$ ,

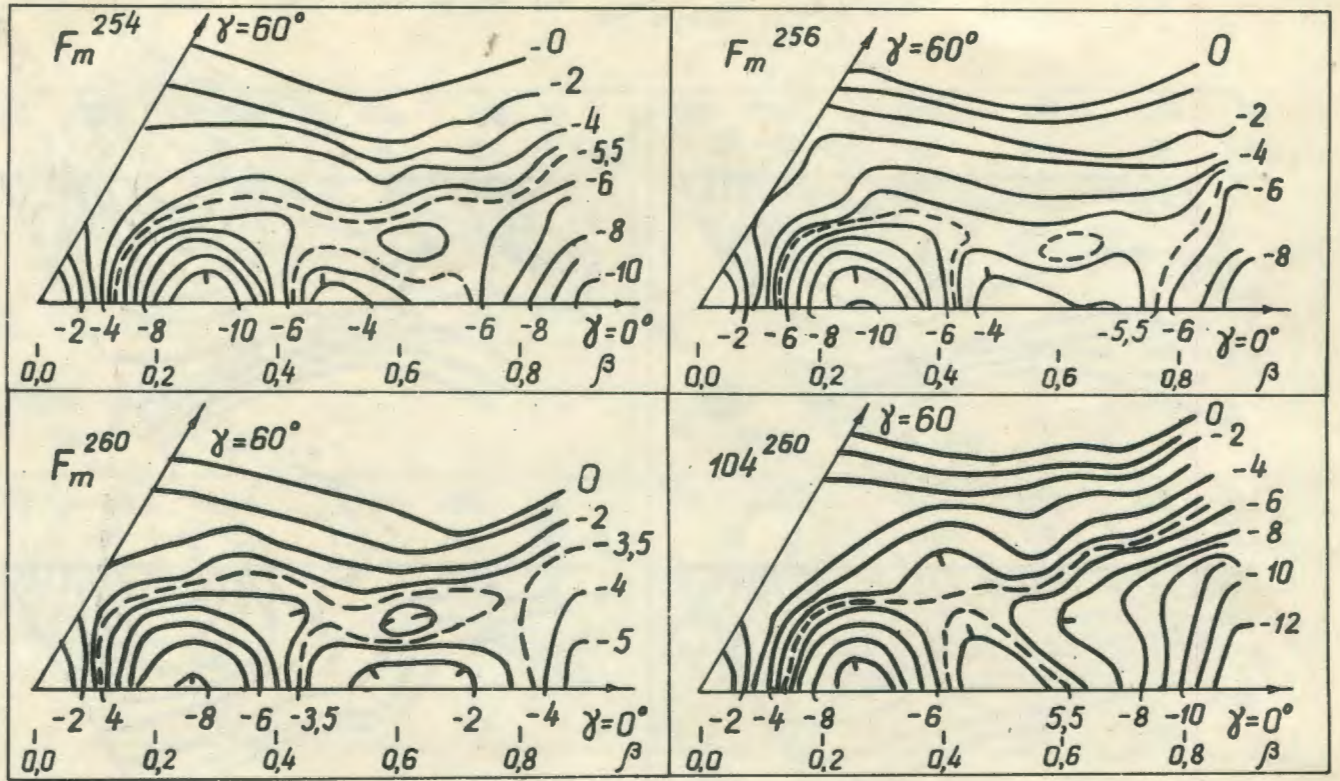


Fig.3. Same as fig.1 for  $^{254}\text{Fm}$ ,  $^{256}\text{Fm}$ ,  $^{260}\text{Fm}$ ,  $^{260}\text{104}$ .

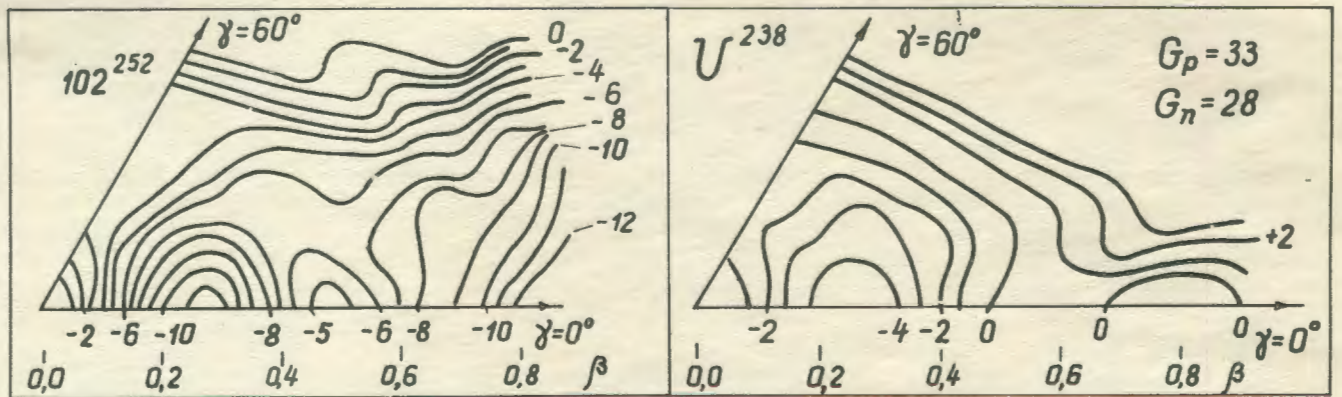


Fig.4. Same as fig.1 for  $^{102}\text{252}$ ,  $^{238}\text{U}$ .  
For  $^{238}\text{U}$  the calculations were made using the constants indicated in the figure (see also the text).